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Comparison of Prospective Mathematics Teachers' Problem Posing Abilities in Paper-Pencil Test and on Dynamic Geometry Environment in Terms of Creativity

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Abstract

This study aims to investigate the similarities and differences between prospective mathematics teachers' creative thinking skills in paper-pencil test and on a Geogebra-supported environment in terms of problem-posing. This case study used purposive sampling method for determining the participants. Findings revealed that the activities carried out in the GeoGebra-supported environment were insufficient to produce creative problems, and GeoGebra's main utility to prospective teachers was in identifying their mistakes related to mathematical concepts and discrepancies among numerical values of the problems posed. The reasons for the low achievement in posing problem were discussed: These were; (i) lack of problem-posing experience, (ii) the structure of problem-posing activity, and (iii) prospective teachers' mathematical content knowledge.

Keywords: Problem-posing, creativity, geometry, GeoGebra

Comparación de las Habilidades para Plantear Problemas de los Futuros Maestros de Matemáticas Usando Papel y Lápiz en un Entorno de Geometría Dinámica en Términos de Creatividad

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Resumen

Este estudio tiene como objetivo investigar las similitudes y diferencias entre las habilidades de pensamiento creativo de los futuros maestros de matemáticas usando lápiz y papel en un entorno apoyado por Geogebra en términos de planteamiento de problemas. Este estudio de caso utilizó un método de muestreo intencional para seleccionar a los participantes. Los resultados revelan que las actividades llevadas a cabo en el entorno de GeoGebra eran insuficientes para producir problemas creativos, y la principal utilidad de GeoGebra para los futuros maestros fue identificar sus errores relacionados con los conceptos matemáticos y las discrepancias entre los valores numéricos de los problemas planteados. Se discuten las razones del bajo rendimiento en plantear problemas: (i) falta de experiencia en la presentación de problemas, (ii) la estructura de la actividad de presentación de problemas y (iii) el conocimiento del contenido matemático de los futuros maestros.

Palabras clave: Planteamiento de problemas, creatividad, geometría, GeoGebra

In recent years, there has been an increasing interest in problem-posing in mathematics education studies. Various researchers (e.g., Lavy, 2015; Singer, Ellerton & Cai, 2013) have discussed the importance of integrating problem-posing into mathematics education courses and indicated that students benefit from this type of activity. Kilpatrick (1987) emphasized the importance of problem-posing in stating that “problem formulating should be viewed not only as a goal of instruction but also as a means of instruction” (p. 123). The National Council of Teachers of Mathematics (NCTM) (2000) asked students to “analyze situations carefully in mathematical terms and to pose problems based on situations they see” (p. 53) and suggested that teachers “ask students to formulate interesting problems based on a wide variety of situations, both within and outside mathematics” (p. 257).

One of the important reasons for interest in problem-posing is its relation to creativity (Leung, 1997; Silver, 1997; Singer & Voica, 2017). Since “creating a problem” is a characteristic of problem-posing and “bring[ing] into being” is seen as the nature of creativity, one might see problem-posing as a kind of creativity skill (Leung, 1997). Singer, Pelczer, and Voica (2011) stated that students who can construct coherent and novel variables in problem-posing activities and understand their results by changing some of the parameters have demonstrated profound creativity approaches. The facts that problem-posing is considered an open-ended cognitive activity (Haylock, 1997; Pehkonen, 1995; Silver, 1997) and a crucial component of inquiry-based learning (Silver, 1997) enlighten why problem-posing is related to creative thinking skills. In addition, since the problems posed give important clues about the mathematical understanding of the participants who pose them (Xie & Masingila, 2017), problem-posing can be seen as an evaluation tool for determining students’ creative thinking skills (Harpen & Sriraman, 2013; Leung, 1997).

The use of dynamic geometry software (DGS) in mathematics education enables students to engage in in-depth cognitive tasks (Ranasinghe & Leisher, 2009). DGS provides opportunities to make generalizations and explore relations by constructing, manipulating, dragging, and re-shaping geometrical objects (Christou, Mousoulides, Pittalis, & Pitta-Pantazi, 2005; Lavy, 2015). In this context, DGS-supported learning activities may provide important opportunities for problem-posing as well as problem-solving. With respect to this, Leikin (2015) stated that DGS has a special place in the

mathematical problem-posing process and allows learners to discover mathematical facts while generating new problems.

Although the mathematics education field's interest in and research on problem-posing has been active, less focus has been placed on the study of the role of technology in facilitating and advancing skills in formulating problems (Abramovich & Cho, 2015). One of the main questions raised in studies involving DGS concerns what insights these kinds of teaching environments provide that traditional teaching environments fail to offer (Artigue, 2002; Christou et al., 2005; Lavy, 2015). Regarding this question specifically for problem-posing and creativity, how do DGS-supported environments promote the participants' creative thinking skills when compared to paper-pencil tests (PPT)? This study aims to answer this question by embracing qualitative approaches. More specifically, this study aims to investigate the similarities and differences between prospective mathematics teachers' creative thinking skills in a PPT environment and a GeoGebra-supported environment (GSE) in terms of problem-posing.

Theoretical Framework

Creativity and Problem-Posing

The concept of creativity has received increasing attention in mathematics education research. The necessity of improving students' creative thinking is a recent conception widely accepted in different instructional documents and curricula (e.g., Ministry of National Education [MoNE], 2018; NCTM, 2000). Despite the indicated importance, there is no single accepted definition for creativity due to its complexity and versatility (Ayllon, Gomez & Balleste-Claver, 2016; Treffinger, Young, Selby & Shepardson, 2002). In essence, researchers' definitions and explanations regarding creativity showed that its most apparent and agreed-upon feature is that producing something new. From the broader perspective, creativity was defined as "the ability to make or otherwise bring into existence something new, whether a new solution to a problem, a new method or device, or a new artistic object or form" (Kerr, 2016).

During the nineteenth and the beginning of the twentieth centuries, creativity, with a narrow understanding, was seen as the common characteristic of people who had revolutionized their fields. Guilford (1950)

indicated that some behaviors, including inventing, designing, composing, and planning, are recognized as the evident characteristics of creative people. This genius view of creativity fostered the ideas that creativity was not affected by instruction and was thought to be “occasional bursts of insight” (p. 75) within the person (Silver, 1997). However, over time researchers began to question this approach. Guilford (1950) indicated that creativity and IQ level do not overlap, so someone who does not have the expected intelligence can be creative. Similarly, Sriraman (2005) stated that although there are people described as mathematically talented, this does not mean that some others are not mathematically creative. In this context, contemporary approaches define creativity as a creative thinking and behaving tendency (Leung, 1997; Silver, 1997). This view of creativity provides a much stronger foundation for building educational applications (Silver, 1997).

This perspective of creativity is characterized by divergent thinking. Convergent thinking is related to the creation of a clearly defined, single correct answer, while divergent thinking includes situations such as producing multiple and alternative responses based on existing data, creating unexpected combinations, establishing connections between distant situations, converting information into unexpected forms, and so on (Cropley, 2006). There are qualitative differences between convergent thinking and divergent thinking: Divergent thinking involves the production of variability, while convergent thinking involves the production of singularity (Cropley, 1999).

Creativity that is seen as synonymous with divergent thinking (Cropley, 2006) is the ability to generate information or ideas from given information or ideas, where the emphasis is on the quantity and quality of output (Balka, 1974). Three factors, which are fluency, flexibility and originality, are used in determining creative thinking skills (Guilford, 1950; Torrance, 1988), and these factors are all seen as aspects of divergent thinking (Cropley, 2006). Fluency is represented by the total number of relevant responses made by the child, and flexibility is represented by the total number of different ideas. Lastly, originality is associated with the uniqueness of the ideas (Balka, 1974). The activities regarding identifying and developing students' creative thinking skills should be designed in such a way as to allow them to examine these three factors.

In addition to problem-solving, problem-posing is also widely used in determining and developing students' creative thinking skills (Balka, 1974;

Harpen & Sriraman, 2013; Haylock, 1997). Singer et al. (2011) stated that students who can construct coherent and novel variables in problem-posing activities and understand their results by changing some of the parameters have demonstrated profound creativity approaches. Silver (1997) indicated that inquiry-based mathematics instruction covers problem-solving and problem-posing activities, and such activities can develop students' creative thinking skills.

The investigations in the studies combining creativity and problem-posing followed different approaches. Some studies classify the problem-posing abilities of the students who were grouped according to their abilities/creativity. For example, talented and less-talented students' problem-posing abilities were compared in Ellerton's (1986) study. It was found in this study that more talented students posed more complex problems and used mathematical language more effectively and fluently. Other studies combining creativity and problem-posing directly focused on the structure of the problems posed. In such research (e.g., Harpen & Sriraman, 2013; Kontorovich, Koichu, Leikin, & Berman, 2011; Yuan & Sriraman, 2010), problems are investigated according to components of creativity, which are originality, flexibility, and fluency. Balka (1974) offered an open-ended situation by providing numerical data to students (for example, the expenditure of an American family of four) and expecting them to pose different problems that could be solved using these data. Haylock (1997), similarly, used one sample of a problem-posing activity in explaining creative thinking skills (for example, posing different problems whose answer is 4). In such problem-posing studies, the total number of problems posed was considered in the fluency category while the total number of problems with different mathematical structures was linked with the flexibility category. According to Balka (1974), a high fluency score does not obviously indicate high creative ability; however, a high fluency score accompanied by a high flexibility score may give a better indication of creative ability in mathematics. Lastly, the frequency of problems observed is considered for the originality category. Problems posed by less than 10% of the participants are embraced as original (Harpen & Sriraman, 2013; Yuan & Sriraman, 2010).

Studies investigating creativity mostly use semi-structured problem-posing activities in the classification proposed by Stoyanova and Ellerton (1996). Bonotto and Santo (2015) stated that semi-structured activities invite

students to stimulate creative thinking. In the semi-structured problem-posing activities, students dealt with the open-ended activities provided and were asked to use their knowledge and experience to pose appropriate problems for these activities. For example, the semi-structured problem-posing activity shown in the Figure 1, adapted from Stoyanova's study (1997), was used in the investigation of creativity in the current study. In this activity, a triangle and a circle inscribing this triangle were given together, and participants were asked to generate as many different and difficult problems as they could. Harpen and Sriraman (2013) investigated the creativity abilities of high school students from America, Shanghai, and Jiaozhou through problem-posing and used the activity shown in Figure 1. This study showed that students' fluency and flexibility scores varied from 2 to 4.9 and from 1.6 to 4.1 respectively. In addition, it was found that students from America, Shanghai, and Jiaozhou posed original problems in 8, 6, and 10 different structures, respectively. In another study comparing Chinese and American students' problem-posing abilities, Yuan and Sriraman (2010) also used the same activity in Figure 1. The researchers indicated that area- and length-type problems were more prevalent among students and that Chinese students were more likely than American students to generate problems by adding auxiliary lines (12% and 2.8% respectively).

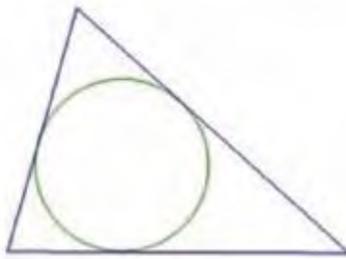


Figure 1. The problem-posing activity adapted from Stoyanova's (1997) study

Technology-Supported Learning Environment and Problem-Posing

The use of technological tools in mathematics education is strongly supported in instructional documents and national and international curricula. According to NCTM (2000), technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and

enhances students' learning. Similarly, the conducted studies emphasized the positive influence of technological tools on discovering mathematical concepts (Liang & Sedig, 2010) and interpreting them (Thompson, Byerley & Hatfield, 2013).

Regarding the positive aspects of technological tools used (e.g., Artigue, 2002; Drijvers & Gravemeijer, 2004; Leung, 2008; Liang & Sedig, 2010), one particular theory gains importance in mathematics classrooms, which is “instrumental theory” (Drijvers, Kieran, & Mariotti, 2010; Verillon & Rabardel, 1995). Briefly, this theory indicates that students may develop different solution strategies due to dual relationship between the tool used and the individual who is using this technological tool, when integrating the technological tools, more specifically DGSs, into the solutions of the mathematical tasks. This theory is rooted from the distinction between the artifact and the instrument, explained by Verillon and Rabardel (1995). Basically, the artifact is the available when the task is presented. However, since individuals do not know how to utilize this artifact when achieving the task, it becomes meaningless for the user. Unlike the direct relation between subject (e.g., the user) and object (e.g., mathematical task), this theory “highlights the intermediary status of instruments (dynamic geometry environment in this case) and takes into account the multiple relationships which bind together the three elements (i.e. subject, object and instrument) constitutive of instrumented activity situations” (Verillon & Rabardel, 1995, p.85). Therefore, the user develops ways using the instrument to mediate the interaction between the user and mathematical task. During this process, the dual interrelation between the user’s cognitive construction and the limits of the intended artifact produces instrumental genesis and this artifact becomes meaningful and useful instrument for the mathematical task provided.

Considering the problem-posing activity on dynamic geometry environment (DGE), this dual relation better explains the possible variation of students’ responses. This is because each student may develop different perspectives to problem-posing activity when engaging it with DGE’s properties including constructing, dragging, measuring, manipulating tools (Christou et al., 2005; Lavy, 2015). For example, one student may drag one component of the shape provided and see the relationship among the other components. So, possible explorations on the shape may trigger student’s thinking and help to produce problem which is completely different than others’ problems. For DGE, one may consider it as whole as single artifact,

while someone else may consider each of its properties as different artefact such as measurement or dragging ones (Leung, 2008). Considering the case of problem-posing activity, students may pose various types of problems for the task provided by taking different aspects of the DGE into account.

One of the important software programs used in technology-supported mathematics lessons is GeoGebra. According to Carter and Ferrucci (2009), GeoGebra, as a dynamic construction tool, relieves prospective teachers (PST) of the limitations of paper-pencil constructions, especially in learning geometry. Hohenwarter and Fuchs (2004) stated that GeoGebra could be used to present demonstrations and visualizations in mathematics education, as a construction tool, and to discover mathematical concepts. In the use of GeoGebra as a construction tool, students can construct shapes based on their existing knowledge. Since the construction of the shapes must be appropriate to Euclidian-type constructions, students can notice their lack of knowledge and conceptual mistakes in this dynamic environment (Öçal, 2017).

Since DGS, such as GeoGebra, provides visualization and allows users to explore concepts while constructing related shapes, it may also have an effect on the quality of the problems posed. As a result of the literature review, it was observed that there were a limited number of studies combining problem-posing and technology. In some of these studies, the aim was for students to share problems with their friends through web-based environments and solve these problems (e.g., Beal & Cohen, 2012; Manuel & Freiman, 2017). Beal and Cohen (2012) provided a way for middle school students to write problems for their friends and solve problems written by their friends through a web-based content-authoring and sharing system. In this study, students were able to successfully pose problems, but problem-solving activities were more dominant among them.

Some other studies (e.g., Conteras, 2007; Leikin, 2015; Leikin & Grossman, 2013; Segal, Stupel, Sigler, & Jahangiril, 2018) investigated how the problems were posed by means of dynamic software such as GeoGebra, Geometry Sketchpad, and Geometry Investigator. In such studies, participants were provided with geometric proof activities, and how they transformed these activities into problems was investigated. Participants' responses were analyzed according to the type of transformations done and the problems posed. The given conditions or the purpose of the activity was changed according to the type of the transformation. Such an approach was not specific to geometry activities, but was emphasized in research on

problem-posing (e.g., [Stickles, 2006](#)). Regarding their types, the problems posed in the DGE were classified as investigation-oriented problems and non-investigation problems. [Leikin and Grossman \(2013\)](#) considered the problems requiring mathematical calculations (such as for angle, length, and area) by means of geometric proof activities as non-investigation problems. At the end, they asserted that these types of problems matched the related problems in the textbooks currently used in classrooms. Investigation-oriented problems were re-classified as verification and discovery types of problems. While it is a question of whether the proof required was correct in the verification-type problems, discovery-type problems require conjecturing, analyzing conjectures, and proving ([Leikin, 2015](#)). The task required introducing new relationships and assumptions by adding auxiliary constructions in generating discovery-type problems.

[Segal et al. \(2018\)](#) provided PSTs with a geometric proof and, based on this, expected them to generate new problems and verify these problems by using a “what if not” strategy and GeoGebra software. In this process, they found that problems were generated by adding new data to the original form, ignoring some features of the original shape/configuration, and discovering other possible features of the original configuration/shape. In addition, researchers determined that PSTs were trying to identify the relationships between the components by measuring the geometric angles and lengths, and thus try to generate new problems. [Conteras \(2007\)](#), on the other hand, stipulated that PSTs used a geometric problem in order to generate problems instead of using a geometric proof activity, and classified the problems they generated as follows: proof problems, converse problems, special problems, general problems, and extended problems. The researcher also emphasized that PSTs provided little room for proof and converse problems compared to other types of problems.

Other studies (e.g., [Christou et al., 2005](#); [Lavy, 2015](#)), however, investigated PSTs’ cognitive processes on DGE. [Lavy \(2015\)](#) classified the cognitive processes (filtering, editing, comprehending, and translating) PSTs applied while posing problems on DGE. In the filtering category, only one of the numerical values was changed (e.g., the given height is 12 cm instead of 10 cm), and the remaining data stayed constant in the problems. If the interval of a single value changed (e.g., the angle may take a value ranging from 67° to 90°), then the problem required a comprehending thinking structure in addition to a filtering one, because whether the values in this

range allowed the production of mathematically valid problems should be inquired in these kinds of problems. Some PSTs wrote problems for changing the structure of the geometric object (e.g., transforming the triangle pyramid into a square pyramid), and these problems require the use of filtering, editing, and comprehending cognitive processes together. In such cases, a different shape should be drawn, the relationships between the data in the new case should be understood, and it should be decided whether the proposed problem allows the production of a new and mathematically valid problem due to the changes in the sketch of the problem. Converting the problem into a proof problem requires the use of all such cognitive processes. Christou et al. (2005) also investigated how PSTs solve the problems on DGE (e.g., what is the figure formed by the angle bisectors of the interior angles of a parallelogram?) and how DGE allows them to produce new problems. PSTs used modeling, conjecturing, experimenting, and generalizing strategies in the problem solutions. At the same time, drawing and measuring tools gave the PSTs opportunities to engage in problem-posing by allowing them to make trials, generalize, specialize, and expand the problem through the modification of the data.

Method

Participants

The participants were composed of 15 PSTs who were in their third year of training in a university in Turkey. Purposively selected participants met the need of two criteria for the study. Firstly, they were expected to be knowledgeable about the use of GeoGebra before the intervention. Secondly, their willingness to participate in the study was sought. Each participant was assigned a pseudonym.

The Turkish education system is composed of five hierarchical periods including kindergarten, primary school (Grades 1–4), middle school (Grades 5–8), high school (Grades 9–12), and university. Standard curricula are applied to each of these periods in teaching mathematics. The application of the curricula does not change according to geographical regions or socio-economic situations. The middle school mathematics curriculum is used to teach mathematics to students of an age group ranging from 11 to 15 years old. There are five learning domains in the curriculum: numbers and

operations, algebra, geometry and measurement, data analysis, and probability. In all of these learning domains, it is recommended that problem-posing should be taught along with problem-solving (MoNE, 2018).

In this study, the participants were enrolled in the third year of the undergraduate teacher training department. They were part of a four-year training program to become mathematics teachers for middle school students, which involves courses on pedagogical knowledge and mathematical content knowledge. The trainees take various courses in the context of mathematical content knowledge, such as calculus, plane geometry, differential equations, analytic geometry, statistics, abstract mathematics, and algebra. They also attend some courses regarding pedagogical content knowledge, such as instructional principles and methods, measurement and evaluation, the use of technology in mathematics education, instructional material design, misconceptions in mathematics education, mathematics instructional methods, teaching mathematics, and school practices. One of the courses in mathematical content knowledge was plane geometry given during their first year of the certificate program. In this course, the content included the construction of plane geometry as well as analysis and proof of geometric shapes and structures. Within the scope of this course, PSTs received instruction on geometric concepts and properties (e.g., as in Figure 1). In their second year of the certificate program, all PSTs took a selective course related to the use of GeoGebra. During the course they were instructed to use the available tools of GeoGebra fluently. In addition, the necessities of geometric constructions in Euclidian geometry were also introduced to them through GeoGebra. PSTs took two courses in instructional methods for mathematics during their third year of the certificate program. These courses involved the basic elementary concepts and theoretical and practical activities to teach mathematical domains and sub-domains found in the national mathematics curriculum for middle schools (MoNE, 2018). Moreover, in these courses, the importance of problem-posing, its relation to problem solving and creativity, and problem-posing types were explained through examples. However, PSTs had not taken a special course on problem-posing activities which were specific to geometry.

Data Collection and Analysis

The process of this study was not conducted as a part of an ongoing course. This study was conducted with the third-year PSTs who voluntarily participated. An application calendar was created and data were collected according to this schedule. The application process of this study was carried out in two phases. In the first phase, the problem-posing activity in Figure 1 was presented on paper to each participant on different days, and they were asked to pose problems for their friends. In addition, it was emphasized that the problems posed would be evaluated considering the number of problems, differences in structures, and their difficulty levels. During the implementation of this activity, PSTs were not subjected to any time constraints, and the implementation times ranged from 26 to 52 minutes with a mean of 40 minutes.

In the second phase, PSTs were asked to construct a shape with its properties (properties of an inscribed circle) on GSE. They were free to analyze the shape on the GeoGebra screen. Then they were asked to pose different problems or make changes to existing ones previously posed in PPT. For the implementation of this phase, no time constraints were made and the activities were concluded in approximately 50 minutes. After these two phases, semi-structured interviews were conducted with each PST. PSTs were expected to explain their thoughts and reasoning about problem-posing processes, and how GeoGebra contributed to the process of posing problems and reorganizing the existing problems. The implementation process was conducted by the first author in two sessions on different days with each PST. PSTs' problem-posing activities on GSE and the interviews conducted were video-recorded. In addition, first author of this study noted his observations during the data collection processes.

In the data analysis process, the problems posed by PSTs in PPT and on GSE were first analyzed according to whether they were viable. This type of analysis was retrieved from the analysis schemas utilized in the related studies (e.g., Harpen & Sriraman, 2013; Kontorovich et al., 2011; Silver & Cai, 2005). In this analysis, the aim was to identify the PSTs' responses (problems posed) that could not be solved with the information provided by them. These problems were coded as non-viable (NV). Then, those in the viable category were analyzed in line with the categories of fluency, flexibility, and originality. The fluency score was calculated by assigning

one point for each problem in the viable category. The flexibility score was determined by counting the total number of problems of different structures in the viable category. Lastly, the uniqueness of the problems in the sample was taken into account to determine the originality score. As stated in Creativity and problem posing section, the types of problems posed by 10% or more of the PSTs were not accepted as original. In this context, since the total number of PSTs was 15, the problems posed by two or more PSTs were not assessed as original. Therefore, to find each participant's originality score, the original problem types were determined and each was considered as one point. The arithmetical mean and median values were utilized in the presentation of the PSTs' fluency, flexibility, and originality scores for the problems they posed in PPT and on GSE.

In addition, reorganizing the existing problems posed in the NV categories and how GeoGebra contributed to them were also analyzed based on the content analysis method. In this process, PSTs' responses, drawings, explanations, and the researcher's observations were subjected to data analysis. As a result of the content analysis, it was determined that how GeoGebra contributed to the PSTs' problem-posing. The detailed explanations are presented in findings section.

Findings

The Distribution of Problems Posed by Psts

The distribution of viable and non-viable problems posed by PSTs in PPT and on GSE are presented in Table 1.

As shown in Table 1, 15 PSTs posed 86 problems in the PPT. Of these, around two-thirds were coded as viable. The PSTs also posed 22 problems on GSE, and the majority of these (17 problems) were in the viable category. Ali, İsra, and Selami's problems, which were in the NV category, were as follows:

NV Problem 1: Let ABC be an equilateral triangle. Let $|AB|=5$ cm and the diameter of the inscribed circle be 4 cm. So what is the area of this triangle in cm^2 ? (Ali)

NV Problem 2: A circle is drawn inside a triangle and a tangent to each side. What is the name of this circle? (İsra)

NV Problem 3: [According to Figure 2] Let the points H, F, and E be tangent to the circle. What is the circumference of the circle in cm? (Selami)

Table 1.

Distribution of PSTs' problems posed on PPT and on GSE

Categories	Viable	Non-viable	Total
PPT	55	31	86
GSE	17	5	22
Total	72	36	108

In Ali's problem, the inscribed circle of an equilateral triangle with an edge of 5 cm could not be 4 cm in diameter. In the case of İsrâ's problem, however, the information already provided was reiterated in the problem-posing activity. According to Figure 2, $|AH|=4$ cm in the problem Selami posed. Based on this information, the length of $|AF|$ must also be 4 cm. Regarding the information given, the radius of this circle cannot be calculated. These types of responses were evaluated in the NV category.

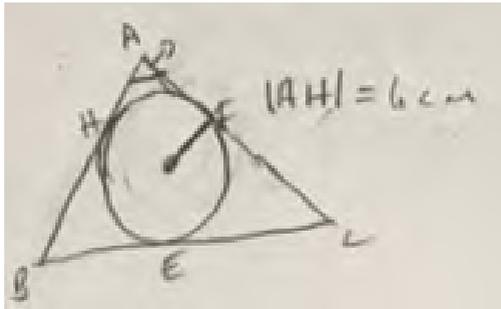


Figure 2. An example of a problem posed by Selami in the NV category

As also indicated in Table 1, there was an observable difference between the total number of problems posed in PPT and on GSE. When posing problems on GSE, PSTs were provided with the problems posed in PPT and expected to pose different problems than those posed in PPT. In this context,

PSTs attempted to pose different problems on GSE than those posed in PPT. This situation was the main reason for the fact that the number of problems posed on GSE was fewer than those in PPT. In addition, according to the researcher's observations during the problem-posing process on GSE, a majority of the PSTs (14) took some of the problems they posed in PPT as a starting point, developed these problems, and posed different ones. This approach contributed to their discovering the properties of geometric shapes during the problem-posing activity and enabled them to determine their errors in the existing problems they posed in PPT. Therefore, they revised for these errors in the existing problems and wrote them as new posed problems. This approach increased the number of problems in the viable category to be higher than those posed on GSE. Dilek's opinions regarding these issues were as follows:

I noticed that some of the knowledge I knew to be correct during the first application was incorrect after GeoGebra application. I used GeoGebra to resolve my mistakes. Then, I changed some problems by using the correct knowledge.

Derya's problem posed in PPT is shown in Figure 3. Derya thought that the center of the inscribed circle was the intersection of median lines. In the interview with Derya, she thought that since the median line divides the edges into equal parts, it also divides each angle of the triangle into two equal parts. In this case, she stated that the arc lengths would also be equal. In addition, there was no numerical value assigned in the problem posed. This problem was evaluated as NV because she incorrectly identified the center of inscribed circle, and there was no numerical value that could be assigned as a solution to the problem.

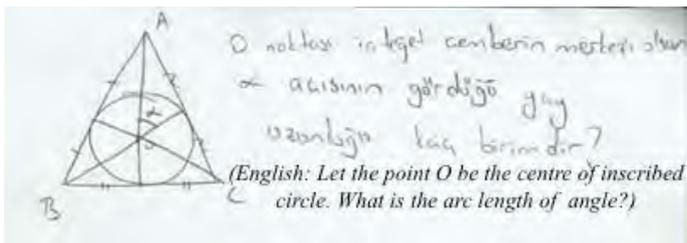


Figure 3. A problem posed by Derya on PPT

Derya chose this problem for starting point of problem-posing on GSE. Derya started her investigation by trying to construct an inscribed circle regarding this problem on the GeoGebra screen. She drew a triangle with the Polygon tool (in GeoGebra software) and continued to determine the midpoints of each edge of the triangle by means of the Midpoint or Center tools. (See points D, E and F in Figure 4.) Then she connected the determined midpoints with the opposite vertices of the triangle by means of the Segment tool. (As shown in Figure 4, the line segments join the points A with F, B with E, and C with D.) Derya determined the intersection point of line segments (\overline{AF}) , (\overline{BE}) and (\overline{CD}) by means of the Intersect tool. Therefore, Derya thought that this intersection point was the center of the inscribed circle and the tangent points were the intersection points of the median lines and the triangle's edges. She formed the shape in Figure 4 by joining the midpoints of the triangle's edges using the Circle through 3 points tool. According to this construction, Derya noticed her mistake regarding conceptual knowledge just after she realized that the center of the inscribed circle was not the intersection of median lines. Then Derya indicated that the center of the inscribed circle was the intersection of angle bisectors, and created the shape in the activity by constructing it for this situation. In addition, she constructed perpendicular lines passing through the central point and discovered that the intersection points of these lines perpendicular to triangle's edges were their tangent points. Derya reorganized the problem as follows: "the length of an ABC equilateral triangle is 4 cm. Let O be the center of the inscribed circle. What is the arc length of α angle?" (She made the same construction shown in Figure 3 and specified the same arc length.) A script of the interview with Derya is as follows:

Interviewer (I): Can you explain how you posed this problem?

Derya: I tried to pose it on the knowledge I already have. But some of the knowledge I have was erroneous. I thought of the intersection points of the inscribed circle with the triangle as the midpoint of its edges. And I incorrectly determined the middle point of circle.

I: How did GeoGebra help you to revise your problem?

Derya: First of all, I drew the shape of the problem I posed. When I dragged some points on the shape, the circle was moving out of the triangle. I realized that my knowledge was incorrect. I tried to rely on the problem I previously posed. I know that both medians and angle bisectors are the same in equilateral triangles. I revised it accordingly.

Ali began to pose problem on GSE by examining the NV Problem 1 written above. In this problem, the edge of the equilateral triangle was 5 cm and the diameter of the inscribed circle was 4 cm. Ali constructed the shape shown in Figure 5 for this problem and found the radius length of this circle to be 1.44 cm. Therefore, he wrote the problem by rearranging the numerical data after determining that the data in the problem were not consistent with each other. A script of the interview with Ali is as follows:

I: Why did you set the diameter to 4 cm in the process of posing the problem?

Ali: I supposed that the length of each side of the triangle was 5 cm, so I thought the [length of] diameter would be less than 5 cm, and I gave it a value of 4 cm. In the drawing, I noticed that the diameter was not 4 cm. At that moment, I did not consider that it would be a single value, depending on the triangle.

I: Could you solve such a problem without knowing the radius of the inscribed circle?

Ali: Actually, we could solve it. I did not think about it when I wrote the problem. In fact, I do not know why I did not think it. At that moment, I focused on the fact that bisector lines divided the triangle into three parts. Then, the total area would be the sum of the areas of each piece [$A(ABC)=A(BCG)+A(AGC)+A(AGB)$ in Figure 5]. Since the area of each triangle is $(\text{edge} \times \text{height})/2$, I thought I should give the radius. But when checking it, the total area could be found because it is an equilateral triangle.

I: Why did you need to rearrange this problem? Did you doubt the correctness of the problem?

Ali: I checked all the problems. Posing problems is hard. It requires a lot of thinking, and you need to explain them. It becomes harder when the subject is geometry, so I felt a need to check them again.

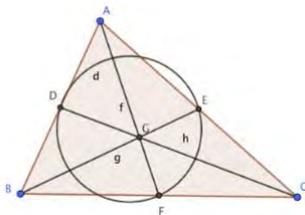


Figure 4. Derya's construction on GSE

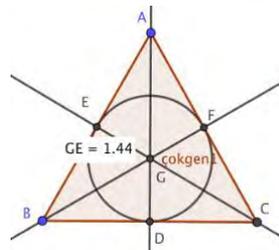


Figure 5. Ali's construction on GSE

The Distribution of PSTs' Scores Regarding Fluency, Flexibility, and Originality

In the viable category, 15 PSTs posed 55 problems in PPT and 17 problems on GSE. The arithmetical mean and median values of PSTs' fluency and flexibility scores for the problems in the viable category are presented in Table 2.

Table 2.

The distribution of PTs' fluency and flexibility scores

Categories	Fluency scores		Flexibility scores	
	Mean	Median	Mean	Media
PPT	3.7	3	2.3	2
GSE	1.1	1	0.93	1

It cannot be considered to compare these two categories (PPT and GSE) because PSTs posed problems on GSE as a continuation of the problems posed in PPT. According to Table 2, low arithmetic mean and median values of PSTs' fluency and flexibility scores point out that there was a low success rate of posing problems both in PPT and on GSE among PSTs. Six categories were identified for the problems posed by PSTs in PPT and on GSE. The distribution of the posed problems according to these categories is given in Table 3.

Table 3.

Distribution of different categories identified among viable problems

Categories	Number of problems	
	PPT	GSE
Length	28	12
Area	11	3
Angle	10	2
Ratio	1	-
Probability	1	-
Identifying the structure of a triangle	4	-
Total	55	17

As shown in Table 3, the PSTs generally preferred to ask for lengths in the problems they posed. Twenty-eight out of 55 problems in PPT and 12 out of 17 problems on GSE were related to length problems. In this respect, it was observed that the majority of the problems posed were related to length. The other two categories that were most preferred were the problems asking for area or angles. Problems posed on GSE were limited to the length, area, and angle categories. Problems related to ratio and probability on the shape provided was the least common problem structures. According to PSTs, the reason for this situation was that posing problems in probability and proof categories requires strong content knowledge. However, GeoGebra, in general, provides trial and error opportunities for checking the correctness of the problems posed. Elif's explanation was as follows:

Since I can draw in GeoGebra, I could give the values in the drawing as given in the problem. When I measure the length or the angle, I calculate it with GeoGebra and used it in the problem. So I had the opportunity to pose errorless problems. Since Length, Angle, and Area are the tools we most actively use in GeoGebra, I generated these types of problems.

The originality of the problems posed by PSTs was determined according to the frequency of occurrence within the group. Three problems posed by PSTs were considered to be in the originality category. All of these problems were written in PPT, whereas no problem posed on GSE was found in the original category. When asked about the reasons, PSTs indicated that posing different problems requires strong content knowledge, and GSE alone was not enough for posing such problems. Another reason for this situation was that they did not know how to make discoveries over the provided shape by means of GeoGebra. Some PSTs have stated that GeoGebra focuses more on the concepts of length, area, and angle because of its dynamic features and experimental environment, which limits them in creating different kinds of problems. Three problems that arose in the originality category in this study were as follows:

There are two race tracks as shown in the figure (see Figure 1). The triangular track is an equilateral triangle with an edge length of 4 cm. A motorcycle and a car with the same speeds are starting to move at the same point from any point of the tracks touching each other. How many rounds does the car run on a triangular track when the motorcycle completes 10 rounds on the circular track? (Ali)

We need a circle and have a triangular piece of cardboard with a perimeter of 30 cm. Find the maximum area of the circle we can obtain from this cardboard. (Ayşe)

The target in a shooting area is as shown in the figure (see Figure 1). The edge lengths of this triangle are 15 cm, 20 cm, and 24 cm. Suppose that a shot falls into this region. What is the probability that the player shoots inside of the circular area? (Akif)

In the problem posed by Ali, the triangle and inscribed circle were presented in figure context by being associated with the race track. This problem requires comparing the circumference lengths as well as calculating the radius of the circle seen in other problems. From this, it was differentiated from other problems by incorporating the rate concept into the process. In the problem posed by Ayşe, different concepts and theorems were included in the process in order to find the maximum value. Lastly, only the edge lengths were given in the problem posed by Akif. In order to solve the problem, it was necessary to calculate the radius and then the area of the circular region using different theorems. In addition, this problem also requires the comparison of these areas by means of probability knowledge.

Conclusion and Discussion

Although worldwide recommendations for the reform of school mathematics suggest an important role for problem-posing (Chen, Dooren & Verschaffel, 2015), studies regarding problem-posing have not yet reached the mainstream of mathematics education research, and there is a need for further research (Singer et al., 2013). This research aimed to expand the problem-posing literature by analyzing the nature of the problems posed in PPT and on GSE in the context of creativity, and to contribute to the effort to integrate problem-posing into technology-supported learning environments. Findings revealed that a considerable number of the problems posed in PPT and on GSE were non-viable-type problems. Parallel to the results of this study, studies conducted in many other areas of mathematics (e.g., Işık & Kar, 2012; Luo, 2009), not only specific to geometry, indicate that the teachers/PSTs have a low ability to pose problems. The reason that problem-posing requires high-level cognitive skills (Conteras, 2007) can be one of the main reasons for PSTs' low success in this study.

When the distribution of the problems in PPT and on GSE were compared, it was seen that the success of generating conceptually valid problems on GSE was higher. According to these results, it was understood that the application of problem-posing on GSE contributes more to producing well-structured problems. As a result of the observations made by researchers and the interviews conducted, the main reason for this difference between PSTs' problem-posing success in PPT and on GSE was the experimentally provided by GeoGebra and that it supported users' trial-and-error approach when dealing with the problems. When posing problems, PSTs checked their problems by drawing the information given in the problems and their other components step-by-step on GSE. During this process, the situation that some PSTs correctly reflected the data given in the problem on their constructions was the most important factor in decreasing the error rate of their posed problems. Particularly on GSE, generating problems related only to the concepts of length, area, and angle (see Table 3) supports the results in a way that PSTs made more use of the measurement capability provided by GeoGebra.

According to the interviews conducted and the observations made, three reasons were identified for the low achievement in posing problems both in PPT and on GSE: (i) lack of problem-posing experience, (ii) the structure of problem-posing activity, and (iii) mathematical content knowledge. PSTs' lack of problem-posing experience led to difficulties in responding to these activities. Although the participants of this study had been introduced to problem-posing, they did not pose problems for the geometric concepts and analyze the posed problems within the scope of a long-term course. Therefore, the lack of experience in problem-posing made PSTs experience difficulties in how to benefit from GeoGebra in generating new problems. For example, Leikin (2015) indicated that new relations and assumptions could be raised by adding auxiliary constructions to the geometric shapes and calling such problems discovery problems. In this study, however, PSTs did not pose problems by using any auxiliary construction on GSE. In addition, Leikin and Grossman (2013) assessed the problems requiring calculations (such as for angle and length) as non-investigation problems and expressed that they matched the problem types found in textbooks. In this study, that the PSTs gave more space to problems requiring the calculations of length, area, and angle (see Table 3) supports that they posed problems depending on their past experiences.

Another reason that PSTs had a low level of problem-posing success was the structure of the problem-posing activity. It was determined that the structure of the activity and the data it contained were other important issues affecting the quality of the posed problems. Considering the fact that PSTs posed more complex problems in a structured mathematical problem-posing environment rather than in a free one, in the study of Silber and Cai (2017), they came to the conclusion that the type of problem-posing activity had an impact on participants' problem-posing successes. This is because the representation of the activity and the relations among the data give participants more opportunities to focus on mathematical concepts and, as a result, they can pose more complex problems. Therefore, in future research, it is suggested that other problem structures mentioned by Stoyanova and Ellerton (1996) should be included into the process. Thus more valid comments can be made about PSTs' creativity skills.

One of the important reasons for PSTs' low success of posing problems in PPT and on GSE is their lack of mathematical content knowledge. One single problem-posing activity was used in this study. A single activity can offer only limited insight about participants' content knowledge, but the observations of the PSTs throughout the process and the insights from the semi-structured interviews provided important ideas about this content knowledge. It was determined that PSTs could not pose well-structured problems, could not correctly use geometric concepts, ignored the consistency of the numerical data in the problems, and could not add new data into problems by using auxiliary drawings (e.g., lines). In addition, PSTs posed problems especially related to area, length, and angle problems on GSE, so this was an important sign of PSTs' lack of knowledge about mathematical concepts. This was because some studies showed that they could pose problems in the categories of analytic geometry, 3-D, involving other figures, and proof and transformation geometry for this activity (e.g., Harpen & Sriraman, 2013; Yuan & Sriraman, 2010). Posing such problems requires strong knowledge of the concepts given in existing problems and other mathematical concepts to which the problem will be related. Therefore, another important result which arose in this study was that GSE alone was not enough for participants to pose different and original problems.

Mathematical creativity is also affected by mathematical content knowledge (Ayllon et al., 2016; Singer et al., 2011; Yuan & Sriraman, 2010). Yuan and Sriraman (2010) indicated that the relationship between creativity

and problem-posing might be related to mathematics knowledge. The findings of the current study support this assumption. PSTs' lack of mathematical content knowledge caused them to have lower fluency and flexibility scores. Failure to associate the existing shape with different mathematical structures led to a low degree of flexibility scoring. Therefore, PSTs generated more problems with similar structure only by changing the numerical data. This situation led to a lower originality score.

Making sketches and dragging the shapes on DGE contributed to producing new problems while evaluation tools gave opportunities to test the correctness of the problems. Christou et al. (2005) indicated that DGE contributes to problem-posing and problem-solving by means of modeling, conjecturing, experimenting, and generalizing strategies. In this study, while modeling and experimenting strategies were widely used, strategies requiring higher-level cognitive skills such as generalizing and conjecturing were less observed. In addition, PSTs tended to pose problems regarding angle, length, and area by assigning numerical values to some parts of the geometric shapes. In this context, they actively used filtering and comprehending cognitive processes during problem-posing. It was observed, however, that PSTs did not pose problems by changing the structure of the geometric shapes or pose proof-related problems on DGE. Posing such problems requires editing and translating types of higher-level cognitive processes. In order to produce such problems, it is necessary to create conjectures and produce generalizations by actively using the drawing, measuring, and dragging tabs of the software. In particular, PSTs' inability to effectively use the dragging tab was the major obstacle to diversifying the types of problems posed. The dimension of making mathematical discoveries by means of auxiliary constructions (Conteras, 2007; Leikin, 2015; Leikin & Grossman, 2013; Segal et al., 2018) has substantially been ignored. PSTs tried to check the correctness of the assigned numerical values on geometric shapes by using the measurement tools on DGE. This type of understanding implies that GeoGebra was predominantly seen as a construction instrument allowing the formation of different geometric shapes (Hohenwarter & Fuchs, 2004). In other words, GeoGebra was seen as a testing tool by means of its construction features rather than a tool to make mathematical discoveries.

Teachers agree that the development of creative thinking is necessary and important, but they have deficiencies in integrating the activities that might develop such abilities into a classroom environment (Silver 1994; Singer &

Voica 2017). Undoubtedly, PSTs' inability to generate creative problems on GSE may also be an obstacle to create such learning environments and implement this in the classroom. The results of this study contribute to the development of the teacher training and mathematics education literature in various ways. First, some studies (e.g., Counteras, 2007; Leikin & Grossman 2013) conducted regarding geometry asked participants to pose problems based on proof problems, while others (e.g., Christou et al., 2005; Lavy, 2015) asked them to pose problems based on a geometric problem. All such problem-posing activities were in structured form. The given and requested facts were already present in the problems, and discoveries can be made on DGE by changing one or more of these situations. The fact that the participants presented generalization and conjecturing activities using the tools on DGE in the research using structured problem-posing activities also supports this result. On the other hand, since the activity provided was opened in form in this study, PSTs needed to structure the problem's given and requested facts by themselves. The absence of the given and requested facts also made it difficult to observe the consequences of changing one or more of the conditions. Therefore, the problems posed on GSE did not differ significantly from the problems in PPT in terms of creativity. From this point of view, starting with structured problem-posing activities may contribute more to the development of creative thinking skills on GSE instead of beginning with semi-structured problem-posing activities. In this context, it is suggested that mathematics educators and researchers should design the learning environment by considering this situation. There are also some results in the literature indicating that the task format (e.g., free, semi-structured, and structured forms) had an impact on participants' problem-posing performance (e.g., Silber & Cai, 2017). On the other hand, no study was observed investigating the effect of the activity format on problem-posing success on GSE. Therefore, since this study investigated the problems posed by PSTs by means of a single activity, further studies are needed to see how the type of task format on GSE contributes to creative thinking skills.

Secondly, PSTs began to pose problems on GSE by examining the problems they previously posed. This can be turned into an advantage in creating GSE-supported learning environments. PSTs can explore the errors in the problems posed by means of measurement tools on GSE and develop the posed-problems by means of dragging tool and auxiliary sketches. Therefore, distinctive features of DGS, which are measurement, drawing and

dragging tools, might be actively embedded into problem-posing process. At the same time, the analysis of the problems posed by PSTs on GSE would increase their interest and motivation towards the lessons. Therefore, PSTs' creative thinking skills could be more strongly supported on GSE.

Lastly, different analytical schemas and available problem examples (e.g., Conteras, 2007; Leikin, 2015; Leikin & Grossman, 2013) regarding problem-posing in geometry can be presented to and discussed with PSTs in order to provide them with structured learning environments while beginning problem-posing on GSE. Thus, PSTs will be able to develop an awareness about alternative types of problems and transfer this to different geometric problem-posing activities. In order to develop PSTs' abilities, it is necessary to give more space to the problem-posing activities and discuss the posed problems in the course of the teacher training program. In the further research, the contribution of such a learning environment to the development of PSTs' creative abilities can be explored. This study adopting the qualitative approach was conducted through an activity. When the influence of the structure of the activity on PSTs' problem-posing successes is taken into account, there is a need to conduct studies with larger participant groups over different geometric shapes and forms. The results of such studies may support the tendency to make problem-posing a more dominant characteristic of classroom instruction (Cai et al., 2013) by means of their contribution to explaining the relationships between problem-posing, technology, and creativity.

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