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Prospective Teachers' Expectations of Students' Mathematical Thinking Processes in Solving Problems

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Abstract: This research aims to describe the expectations of prospective teachers for students' mathematical thinking processes in solving problem-based on the Polya model. This model is perceived by the theory of mathematical thought processes proposed by Mason. A descriptive method with a qualitative approach was used in this research. The research subjects were 25 students from the Department of Mathematics Education, Ibrahimy University. The test was given to collect data related to mathematical thinking processes expected by prospective teachers to students. Collected data including observations, tests, and interviews were tested in the aspect of their validity by triangulation. The qualitative descriptive was used to analyze the data. The results indicated that: (1) The average GPA (Grade Point Average) of the high, medium, and low group prospective teachers' were 93.25; 89.89; and 83.63 with a standard deviation of 1.754 each; 1.054; and 5.370, respectively (2) The prospective teachers expected that the students' mathematical thinking processes were able to carry out all of four mathematical thinking processes based on Mason Theory; (3) The prospective teachers expected that students were able to use Mason Theory on every stage of the Polya model problem solving; and (4) The expectation of prospective teachers were specializing (89%), generalizing (75%), conjecturing (62%), and convincing (59%). The results suggest for following up in a teachers or lecturer's meeting in order to find out the expectations of their students' mathematical thinking processes, both in mathematics or other disciplines.

Keywords: *Prospective teachers', expectations, mathematical thinking processes, Polya models, Mason theory.*

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Introduction

Education is a conscious effort in order to create people thinking wisely. Advanced nations always put the education sector on their priority budget and policy. Because through that, every single person can easily access the schools, getting more knowledge, and improve his or her thinking skills. According to Tohir et al. (2018), the think ability to identify and construct the formulas in mathematics is urgently needed to foster a student's understanding in terms of the material and generate a meaningful learning process (Reyes-Cedeno et al., 2019). Thalhah et al. (2019) stated that mathematic is the bedrock of any contemporary order of science. Chasanah et al. (2020) said that in mathematics learning, collaboration skills have become an intake for both elementary and university students. Therefore, effective mathematics learning is a step within creative thinking for each person which will have different abilities to solve a problem.

Tohir et al. (2018) reported that a mathematical problem contains mathematical concepts requiring an indirect process to solve it. Problem-solving is the heart of mathematics education, accordingly, every student is acquired to have the problem-solving skills (Barham, 2020). Nowadays, in the 4.0 industrial revolution, the problem-solving processes have been replaced massively by several applications where the users or students are not required to concern about its mathematical processes anymore. It is not sufficient for authentic teachers to just look at the final result, instead of the ability of the student to solve problems. The student's mathematical thinking process is a very important thing to be

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understood and known by prospective teachers. Because knowing students' mathematical thinking processes is the initial provision of a teacher (especially teacher candidates) to become a professional one and to be ready to be a good facilitator in the process of learning mathematics at school.

Thus, the important part to prime prospective teachers in knowing the student's mathematical thinking process is the aspect of preparation done carefully by the teachers themselves. One effort that can be done is to provide problem-solving test questions by describing the expected mathematical thinking process to students. Because, teacher plays an important role on mathematics learning in the school. Hence, students 'mathematical thinking processes must be sharpened continuously and constantly in order to enhance their performance. As it is suggested by Tohir (2017) that one of the roles of the teacher in mathematics learning is to help students express how the process runs in their minds when solving problems, for example by asking students to tell the steps in their minds. In line with this statement, Mason et al. (2010) said that mathematical thinking itself is an individual activity based on personal experience which can emerge as a focus on associating the main ideas possessed.

As for this matter, Primasatya (2016) reported that the prospective teacher's mathematical thinking ability is still quite low and needs to be improved. Meanwhile, Santos-Trigo and Reyes-Martinez (2019) show that prospective high school teachers relied on a set of tool affordances (dragging objects, looking and exploring object's loci, using sliders, quantifying and visualizing mathematical relations, etc.) to formulate, explore and identify properties or relations to share, discuss and support mathematical conjectures. In addition, Xu et al. (2019) found that there was a mismatch between the problems constructed by students' thoughts and the results predicted by teachers towards their thoughts. Thinking problems predicted by teachers are more likely to involve functional relationships about patterns and are less likely to occur.

Based on those previous reports, it is necessary to further investigate the prospective teachers regarding how their expectation to their student's thinking processes. Therefore, the purpose of this study is to describe the expectations of prospective mathematics teachers for students' mathematical thinking processes in solving mathematical problems based on Polya's Model, such as: *understanding the problem*, *devising a plan*, *carrying out the plan*, and *looking back* (Hulaikah et al., 2020; Saiful et al., 2020; Son et al., 2020; Tohir, 2017; Yusnia, 2018); Then the problem solving used is related to the mathematical thought process theory expressed by Mason et al. (2010), consisting of *specializing*, *generalizing*, *conjecturing*, and *convincing* (Farib et al., 2019; Iswari et al., 2019; Lane & Harkness, 2012; Lesseig, 2016; Sumarna & Herman, 2017). The following is a summary of the book "*How To Solve It*" concerning the Polya Model (Polya, 1973) in Table 1, it is stated that there are several stages to solve the problem.

Table 1. Polya Models Problem Solving Steps

Polya Models	**Problem Solving**
Understanding the Problem	What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory
Devising a Plan	Have you seen it before? Or have you seen the same problem in a slightly different form? Do you know a related problem? Do you know a theorem that cloud be useful?
Carrying Out the Plan	Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?
Looking Back	Can you check the result? Can you check the argument? Can you derive the result differently? Can you see it at a glance? Can you use the result, or the method, for some other problem?

Through this problem solving, prospective teachers can find out the expectations of their students' mathematical thinking processes based on Mason Theory. So That the indicators of the mathematical thinking process in solving problems is shown in Table 2 below.

Table 2. Indicators of the Mathematical Thinking Process

Mathematical Thinking Process	Indicators
Specializing	Identify the problem Devise and try various possible strategies
Generalizing	The reflect on the ideas made Expanding the scope of the results obtained
Conjecturing	Analyze in similar cases
Convincing	Look for reasons why the results can appear Form a pattern from the results obtained Make of the reverse an already formed pattern

The mathematical thinking process that has been put forward by Mason et al., (2010) is hierarchical so that it cannot go backward or jump up and down. For example, if someone can think specializing and generalizing, but the ability to think conjecturing has not yet emerged, then the ability to think convincingly emerges either. This has an impact on the indicators that have been prepared. The mathematical thinking indicator is also hierarchical, so it must be in the appropriate order from low to high levels.

Methodology

The method used was descriptive research with a qualitative approach. Qualitative research follows the natural background in which a study takes place (it views the context as a whole). In this case, humans function as instruments. Qualitative methods require inductive data analysis and theory development, all of which are based on data, which are descriptive and more concerned with the process being investigated. This research is limited by its focus, specific criteria to ensure data validity and provisional design. Also, qualitative data is generated through joint decisions (Munawwarah et al., 2020; Tohir et al., 2018; Tohir, 2019). Yin (2017) argued that a qualitative research design is used for in-depth investigation of the current situation in real-life contexts.

Participants and Data Collection

The subjects of this research were 25 students of the Mathematics Education Study Program at Ibrahimy University. The students, as teacher candidates were expected to describe the mathematical thinking process for their future students in solving of mathematical problems. Data was collected by giving one question obtained from National Examinations for Junior High Schools. Those questions were issued by the National Examination committee team from the Ministry of Education and Culture of the Republic of Indonesia. The collected data was reduced, presented, summarized, and verified. Data validation was ensured by triangulation, peer checking, and extended observation.

Analyzing of Data

Data analysis technique was carried out by: (1) analyzing of each mathematical thinking process based on Polya and associated with the theory of Mason, Burton, and Stacey, (2) grouping, and (3) analyzing of mathematical thinking processes expected by prospective teachers.

Data and sources in this study were obtained from students' GPA (Grade Point Average), observations, problem-solving tests, and data interview. Observations were made during the learning process in for 3 meetings, the test questions were given to the subject after the researcher analyzed several National Exam questions, then one question was selected and re-validated by the experts with valid results, then given to students. Interviews were conducted directly with all subjects to find out real information on the expected thought processes of students. Then, the data collected through observations, test results, and interviews were tested for the aspect of the validity by using triangulation. Triangulation was carried out to check the suitability of the data obtained between observations, test results (documents), and interview results. Triangulation is an attempt to check the truth of data or information obtained by researchers from various different points of view by reducing the bias as much as possible that occurs within the process of collecting and analyzing data (Tohir et al., 2018). A qualitative approach was used to determine the prospective teacher's mathematical thinking process. The presentation of data in qualitative research can be made in brief descriptions and relationships between categories. While, most of the time, the data presentation used in qualitative research was in the form of narrative text. The presentation of data includes the process of classifying, identifying, and writing down the organized and categorized data sets so that conclusions can be drawn (Tohir, 2019). The last stage was drawing conclusions based on data groups; in this case, it was about mathematical thinking processes in solving mathematical problems.

Findings / Results

The mathematical thinking process of the subjects was illustrated as a problem descriptions based on Mason et al. (2010) and reviewed based on the problem-solving method proposed by Polya (1973). Then, the completion of indicators for high, medium, low group categories were also analyzed The results of prospective teachers GPA scores for the high, medium, and low group categories can be seen in Table 3.

Table 3. The results of prospective teachers GPA scores

Categories	N	Minimum	Maximum	Mean		SD	Variance	Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error
High	8	91	95	93.25	0.6196	1.7536	3.071	-1.914	1.481
Medium	9	88	91	89.89	0.3514	1.0540	1.111	-0.546	1.400
Low	8	75	88	83.63	1.8987	5.3702	28.839	-0.800	1.481

Table 3 shows that the test value of the subject for high, medium, and low group categories are 93.25; 89.89; and 83.63, respectively. From these data, it explains that there is a difference in value between the three subject groups, namely 3.36; and 6.26. The results of this test indicated that the abilities between the subject groups as appeared the significant differences. The second data obtained by researchers was the result of observations of prospective teachers' activities and lecturer performance. The observation result is shown in Table 4.

Table 4. The observation results distribution

Activities	Meeting I	Meeting II	Meeting III
Prospective Teachers	68	73.6	86.4
Lecturer	73.6	77.4	84.2

At the first meeting, the prospective-teacher activity score was 68, while the lecturer performance got 73.6. At the second meeting, the prospective-teacher activity score was 73.6, while the lecturer performance got a score of 77.4. At the third meeting, the prospective-teacher activity score was 86.4, while the lecturer performance got a score of 84.2. The results of these observations indicate that the activities of prospective teachers and lecturer performance in 3 meetings increased significantly.

Subject's Mathematical Thinking Process in Understanding the Problems

The results of indicators within mathematical thinking process data expected by prospective teachers to students at the stage of *understanding the problem* were obtained based on test results. Figure 1 presents the results of the mathematical thinking process expected by the subject in solving problems.

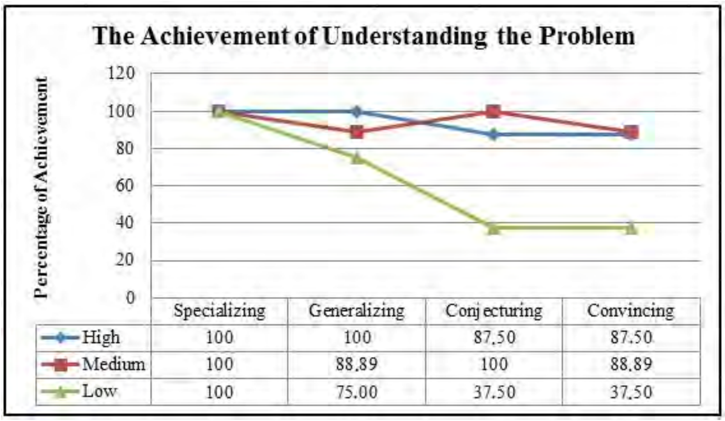


Figure 1. Percentage of Completion in the Problem Understanding Stage

As it is seen in Figure 1, the expected mathematical thinking process between the high, medium, and low group subjects seems to have significant difference within the aspect of the mathematical thinking process of "*specializing*", "*generalizing*", "*conjecturing*", to "*convincing*" at "*understanding the problem*" stage. The results also suggested that at the stage of "understanding the problem", all subjects were able to do "*specializing*" well, on average, making depiction illustrations based on the information enclosed in the problem to make it easier for them to find the appropriate strategy.

Only high group was able to do "*generalizing*" perfectly in the "*understanding the problem*" stage, in which the subjects were able to select the proper depiction to make an appropriate illustration based on the information and the purpose of the problem. Likewise, the middle group did the "*conjecturing*" very well, such as choosing depiction illustrations by the information enclosed in the problem, the selection of depiction illustrations itself was based on the perspective of each subject by their knowledge that has been obtained previously. About 72% of subjects (Percentage of all research subjects) were able to do "*convincing*" well at the stage of "*understand the problem*"; which was able to provide reasons and precise explanations in choosing the illustration of images in the form of cuboids for the hall to be painted on the inside. Figure 2, presents the expectations of the high group within prospective teachers on students' mathematical thinking processes at the stage of "*understanding the problem*".

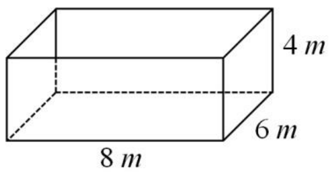
Description:

Step 1 : I hope that after reading the questions, the students are about a hall in the form of a block of space with known length, width, and height.

Step 2 : It turns out that in this case, if the interior of the hall will be painted, it costs Rp. 50,000/m^2, and must determine all painting costs.

Step 3 : Because it is expected that students can think critically, then they will think that there is something on the wall or not. It could be that on the wall there is a door or window. Then, if there is a door or window of course it must also be painted, then does the paint cost the same?.

It should be in that question, the part of the unpainted wall should be added and if the unpainted wall costs the paints differently, the cost must also be stated.
because I also hope students can think about mathematics.



because it is known and the painting is the inner wall, what is calculated is the front and rear walls as well as the right and left sides.

Build 1st　　Build 2nd

Figure 2. Subject Expectations in the Problem Understanding Stage

According to the results of the answers from prospective teacher A in Figure 2, the writer conducted a distinctive interview with the prospective teacher to find out more about what was actually expected from students' mathematical thinking processes when "*understanding the problem*" in the given problem. The result of the interview is as follow:

Researcher : *What exactly do you expect your students to see about this problem*?

Subject A : *When students face questions like this, I hope that they can read the questions first until they understand it, then they can imagine that the hall is in the form of cuboids, if it is difficult for them to imagine it, at least the student can illustrate it in the form of pictures*.

Researcher : *Do you expect the student to re-communicate the main ideas enclosed in the problem, either communicating them in their own words orally, in writing, pictures, or diagrams*?

Subject A : *yes, I hope so, as I said previously that students can communicate in the form of illustrated images. However, it is still tentative related to the sizes and how many windows and doors for the hall*.

Researcher : *why do you expect students to choose such picture illustrations*?

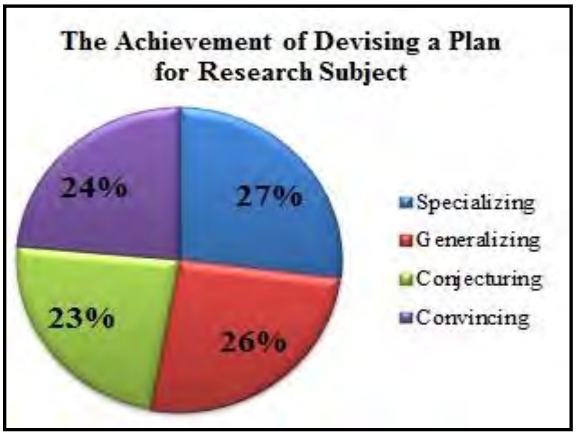
Subject A : *So that students find it easier to understand in looking for answers to these problems, then students should also be able to illustrate in the form of webs of cuboids*.

Researcher : *Have you written down what is known and asked about the problem*?

Subject A : *yes, I have. Because to work on these questions we must understand what is known and what is asked, so that the students do not use any incorrect solution or formula*.

Subject's Mathematical Thinking Process at the Stage of Devising a Plan

The data on the results of the indicators completion of the mathematical thinking process expected by the prospective teacher for students at the "*devising a plan*" stage were obtained based on the test results. The completion results about the mathematical thinking process expected by the subject in solving problems is presented in Figure 3.



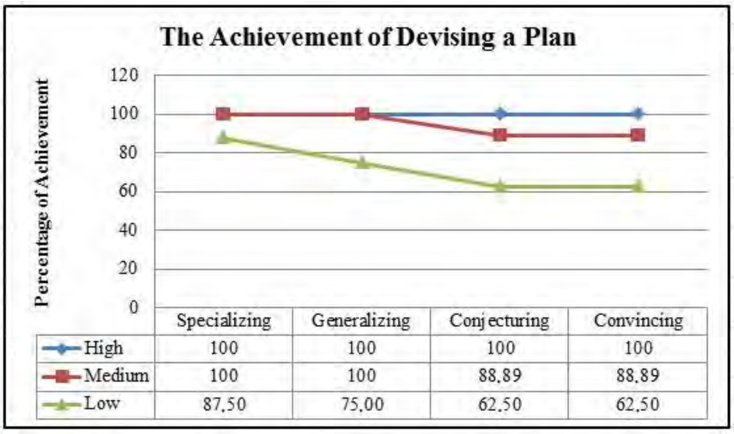
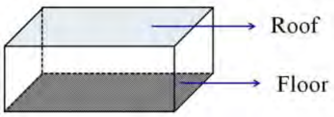


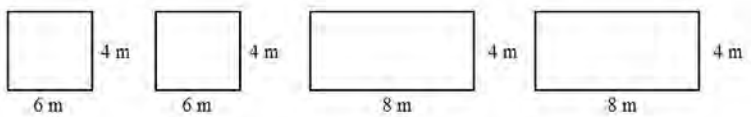
Figure 3. Percentage of Indicator Completion in the Stage of Devising a Plan

Based on the results of the scores indicators completion percentage of the mathematical thinking process expected by prospective teachers to students in Figure 3, it shows that the expected mathematical thinking process between the high and medium group subjects had insignificant differences within the mathematical thinking process of "*specializing*", "*generalizing*", "*conjecturing*", and "*convincing*" at the "*devising a plan*" stage. As for the low group subjects, there were found significant differences with the high and medium group subjects. The results of this study also suggested that at the stage of "devising a plan", the subjects within the high and medium group were the only ones who were able to "*specializing*" well and perfectly, where, most of them can make the parts of the wall to be painted by taking 4 sections of the wall and remove the floor and roof sections. The high and medium group subjects were able to do "*generalizing*" perfectly, i.e. the subject considers that it is important to draw pictures from all four parts of the wall and remove the floor and roof sections to make it easier for them to solve the problem so that they cannot be misled by the formula of the entire surface of the cuboids. Only those who belong to the high group subjects were able to do "*conjecturing*" well, i.e. the subject makes an analogy within the formula of the surface area of the cuboids with a rectangular area and number operations, because there were only four parts painted in the hall. Furthermore, all of the subjects from the high group were the only ones who were able to do "*convincing*" well and perfectly, which they could provide an explanation and the exact reason for the proposed completion plan, even though some of them were found to have errors in the selection of the plan. Figure 4 illustrates one of the expectations of the prospective teacher of the medium group to the students' mathematical thinking process at the "*devising a plan*" stage.

The third, because the roof and floor do not have to be painted. I also hope that students also think like that. Because the hall is shaped like a beam. So, the walls that must be painted are only the edges that are rectangular ($p \times t$), the front, and the back, which are small rectangles ($p \times l$).



So that students also have to understand which parts include the length, width and height.



Furthermore, if the students already understand the description, I hope the students will be good at number operations mathematics.

- $2\ (p \times t) = 2\ (8 \times 4) = 2\ (3 \times 2) = 64$ for two large (equal)
- $2\ (l \times t) = 2\ (6 \times 4) = 2\ (2 \times 4) = 48$ for two equal small (equal)

Figure 4. Expectations of Subject B at the Devising a Plan Stage

According to the results of the answers from prospective teacher B in Figure 4, the researcher conducted a distinctive interview with the prospective teacher to find out his expectations for students on the mathematical thinking process within the stage of "*devising a plan*" on the problem. The following are the results of the interview:

Researcher : *What strategies do you expect from students in planning for the answer to the problem? Please elaborate.*

Subject B : *The possible strategy to answer the problem by using the surface area of the cuboids without having the roof or floor section or using the total area of each wall.*

Researcher : *Have you found the appropriate and reasonable strategy in the matter?*

Subject B : *The strategy that I use is to illustrate the problem in the form of 4 rectangular images as the representative of the wall to be painted, or in other words, to find the surface area of the cuboids by using an area without the floor and roof section which can make it easier to work on*.

Researcher : *Why did you choose such strategy*? *Can you state the reason*?

Subject B : *Because such strategy is easier to simply reduce existing formulas, or based on the four rectangles that I have illustrated in the form of drawings, it is easier for me to determine the area, then all that remains is to add between two congruent rectangles*.

Subject's Mathematical Thinking Process at the Carrying out the Plan Stage

The data on the results of indicators completion of the mathematical thinking process expected by prospective teachers to students at the "*carrying out the plan*" stage was collected based on the test results. The completion results of the mathematical thinking process expected by the subject to students in solving problems at the "*carrying out the plan*" stage is presented in Figure 5.

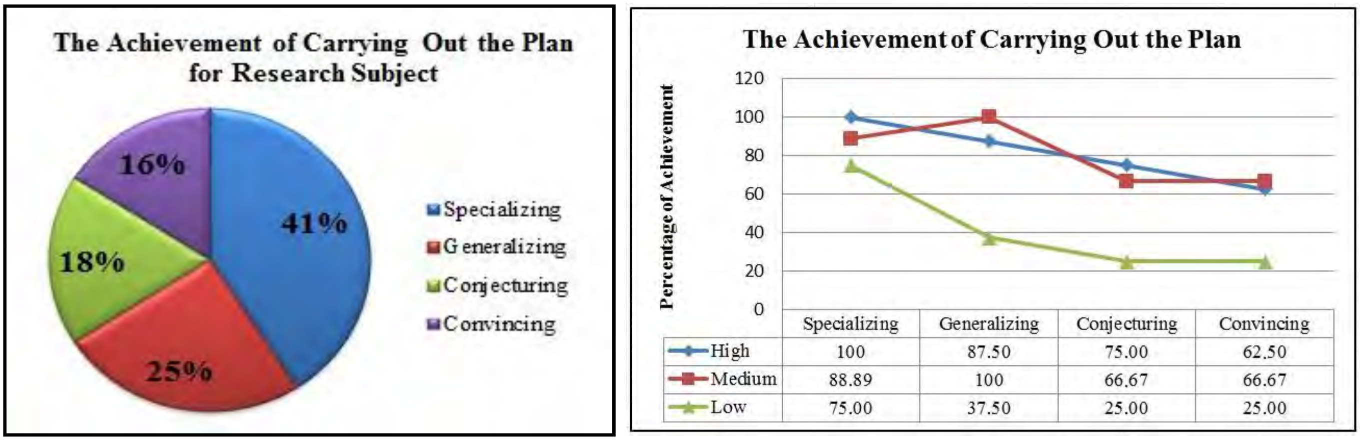


Figure 5. Percentage of Indicator Completion in the Carrying Out the Plan Stage

Based on the results of the scores indicators completion percentage of the mathematical thinking processes expected by prospective teachers of students in Figure 5, it showed that the mathematical thinking process expected by the high group subjects decreased and form linear lines starting from mathematical thinking of "*specializing*", "*generalizing*", "*conjecturing*", to "*convincing*" at the "*carrying out the plan*" stage. As for the medium group subject, they tended to be inconsistent between the four mathematical thinking processes. Whereas, as for the low group subjects, there were some very significant differences found compared to the high and medium group subjects. The results of this study also showed that at the stage of "*carrying out the plan*", the high group subjects were able to carry out mathematical thinking processes in "*specializing*" well and perfectly, which most of them could make parts of the wall to be painted by seizing those walls in 4 parts and removing the floor and roof section. As for all of the group subjects, the medium group subject was the one who can only carry out the mathematical thinking processes on "*generalizing*" well, i.e. the subject considered that it was important to generalize the cuboids' surface formula into the hall wall area so that valid results could be obtained.

The subjects considered that it was important to also find the area of the hall by dividing it into four rectangular sections so that the answer could be found by using the rectangular area, and then utilize number operations. Whereas, none of the subjects within the low group one were able to do all four mathematical thinking processes as a whole. There were as many as 56% of subjects (percentage of all research subjects) who could do "*conjecturing*" well at the stage of "*carrying out the plan*", i.e. subjects used the surface area of the cuboids without a base and cover which then make an analogy to the cuboids surface area with a square area and number operations. There were as many as 52% of subjects (percentage of all research subjects) who could do "*convincing*" well at the stage of "*carrying out the plan*", which they provided an explanation and the right reasons for the completion of the compiled and the answers obtained, although among them some subjects were having some errors in using the plan set up in which it resulted for having the wrong answer. Figure 6 presents one of the expectations of the prospective teacher from the high group for the students' mathematical thinking process at *carrying out the plan* stage.

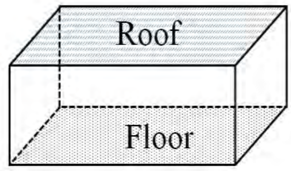
To find the area of the block, which is 2 ($p \times l + p \times t + l \times t$), it has been taught that that's how to find the area of the block. When students have calculated, the following results are obtained:

$p = 8$, $l = 6$, and $t = 4$. Then,
Surface area $= 2\,(p \times l + p \times t + l \times t)$
$= 2\,(48 + 32 + 24)$
$= 2\,(104)$
$= 208$

After finding the area of a hall, students will continue to look for the cost of interior wall paint and this is where I want students not only to multiply the area of the hall by the cost of paint/m^2.

Because in the matter of wanting him to look for the cost of the inner wall paint, not just the entire space. known: All costs Rp 10,400,000

So here it must be reduced by the roof and floor.



$p \times l = 48 \times 2 = 96$ then multiplied by Rp 50,000 = 4,800,000

Then the final result is found:
10,400,000 – 4,800,000 = 5,600,000

Figure 6. Expectations of Subject C at the Carrying Out the Plan Stage

Based on the answers from prospective teacher C in Figure 6, the researcher conducted a distinctive interview with him, to find out what the teacher expected of the students' mathematical thinking process when implementing the plan for solving the problem. The following are the results of the recording of the interview. The result of the interview is as follow:

Researcher : *Did you really carry out the completion procedure as you have planned*?

Subject C : *yes, I did sir. I carried out the completion procedure in accordance with what I had planned by making the assumption in advance that the hall to be painted inside as a whole by ignoring the doors and windows (like the hall in general)*.

Researcher : *what kind of completion procedures do you expect for your students to get valid answers*?

Subject C : *the procedure that I expect for students is that they can use the cuboids surface area formula and the one without having the base and cover part*.

Researcher : *why do you use that kind of completion procedure*?

Subject C : *I use such a procedure so that students can use their mathematical thinking processes as smooth as like water flow systematically and measurably, hence the initial concept of the surface area of the cuboids is still used that they have to think critically in getting the valid answers*.

Researcher : *Are you sure you can find the correct answer to the problem? What do you think so*?

Subject C : *In my opinion, by using this method, the answer is correct but within the test items, the windows and air vents part is not included*.

Subject's Mathematical Thinking Process at the Looking Back Stage

The data on the results of the indicators completion of the mathematical thinking process expected by the prospective teacher for students at the "*looking back*" stage were obtained based on the test results. The completion results about the mathematical thinking process expected by the subject in solving problems is presented in figure 7.

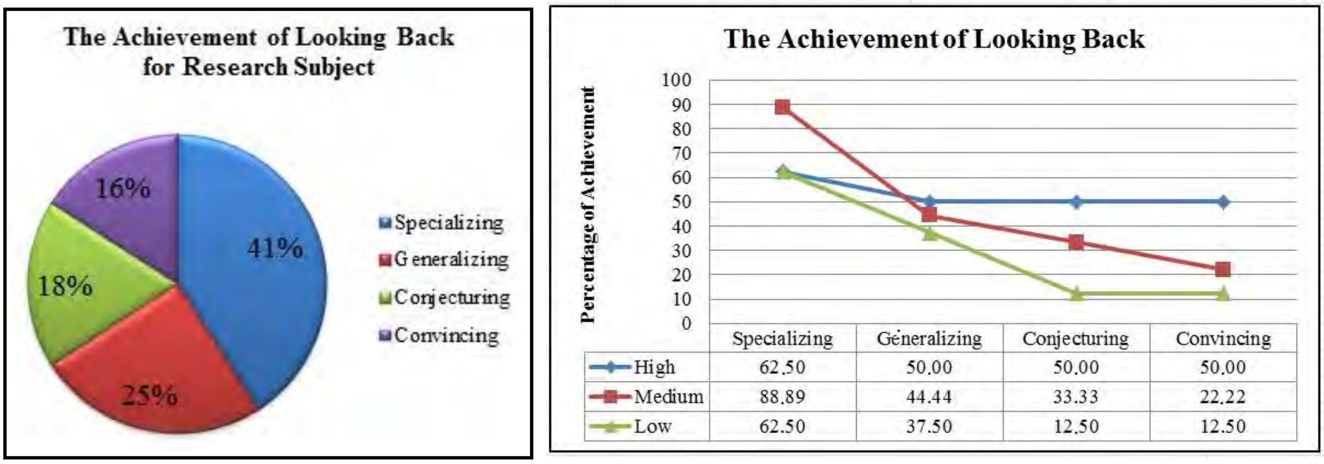


Figure 7. Percentage of Indicator Completion in the Looking Back Stage

Based on the results of the scores indicators completion percentage of mathematical thinking processes expected by prospective teachers of students in Figure 7, it indicated that the mathematical thinking process expected by all of the group's subjects decreased from mathematical thinking process of "*specializing*", "*generalizing*", "*conjecturing*", to "*convincing*" at the stage of "*looking back*". The results of this study also showed that there were 88.89% of the medium group subjects who were able to "*specializing*" well in the "re-reviewing" stage, i.e. when they re-examine the results of their work and utilize other strategies to ensure that the answers were the same or not using the surface area formula of cuboids without the base and cover part. There were as many as 50% of the high group subjects who were able to do "*generalizing*" well in the "*looking back*" stage, i.e. the subject considers that it is important to use another strategy to ensure the answer is correct or not that they compare the answers that have been obtained previously with the answers from other strategies used. There were as many as 50% of the high group subjects who were able to do "*conjecturing*" well in the "re-reviewing" stage, i.e. the subjects got the same answer as the initial calculation, so they assumed that the answer was correct. There were as many as 50% of the high-group subjects who were able to do "*convincing*" well in the "*looking back*" stage, in which they could provide an explanation and the exact reason for the completion procedure that they got the same answer as the previous one and the subject believed that the answer was correct. Figure 8 demonstrates one of the expectations of the prospective teacher from the high group for the students' mathematical thinking process at the "*looking back*" stage.

I hope that students can proofread their work so that these students can get the formula without a base and without a roof.

$$
\begin{aligned}
2\,[(p \times t) + (l \times t)] &= 2\,[(8 \times 4) + (6 \times 4)] \\
&= 2\,[32 + 24] \\
&= 2 \times 56 \\
&= 112 \text{ m}^2
\end{aligned}
$$

Because every 1 m^2 it costs Rp 50,000,
then students can think that 50,000 × 112 = 5,600,000

Figure 8. Expectations of Subject D at the Looking Back Stage

According to the results of the answers from prospective teacher D in Figure 8, the researcher conducted a distinctive interview with the prospective teacher to find out his expectations for students on the mathematical thinking process within the stage of "*looking back*" on the problem. The following are the results of the interview:

Researcher : *Did you double check the results of your answer*?

Subject D : *I use the re-checking by implementing the second strategy by adding the total area of the entire side of the hall's walls*.

Researcher : *Have you concluded that your answer is correct? How to prove it*?

Subject D : *If both of them are the same, then the area asked is correct, but from the beginning the constraints aspect of the problem is that the size and number of windows and doors are not informed*.

Discussion

Understanding the Problems

From the results of the interview with subject A, it shows that the subject could do all of four mathematical thinking processes properly and perfectly. The results of this study indicate that in "*understanding* the problem" is a thinking process involving "*specializing*", "*generalizing*", "*conjecturing*", and "*convincing*". This is in line with the statement of Mason et al. (2010) that at the entry, attack, and review stages there are important processes in problem-solving, such as "*specializing*", "*generalizing*", "*conjecturing*", and "*convincing*". *Specializing* is the stage of specifying problems such as making a pattern or picture and arranging parts of the problem into "the known and asked one". *Generalizing* is an activity to find steps for completion and how to test the steps of truth on the alleged settlement that has been made. *Conjecturing* is an activity to make guesses from a pattern that has been made. Allegations that have been made are tested to see if it is true, and when the allegations are false, then new allegations must be made until the proper solution is found. *Conjecturing* involves reasoning about mathematical relationships to develop statements that are tentatively thought to be true but are not known to be true (Lesseig, 2016). *Convincing* is an activity explaining the reason for completion based on mathematical concepts (Mason et al., 2010). "While developmental trajectories of students' learning of more advanced mathematics topics is less detailed, high school teachers learn in real-time by going with the students' thinking to gauge their understanding of the content" (Kent, 2017, p. 94).

Devising a Plan

From the results of the interviews with subject B, it showed that the subject was able to do all of four mathematical thinking processes well even though the imperfect form, because he ignored the general hall shape of the wall consisting of doors and windows. According to Polya, he said that the stage of preparing a plan had a higher level of difficulty than the other stages of problem-solving; this is because, at the stage of preparing the plan, students were required to determine the right steps and related concepts to find solutions of problem-solving (Tohir, 2017). The results of this study also suggested that in compiling a problem-solving plan, there were mathematical thinking processes involve such as specializing, generalizing, conjecturing, and convincing. "Teachers, through a general knowledge of their students' mathematical thinking, can find and/or create problems or tasks that will give them access to the underlying mathematical concepts" (Kent, 2017, p. 95). This supports the statement of Mason et al. (2010) that in the process of thinking, it includes the process of *specializing*, *generalizing*, *conjecturing*, and *convincing* (Ariefia et al., 2016). The results of this study reinforced the research results obtained by Ariefia et al. that showed the following thought processes (*specializing*, *generalizing*, *conjecturing*, and *convincing*) took place at each stage of problem-solving, but not all of the stages emerged simultaneously at each stage of problem-solving.

Carrying Out the Plan

From the results of the interviews with subject C, it showed that the subject was able to do all four mathematical thinking processes perfectly. The results of this study indicated that in the stage of "plan implementation", there were mathematical thinking processes involved, such as "*specializing*", "*generalizing*", "*conjecturing*", and "*convincing*". This is in line with Polya's statement that in solving the problem, it contains the aspect of "*specializing*" and "*generalizing*" (Ariefia et al., 2016). This is also supported by Lin (2006) who states that the theory of thinking proposed by Mason et al is based on Polya's thinking. According to Mason et al. (2010), they stated that mathematical thinking is a very complex high-level activity, which is the result of great work that has been written or studied. At the stage of "*carrying out the plan*" in this particular study, it was found that; (1) there were as many as 25% of the low-group subjects who were not doing the thinking process of "*specializing*"; (2) there were as many as 12.50% of the high group subjects and 50% of the low group subjects who did not do the process of "*generalizing*" thinking; (3) there were as many as 15% of the high group subjects, 33.33% of the medium group subjects, and 75% of the low group subjects who did not doing the "*conjecturing*" thinking process; (4) there were 37.50% of the high group subjects, 33.33% of the medium group subjects, and 75% of the low group subjects who did not make the process of "*convincing*" thinking; and (5) the total average of the subjects who did not do the four mathematical thinking processes was caused due to the inaccuracy in the completion procedures used, so that they got an incorrect and forgot to double check the results of their work and did not re-do the work by using other strategies to ensure whether the answer was correct or not.

The results are also in line with Sfard (1991) opinion that there are three reasons why students' thought processes can be said to be interesting; (1) students do not always logically express their thought processes; (2) although students do not seem to understand mathematical concepts, there is a possibility that they have the knowledge in their thinking process; (3) the process of identifying the conceptual problems experienced by students in trying to understand a knowledge can be a difficult thing to do especially when they are thinking abstract things. According to Polya (1973), problem-solving is defined as an effort to find a way out of difficulty to achieve a goal that is not easy to achieve immediately. Therefore, the process of thinking mathematically between the high, medium, and low group subjects has different capabilities in carrying out the plan of solving a problem depending on the knowledge that has been obtained beforehand and how much they practice to solve any problem. These processes occur naturally as one first explores the meaning of a question or problem by examining particular examples (specializing) and begins to conjecture about the relationships that connect these examples (Lesseig, 2016).

Looking Back

The results of the interviews with subject D, it showed that the subject was able to do all four mathematical thinking processes well. The results of this study indicated that at the stage of "*looking back*" there were mathematical thinking processes involved such as *specializing*, *generalizing*, *conjecturing*, and *convincing*. This supports Polya's statement that in solving the problem, it involves the process of *specializing* and *generalizing* (Ariefia et al., 2016). This also supports the statement proposed by Mason et al. (2010) that in the process of thinking, it includes the process of *specializing*, *generalizing*, *conjecturing*, and *convincing*. The results of this study are in accordance with the results of research conducted by Tohir (2017) which showed that the thought process of subjects with high mathematical ability involves examining the work that has been made called as an assimilation process, in which to check the work before and after arriving at the completion by tracing back the calculations that have been done through mental calculations efficiently.

The results of this study also strengthen our previous observation Tohir (2017) which suggested that subjects uncompleting the steps of the Polya model were mostly categorized to moderate and low groups. Moreover, Rangka et al. (2019) said that the an imbalance problem was faced by students when the did not understand the mathematical questions. Tabak (2019, p. 372) said "that teaching will be more effective if it is based on the links between mathematical statements and real-life". The chore of mathematics education is clarifying the thinking process of students in learning mathematics and how mathematical knowledge is interpreted within their mind (As'ari et al., 2017; Nurhidayah, 2020; Tohir, 2017). Therefore, individual with mathematical understanding will be able to use this ability to learn mathematics without any significant difficulties and can think mathematically where the way of thinking is very useful in everyday life (Afgani et al., 2019). Generalizing is widely acknowledged to be a critical component of mathematical activity (Ellis, 2011). Thus, the result of this study is important for educators in order to know and allow them to apply it to their students; to find out their expectations of the mathematical thinking process when their students solve math problems. The results of this research can also be applied and acted upon in other disciplines.

Conclusion

From the research results and discussion about expectations of prospective teachers for students' mathematical thinking processes, it can be concluded that; (1) generally, no significant difference was observed on the mathematical thinking process expected by the high and medium group subjects, however the significant differences was confirmed on the low group subjects; (2) the theory of mathematical thinking processes proposed by Mason, namely "*specializing*", "*generalizing*", "*conjecturing*", and "*convincing*" taken place on each stage of solving process based on the Polya model; (3) at the "*understanding the problem*" stage, the four mathematical thinking processes were well carried out, even though the mathematical thinking process of "*generalizing*" and "*conjecturing*" were mostly missing within the low-group subjects; (4) at the "*devising a plan*" stage, the four mathematical thinking processes were done perfectly; (5) at the "*carrying out the plan*" stage, the four mathematical thinking processes were done well by the subject, even though the mathematical thinking processes of "*generalizing*", "*generalizing*" and "*conjecturing*" were mostly disregarded by the low-group subjects; (6) at the stage of "*looking back*", the four mathematical thinking processes were not done well, but in the process of mathematical thinking within "*specializing*" processcould be carried well by the high group subjects; and (7) the development of prospective teacher's expectations toward students' mathematical thinking processes could be carried out by increasing their learning motivation, implementing the HOTS (*Higher Order Thinking Skills*) learning processes leading to support the improvement of mathematical thinking processes, as well as giving some questions HOTS-based and accordingly it could enhance their mathematical thinking skills in accordance with 21st century competencies.

Recommendations

Recommendations for prospective teachers, teachers, lecturers, and researchers, or other disciplines are expected to be able to carry out follow-up research on the results of this study because it still needs to be tried in experimental research. This research also needs to be tested on subject teacher forums in particular training to find out their students' mathematical thinking processes, either for regular teachers or national instructors. This is intended so that the mathematical thinking process expected by prospective teachers or teachers can be trained and developed continuously. In addition, educators or researchers should be able to apply HOTS learning that can support prospective teachers or improving their mathematical thinking skills. Then, teachers can be developed if the learning process in the classroom also supports the improvement of the thinking skills of prospective teachers, researchers, etc. So, in the learning process in the classroom, an educator should always provide structured and unstructured assignments as well as HOTS-oriented problem-solving. The HOTS learning is learning-oriented towards higher-order thinking skills, the role of the teacher does not explain much, on the contrary, the teacher stimulates many questions to encourage the release of students' original thoughts. Thus, students' high-order thinking processes will increase and develop by themselves along with mental activity in their metacognitive knowledge obtained in HOTS learning.

Limitations

This research is limited only to prospective teachers or educators to find out the expectations of their students' mathematical thinking processes, but it is not sufficient that the parents of students also have expectations of their children's thought processes when facing a certain problem. Therefore, the results of this research must be tested in all fields of science, technology, and social life.

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