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Arithmetic and algebraic problem-solving approaches of prospective teachers and teachers in service*

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Abstract

The aim of the study was to determine the arithmetic and algebraic problem-solving approaches of prospective mathematics teachers in the Department of Elementary Mathematics Education, and mathematics teachers in secondary schools. The study adopted mixed methods research design using exploratory sequential pattern. The participants of the study were composed of 128 prospective teachers (59 sophomores, 41 juniors and 28 seniors) in the Department of Elementary Mathematics Education in a state university and 22 secondary school mathematics teachers in different cities of Eastern Anatolia Region of Turkey. Clinical interviews were conducted with 12 prospective teachers and 5 teachers. The data were obtained via the Problem-solving Test [PST] and a semi-structured interview form [SSIF]. The PST was prepared to examine the problem-solving processes of the teachers and prospective teachers participating in the research. He findings of the study revealed that both groups of the participants preferred the algebraic approach while solving problems, and could not make use of the arithmetic approach. It was also understood that they had tendency to use algorithms they knew by heart while conducting algebraic solutions. As a consequence, these results showed that the participants had some deficiencies in terms of bringing both arithmetic and algebraic solutions to mathematical problems.

Keywords: Problem solving; arithmetic approach; algebraic approach; teachers; prospective teachers

1. Introduction

Individuals may encounter different problems at every stage of their lives. Today, providing individuals with the knowledge and skills to overcome these problems has become one of the main objectives of the curriculum. In this way, it is aimed that individuals overcome the problems they encounter in daily life (Doruk & Umay, 2011; Silk, Steinberg & Morris, 2003; Türnüklü & Yeşildere, 2014). Individuals who encounter problem situations are expected to make moves to solve the problem by approaching the problem systematically, based on the available data (Cai & Nie, 2007). Although the solution of problems faced by individuals throughout their lives is not equivalent to directly problem-solving in mathematics, some problem-solving skills aimed at acquiring

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in mathematics classes can help individuals overcome the problems they face in their lives (Albayrak, Şimşek & Yazıcı, 2018). For example, the education given to the individual to acquire the ability to solve mathematical problems can help him/her gain analytical thinking skills (Blum & Niss, 1991).

The development of problem-solving skill is related to problem-solving with different approaches (Yerushalmy, 2000). Two different approaches, arithmetic and algebraic, appear to occur in the solutions of verbal mathematics problems (Van Dooren, Verschaffel, Onghena, 2002). The ability to use these approaches differs according to the grade level in which students are studying. According to the Common Core State Standards of Mathematics [CCSS-M], there are preparatory activities for algebraic thinking from preschool to fifth grade, and students write, interpret and use equations for the first time in problem-solving (NGA, 2010). Similarly, there are five learning areas in the Secondary Mathematics Teaching Curriculum of Turkey (MEB, 2018): *Numbers and Operations*, *Algebra*, *Geometry and Measurement* and *Data Processing*. While the acquisitions for geometry and measurement, numbers and operations, and data processing learning areas take place in primary school, the introduction of algebraic expressions about algebra learning area is included in the secondary school curriculum starting from the sixth grade. In addition, acquisitions for first degree equations with one unknown are included in the algebra learning area from the seventh grade (MEB, 2018). Therefore, students lacking algebra knowledge before the sixth grade are expected to use arithmetic operations while solving problems, and they are expected to use this information in the problem-solving process after acquiring algebraic information. Starting to learn algebra, problem-solving is perceived to be largely equivalent to writing equations by the students (Altun, Memnun & Yazgan, 2007). Researches indicate that there is a complexity during the transition from arithmetic to algebra, and this period is called pre-algebra period (Akkan, 2009; Kaya, Keşan, İzgiol & Erkuş, 2016; Koedinger, Alibali & Nathan, 2008; Van Amerom, 2002; Yıldızhan Şengül, 2017) . In this context, it is expected that the way students think in the problem-solving process will also change (Akkan, Baki & Çakıroğlu, 2012). In addition, the researches (Akkan, 2009; Akkan *et al.,* 2012; Gürbüz & Toprak, 2014; Van Amerom, 2002) show that the transition of students from arithmetic to algebra is carried out in a longer period. In order to overcome the problems related to the long process of transition from arithmetic to algebra, mathematics teachers and prospective teachers must have knowledge and skills about how they can help students solve problems with arithmetic and algebraic approach. All of these knowledge and skills are included in the subject area knowledge and pedagogical content knowledge. According to Shulman (1986), there is an interaction between pedagogical knowledge and subject area knowledge and teachers' subject area knowledge plays an important role in teaching. Therefore, mathematics teachers and prospective teachers may need to be able to successfully solve mathematical problems arithmetically and algebraically. In spite of this, in the studies (Nathan & Koedinger, 2000; Schmidt &

Bednarz, 2002; Şener, Ünal & Bulut, 2017), the problems faced by teachers and prospective teachers while solving math problems with an arithmetic and algebraic approach were determined. Some of them are not being able to connect arithmetic and algebra, seeing the arithmetic solution unnecessary and not being able to find the arithmetic or algebraic solution.

If the definitions of arithmetic and algebraic solutions are examined, in arithmetic solutions students try to solve the problem by using one or more of the mathematical operations with the given data to reach the unknown (Kieran, 1992; Linchevski & Herscovics, 1996). In algebraic solutions, students can reach the solution of the problem by writing an equation with symbols representing the unknowns as a result of the solution of the equation they obtained (Reed, 1999). Let's consider the following problem; *there are only yellow and red beads in a box. The number of yellow beads is 3 more than red beads and the total number of beads is 21. So how many yellow beads are there in this box?* Let's investigate the following two solutions. Solution 1: *If the number of yellow and red beads were equal, that is, if the number of yellow beads was not 3 more than red beads, there would be a total of 18 beads. 9 of them would be yellow and 9 of them would be red. Since the number of yellow beads is more than 3, there must be 12 yellow beads.* Since only four operations and numbers are used in this solution, the solution was made with an arithmetic approach. Solution 2: *Let red beads be x, yellow beads $x + 3$. In this case, $x + x + 3 = 21$ equation is obtained, and $x = 9$. Since the yellow beads are $x + 3$ there are 12 yellow beads.* Those given in the solution are expressed as variable and the solution is reached by establishing an equation. Therefore, this solution was done with an algebraic approach.

In order for students to solve mathematical problems with an arithmetic and algebraic approach, mathematics teachers whose guidance is needed on this subject should also have the ability to solve problems with both arithmetic and algebraic approach (Schmidt & Bednarz, 2002). On the other hand, Van Amerom (2002) stated that individuals forget the arithmetic thinking when they achieve algebraic thinking in mathematics learning process. Therefore, it is possible that teachers' and prospective teachers' deficiencies related to arithmetic thinking may occur with the development of their algebraic thinking. So, the fact that the development of algebraic thinking causes forgetting of arithmetic thinking is a situation that contradicts the need for teachers to be able to solve problems both arithmetically and algebraically. In this research, this created a rationale for examining prospective teachers' arithmetic or algebraic problem-solving processes.

In their research on high school students, Koedinger et al. (2008) stated that they obtained evidence that even high school students who are expected to have completed the transition to algebra theoretically has not been fully achieved. According to the results of the research, it can be said that the transition from arithmetic to algebra was not completed in a short time as expected; this process was quite slow and took a long time. For this reason, it is thought that mathematics teachers should have the ability to solve mathematical problems with both arithmetic and algebraic approaches in order to provide the necessary guidance in the process of mathematical problem-solving in order to enable students to move from arithmetic to algebra. During the transition from arithmetic to algebra, teachers and prospective teachers should be able to choose the problems that can maintain the relationship between arithmetic and algebra, have knowledge about the connections between arithmetic and algebra, and be aware of the differences between arithmetic and algebraic thinking (Schmidt & Bednarz, 2002). This research can contribute to the literature by identifying and improving the current knowledge and experience of teachers and prospective teachers on this subject.

For this purpose, it was aimed to determine mathematical problem-solving approaches of elementary school prospective mathematics teachers and elementary school mathematics teachers. Accordingly, the problem of the research is shaped around the question *"What are the arithmetic and algebraic problem-solving approaches of prospective mathematics teachers and teachers in service?"*

2. Method

The study was based on the mixed methods, and was carried out using the exploratory sequential pattern. In the study, the arithmetic or algebraic approaches of prospective teachers and teachers were examined in relation with the situations related to the variety of approaches with the sequential explanatory pattern. In the research, the exploratory pattern was used to determine and compare the situations in terms of the variables via the quantitative and the qualitative aspects.

The participants consisted of 128 prospective teachers (59 sophomores, 41 juniors and 28 seniors) in the Department of Elementary Mathematics Education in a state university located in the Eastern Anatolian Region of Turkey, and 22 secondary school mathematics teachers working in different cities of Eastern Anatolia Region at the same period. In addition, for clinical interviews, studies were conducted with the sub-group of the participants with the maximum diversity method for the qualitative part of the research. Accordingly, clinical interviews were conducted with 12 prospective teachers and 5 teachers. In the maximum diversity sampling method, the aim is to increase the diversity of individuals who are parties to the problem (Grbich, 2012; Yıldırım & Şimşek, 2008). In this way, it was aimed to provide the highest level of diversity according to the arithmetic or algebraic approaches of the participants.

The data were obtained by Problem-solving Test [PST] and Semi-Structured Interview Form [SSIF]. The PST was prepared to examine the problem-solving processes of the teachers and prospective teachers participating in the research. The PST, consisting of 8 open-ended questions, was developed by the researcher conducting the research. In the development phase, it was paid attention to the fact that the problems are suitable for algebraic and arithmetic solution in order to provide measurement in accordance with the purpose of the research. Except for the researcher, two experts checked whether each problem could be solved by both arithmetic and algebraic approaches and confirmed that they could be solved by both approaches. SSIF was prepared to be used in clinical interviews as a qualitative data collection tool of the research. Thanks to clinical interviews, it was possible to collect data for the algebraic approach from the participants who made arithmetic solutions, and from those who made algebraic solutions in a similar way. During the clinical interviews, teachers and prospective teachers were asked some problems and asked to solve these problems, and meanwhile, questions were asked in order to determine their views on the research, while making directions. In the selection of these problems, the arithmetic-algebraic approaches that are the focus of the research are taken into consideration. For example, if a participant always made an algebraic solution in the PST, one of these problems in the PST was asked again and the participant was asked to solve this problem arithmetically. In other words, when the participant selected for the interview was asked for more detailed information about a solution in the PST or the solution that may arise when guidance was made in that problem, the problem in the PST was asked again and the interview continued through this problem. On the other hand, if the data obtained from the participant in the PST was to be enriched with a clinical interview, then the interview continued on the new problem. In the clinical interviews, some of the interview questions asked to the participants varied according to the situation of the participants, which were formed according to the maximum variation criteria. For participants who solved a problem only with an arithmetic or only algebraic approach, whether they can solve the problem with the other approach, whether there can be another solution or not, can you solve this problem in another way? Questions such as “If it is solved, how is it solved?” were asked. For both arithmetically and algebraically solved problems, the participants were asked to explain how they compared these solutions with questions such as "Why did you use different approaches?", "What differences did you consider while solving the problems with these approaches?" In addition, the participants were asked to verbally explain the solution and explain the sources of some of the operations they performed in order to determine what they thought about the solution. While the data is being digitized, the value of 1 was entered to the categorical variable of arithmetic-algebraic approach to problems for each solution of each participant if an algebraic solution was made, the value of 2 was entered if arithmetic solution was made, and in one question, the value of 3 was entered if both arithmetic and algebraic solution was made. In the arithmetic-algebraic analysis of the problem solutions, the Chi-square test was carried out, and it was investigated whether there was a statistically significant difference in the arithmetic-algebraic approaches of teachers and prospective teachers studying in different grades. By applying the chi-square test to these groups in pairs, it was tried to determine which two groups had this difference (if any). Since the themes and categories of the research also included the items of the quantitative data, quantitative and qualitative data were presented together, and qualitative data were supported with quantitative data. During the reporting of the data, information such as school, participant name, etc. were hidden and the data were presented by assigning codes to the participants. In this coding, 'T' is used for teachers and 'PT' for prospective teachers. The number indicating the grade was added before the abbreviation of 'PT' in order to indicate which grade the prospective teachers were. The number after 'T' and 'PT' indicates the sequence number of the participant. Accordingly, the participant coded as T12 refers to the person ranked at 12 among the teachers from which the data was obtained. The code 3PT9 indicates the prospective teacher ranked at 9 from which the data was obtained and and his grade as junior (3rd grade).

3. Findings

First of all, it would be appropriate to examine how many of the problems were solved by the prospective teachers and teachers participating in the research. Therefore, respondents' status of answering the problems in the PST is indicated below.

Table 1. *Participants' Problem-solving Rates*

	The Number of Problems Answered		The Number of Problems Left Blank	
	f	%	f	%
Sophomore A.	410	86.9	62	13.1
Junior A.	293	89.3	35	10.7
Senior A.	235	94.8	13	5.2
Teachers	174	98.9	2	1.1
Total	1112	90.8	112	9.2

According to Table 1, when the response rate of the participants is examined, it is seen that 9.2% of the problems are not answered. It has been determined that the rate of leaving the problems blank is the highest (13.1%) in the sophomore prospective teachers and the lowest in the teachers (1.1%). In addition, it is observed that the rate of problems left blank decreases with the increase in the grade level. In general, it can be said that the majority of the participants solved the problems.

In order to reveal the statistical distribution of the arithmetic and algebraic approaches of the participants, the distribution of algebraic, arithmetic or both ways of solving the problems in the PST is given in Table 2.

Table 2. *Arithmetic-Algebraic Examination of Participants' Approaches to Problems*

	Algebraic		Arithmetic		Algebraic and Arithmetic	
	f	%	f	%	f	%
Sophomore A.	285	69.5	55	13.4	70	17.1
Junior A.	190	64.9	37	12.6	66	22.5
Senior A.	135	57.5	39	16.6	61	25.9
Teachers	84	48.3	23	13.2	67	38.5
Total	694	62.5	154	13.8	264	23.7

According to Table 2, it can be said that 62.5% of the teacher and prospective teachers' problem solutions in PST were carried out with an algebraic approach. With this finding, it can be said that teachers and prospective teachers prefer solving questions with the help of algebra in problem-solving. Moving from the sophomore prospective teachers towards the teachers, algebraic solutions decrease, while conducting arithmetic and algebraic solutions together increases. In other words, undergraduate education and teaching contribute to solving problems both arithmetically and algebraically. In order to determine the statistical significance of prospective teachers and teachers at different grade levels in terms of arithmetic-algebraic approach to problems, Chi-square test results were calculated. According to the chi-square test results, it is understood that there is a significant difference between the groups participating in the research in terms of their arithmetic-algebraic approach to problems (sd [degree of freedom] = 6, p = .000 <.05). Accordingly, arithmetic-algebraic approaches of prospective teachers and teachers at different grade levels differentiate. In order to examine this situation in detail, the results of the double chi-square test of the groups are given in Table 3.

Table 3. *One-to-One Comparison of Difference Between Groups in Arithmetic-Algebraic Analysis of Participants' Approaches to Problems*

	Junior A.		Senior A.		Teachers	
	χ^2	p	χ^2	p	χ^2	p
Sophomore A.	3.18	0.204	10.18	0.006	32.64	0.000
Junior A.			3.30	0.189	15.13	0.001
Senior A.					7.35	0.025

When Table 3 is analyzed, it is seen that there is no significant difference between sophomore pre -service teachers and junior pre -service teachers on arithmetic-algebraic approaches to problems (sd = 2, p = .204> .05). In the comparisons of sophomore prospective teachers with senior prospective teachers (sd = 2, p = .006 <.05) and teachers (sd = 2, p = .000 <.05), a significant difference was found between the participants' arithmetic-algebraic approaches to problems in favor of senior prospective teachers and teachers. There is no significant difference between junior prospective teachers and senior prospective teachers in terms of arithmetic-algebraic approaches to problems (sd = 2, p = .189> .05). However, there is a significant difference between junior prospective teachers' and teachers' arithmetic-algebraic approaches to problems in favor of teachers (sd = 2, p = .001 <.05). According to this, although there is no difference in terms of arithmetic-algebraic approaches to problems when the grade level difference is one during undergraduate education, when the difference in education level reaches two years, there is a significant difference between the education levels in terms of arithmetic-algebraic approaches to the problems. In addition, a significant difference was found between the teachers and prospective teachers at all grade levels in terms of their arithmetic-algebraic approach to problems.

It was observed that while some of the participants (f = 11) solved the problem with the strategy of setting an algebraic equation when they could easily solve the problem., they tended towards an arithmetic approach in problems that they couldn't easily solve. The solutions to this situation for the first, fourth and sixth problems of the PST given in Annex-2 of T20 are given in figures 1, 2 and 3.

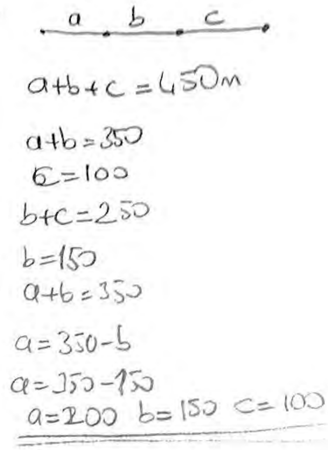

Figure 1. The solution made by the T20 participant to the first problem of PST.

In the solution given in Figure 1, T20 aimed to find the lengths of the parts of an athletics track. T20 set up an equation by assigning variables to the parts of the track and solved the equation and reached the correct answer to the problem. The fact that variables were used in the solution by T20 shows that algebraic approach is used in problem-solving.

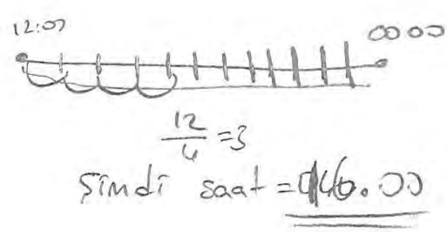

Figure 2. The solution made by the T20 participant to the fourth problem of the PST.

In Figure 2, the solution made by T20 to the problem in which the ratio of time remaining from afternoon to night is given. It seems that T20 tried to solve the problem with the help of numbers by using the help of understanding the problem from the drawing, but did not succeed because he made a mistake while making the operation. In this case, it can be said that he made the solution with an arithmetic approach since he used the numbers.

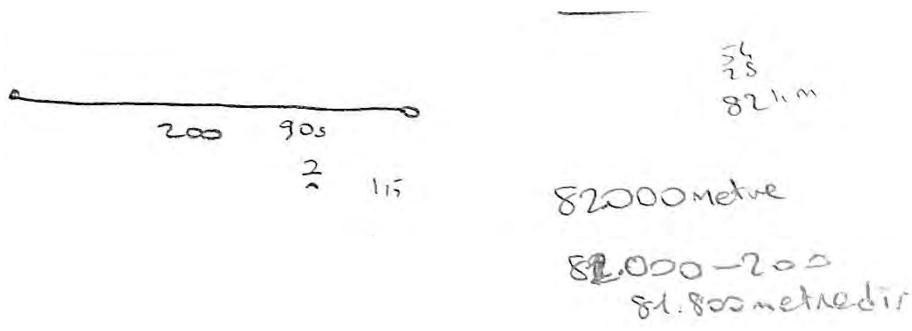

Figure 3. T20 participant's solution to the sixth problem of the PST.

In Figure 3, the solution for the problem in which T20 tries to find the length of the tunnel through which a train passes is given. In the solution, it is seen that he made the operation with number by using the drawing to understand, but did not get the correct result. Trying to do the solution with the help of numbers and four operations shows that he tried to solve the problem with an arithmetic approach. It is seen that T20 successfully solved the first problem of the PST with an algebraic approach by establishing an equation, and made an arithmetic attempt to solve the fourth and sixth problems, but was not successful. Therefore, T20 made an algebraic solution when he was able to solve the problem, and if he did not, he tried to solve the problem with an arithmetic approach, but failed. Similarly, part of the conversation about the comparison of problem solutions with the junior prospective teacher coded 3PT18 is as follows;

A: *When we examine your solutions in general, it is seen that you have tried different approaches. For example, you solved this first problem by establishing an equation.*

3PT18: *Yes, it was given the problem; I made an equation from them.*

A: *So, how did you think of the third problem (the problem that he tried to do arithmetic solution and failed)?*

3PT18: *I compared the time they went with the time needed to close the distance between the child and his brother. I tried to figure out how the time they went provided getting off the track and catching up.*

A: *You didn't use an equation here.*

3PT18: *I thought an equation but did not occur to my me.*

A: *We call algebraic solutions to solutions you use equations or variables, and arithmetic solutions to solutions you use numbers. You made arithmetic in some solutions and algebraic in others. What does this depend on, when do you prefer arithmetic and when algebraic approach?*

3PT18: *I read the problem, set up the equation, if not, I think about other things.*

In the above interview, it is seen that 3PT18 tried to solve the problems with an algebraic approach first, and if he couldn't, he looked for different solutions. The fact that T20 solves the problems algebraically and tries to solve the problems with the arithmetic solution approach supports this situation. These findings show that some of the teachers and prospective teachers are conditioned to algebraic approach in problem-solving and consider problem-solving as a routine with an algebraic approach. In addition, it is seen that when the routine doesn't work, they tend to think and arithmetic solutions. In addition, in clinical interviews, it was observed that approximately 80% of the participants made experiments to establish an equation right after reading the problem. These attempts are generally to make arrangements with symbols in line with the numbers in the problem. The interview that examines the solution made by establishing an equation by the sophomore prospective teacher coded 2PT35 showing this situation was as follows:

A: *Can you explain how you made the solution?*

2PT35: *I handled the year it asked us as x. So I set up an equation and found x.*

A: *Can you solve this problem in another way?*

2PT35: *(After thinking a little) It doesn't occur to me.*

A: *Can it be solved without using letters like x, y?*

2PT35: *(After thinking a bit) No, I couldn't find.*

A: *For example, imagine that you have solved a problem from the beginning. What do you think first after reading the question?*

2PT35: *While reading the question, I think about how to set up the equation. I try to take something as x and create an equation.*

A: *Can't there be a solution other than the equation?*

2PT35: *Maybe sometimes, but always equation.*

A: *How so?*

2PT35: *I mean, every problem can be solved by an equation.*

During the meeting with 2PT35, upon the tendency to establish an equation immediately after reading the problem for the third problem of the PST given in Annex-1, he was asked "What would you think of doing first after reading the question?" The answer of 2PT35 as *I think how to set up the equation* shows that prospective teachers try to solve the problem with algebraic approach by establishing an equation directly after reading the problem. In addition, when 2PT35 explains how he solved the problem in the interview, the fact that first he wrote the x variable and searched for the solution through this variable shows that he made adjustments in accordance with the numbers in the problem with symbols.

Considering that it would be a preference for teachers and prospective teachers to be able to solve the problem algebraically or arithmetically, the participants were asked to solve a problem in as many ways as possible in PST. If the participants comply with the instructions (Solve the problems with different solutions as much as you can), there is no preference because they have to do all the solutions they can do. An interview was made with the sophomore prospective teacher coded as 2PT30 to control the participants' ability to make solutions other than the solutions they made in the PST. After the prospective teacher solved the problem in which the number of chickens and sheep was asked by giving the total number of animals and feet, the following interview was held to examine the solution;

A: *Can you explain how you solved the problem?*

2PT30: *I took the number of chickens as x, and the number of lamb became 12-x. I established this equation (by showing the equation ((2.x + 4 (12-x) = 36)) and found x.*

A: *Can you solve this problem in another way?*

2PT30: *(After thinking a little and trying to set up another equation, he scribbles) I don't think there is another solution.*

A: *For example, cannot it be solved without using letters like x, y?*

2PT30: *There is an unknown here and if there is an unknown, it cannot be solved without establishing an equation, so I take the unknown as x.*

A: *Doesn't every problem ask something, then no problem can be solved without equation?*

2PT30: *Isn't the name of this subject in high school equation problems?*

A: *Where is it written?*

2PT30: *The title of the problems are equation establishing problems, these are number problems, age problems, motion problems, etc.*

A: *I did not know that equation was written in the title.*

After thinking a few seconds, 2PT30 responded that *he didn't think there was no other solution to the problem* to the question whether he can solve the problem without establishing an equation and explained the reason as *there is an unknown here and if there is an unknown, it cannot be solved without establishing an equation, so I take the unknown as x,* which shows that he thinks that the problems can be solved algebraically only. It is also understood from the interview that he could solve the problem in only one way, that is, it was not an option to solve the problem in one way.

In the clinical interview, the question "What similarities or differences do you see when comparing these two solutions?" was asked to the participants who managed to solve the problems in the PST with both an algebraic and arithmetic approach The relevant part of the interview with the teacher T3 is as follows;

A: *In some of the solutions to the problems you used variables and set up equations, in others you solved it only by numbers. We call algebraic solutions to solutions you use equations or variables, and arithmetic solutions to solutions you use numbers. What makes you decide whether an arithmetic or algebraic solution?*

T3: *I normally solve with equation. I tried others because it was reported in the statement to use different ways.*

A: *Why do you solve with equation?*

T3: *Because the equation is the easiest.*

A: *Are others difficult?*

T3: *Yes, others are not like equations. For example, it is necessary to write the following question at length (by showing the solution with the strategy of making a list for the third problem of the PST given in Annex-1).*

A: *So would you compare the algebraic solution to the arithmetic solution?*

T3: *The equation is the short way, the other way is the long solution for those who cannot solve the problem (indicating the students who will not solve the problem by setting up an equation).*

A: *Won't it have any other advantage?*

T3: *The purpose in solving a question is to solve it quickly.*

It is seen in the interview that T3 perceives the equation as a short and easy way when comparing the arithmetic and algebraic approach. Similarly, the following interview was held on the problem of arithmetic and algebraic solution with the sophomore prospective teacher coded as 2PT22.

A: *You have made two different solutions to the problem* (his solution for the 1st problem of the PST given in Annex-1 is coded as A+C-2-Equation, Drawing) *Would you compare these solutions?*

2PT22: *My first solution is equation, the other is drawing.*

A: *Which one would you do if you were to make one solution?*

2PT22: *The equation*

A: *Why?*

2PT22: *I normally solve the problem with an equation, as you said another solution, I thought about another way by looking at the equation after solving it with the equation.*

A: *Did you make the drawing solution from the equation?*

2PT22: *Yes, the figure explains how the equation occurs, the equation gives the exact solution.*

A: *Wouldn't it be solved with drawing without establishing an equation?*

2PT22: *I don't know, I couldn't find it, maybe it is solved.*

It was understood from the interview with 2PT22 that he first solved the problem by establishing an equation and then derived a new solution through this solution.

Similarly, the following conversation was held with the second grade prospective teacher coded as 2PT12 on the problem that he made an arithmetic and algebraic solution.

A: *Would you compare the solutions you made in the problem (coded as A + C-2-Equation, Reasoning)?*

2PT12: *I solved this with an equation (showing the solution made with the strategy of* *establishing an equation) and the other one without an equation.*

A: *Which one would you choose if you were going to do one?*

2PT12: *I'd choose the equation.*

A: *Why?*

2PT12: *Problems are always solved by equation.*

A: *Why, what are the pros of the equation?*

2PT12: *There's little chance to make a mistake in an equation, in the other solution you can always make a mistake.*

A: *Can the other solution have no other advantage?*

2PT12: *No… I think the equation is easy.*

In the interview with 2PT12, it is seen that the prospective teacher perceives the algebraic solution as advantageous in terms of preventing making mistakes and being easier. The interviews show that the participants who solved the problem with both arithmetic and algebraic approaches consider the algebraic approach as a more precise, easy and essential solution, while considering the arithmetic solution as longer and suitable for making mistakes.

4. Results, Discussion and Recommendations

According to the findings of the research, it was observed that the participants preferred the algebraic approach while solving the problems and could not solve the problem with an arithmetic approach. It is also understood that they tend to use algorithms they know by heart while making algebraic solutions. These results show that the participants have some deficiencies in terms of bringing both arithmetic and algebraic solutions to the problems. There are studies in the literature stating that teachers and prospective teachers have deficiencies related to arithmetic and algebraic approach to problems (Schmidt & Bednarz, 2002; Siswono et al., 2017; Turgut & Doğan Temur, 2017). In these studies, it was stated that teachers and prospective teachers are inadequate in solving problems arithmetically and algebraically. In this study, the fact that the participants to tend to the algebraic solution and their inability to make arithmetic solutions is in parallel with the literature. For an effective mathematics teaching, the teachers and prospective teachers should be able to solve problems both arithmetically and algebraically, and this directly affects student success (Akkan, 2009; Turgut & Doğan Temur, 2017; Van Amerom, 2002). In the study, it was determined that there was a difference between the sophomores and the seniors in terms of arithmetic and algebraic approach to problems. It can be understood that as the education level of prospective teachers increases, their success of solving problems both arithmetically and algebraically increases. Such results indicate that undergraduate education activities have a positive effect on the problem-solving approaches of the prospective teachers. It was also determined that the teachers were more successful than the prospective teachers in solving problems with both arithmetic and algebraic approaches. Considering these results, it can be said that teaching experience and knowledge of students are important factors on the variety of problem-solving approaches. Knowing the student means that the teacher knows the readiness, deficiencies and misconceptions of the students (Ball, Thames & Phelps, 2008). Driel and Berry (2010) stated that prospective teachers' knowledge on students is generally lacking and this is eliminated as they teach. Therefore, it is probable that the teachers' deficiencies in combining arithmetic and algebra in problem-solving arise from their lacking knowledge about the students. Besides, considering the current situation of teachers, it was determined that they have some deficiencies regarding arithmetic and algebraic approaches. These deficiencies are that they prefer algebraic solution in problem solutions, find the arithmetic solution useless and fail in arithmetic solutions.

The researches show that secondary school students have difficulties in transition from arithmetic to algebra (Akkan, 2009; Akkan *et al.,* 2012; Gürbüz & Toprak, 2014; Van Amerom, 2002), and these difficulties can continue even in high school period (Koedinger *et al.,* 2008). It is also known that teachers need to master arithmetic and algebraic solutions in order to manage this process well (Yıldızhan & Şengül, 2017). The fact that teachers' deficiencies in arithmetic and algebraic solutions have been identified may be an explanation for the disruptions in students' transition from arithmetic to algebra. In addition, Koedinger *et al.* (2008) 's conclusion that high school students' deficiencies related to transition from arithmetic to algebra continues reveals the possibility that this transition may not have been achieved in the future. Therefore, according to the results of this research, it can be said that these shortcomings of prospective teachers may have continued and decreased gradually throughout the undergraduate education.

In this study, it was determined that prospective teachers had deficiencies in solving problems both arithmetically and algebraically. Demonstrating both arithmetic and algebraic solutions of the problems to prospective teachers in solving problems in elective or compulsory courses for teaching problem-solving and asking the prospective teachers to try solutions in this direction can improve their problem-solving skills so as to master both approaches.

In this research, the problem-solving processes of teachers and prospective teachers were examined, and deficiencies were revealed. As a continuation of the quantitative dimension of the research, a broader research can be carried out with teachers and prospective teachers from different regions and cities to make the findings more generalizable. Thus, it may enable the results found in this study to be compared with the situation of teachers and prospective teachers in Turkey.

In addition, given that teachers are more successful than prospective teachers, by doing this research only for teachers, examining the problem-solving processes in terms of experience may be useful in explaining the difference in success.

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