

Teachers' understanding about the characteristics of realistic mathematics education

Cut Morina Zubainur¹, Rahmah Johar², Rahmi Hayati³, M. Ikhsan⁴

^{1,2,4}Mathematic Education Department, Universitas Syiah Kuala, Indonesia

³Mathematic Education Department, Universitas Almuslim, Indonesia

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ABSTRACT

The application of Realistic mathematics education (RME) in schools has not satisfied. The efforts of educators to help teachers implement RME are an ongoing process, including by providing examples of learning process through the video lessons. This study aims to investigate teacher understanding of the characteristics of RME through watching video lessons. Three elementary school teachers in Aceh, Indonesia, were involved in this research. Data was collected through questionnaires and semi-structured interviews. Data were analyzed descriptively. The results showed that the teacher did not understand three of the five characteristics of RME. Teachers' understanding of real problems is only limited to the use of concrete objects. The teacher could not identify activities on the video about using models or symbols to scaffold students from concrete to more formal knowledge. The teacher also did not realize that the relationship between mathematical concepts can help students understand mathematics well. The implication of this study is that researchers need to guide the teacher design the learning trajectory before watching learning from other teachers. After watching the whole video lessons, the teacher also needs to be moved to identify important events related to the characteristics of RME in the learning in the video.

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Corresponding Author:

Rahmah Johar

Mathematic Education Department,

Universitas Syiah Kuala,

Darussalam, Banda Aceh, 23111, Indonesia.

Email: rahmah.johar@unsyiah.ac.id

1. INTRODUCTION

For many years, many students in Indonesia do mathematical procedures or algorithms without the understanding of "why they fix to solve the mathematics problem" [1-3]. Contextual problems, which are considered as the starting point to motivate students to solve mathematics problems creatively, are rarely found in the Indonesian mathematics textbooks [4-6]. Although there are textbooks present words problems, however these problems are fictitious or meaningless due to they do not correspond to the students' daily life or the student's imaginations [7]. This situation has an impact on teaching that directs students to procedural knowledge rather than conceptual knowledge so that students do not understand mathematics well [8]. Teachers need assist to design lessons in order to overcome their students' problem. One of solutions is to implement realistic mathematics education (RME).

Treffers [9] explained that there are five characteristics of RME; (i) using a real life problem for starting a lesson, (ii) using models or symbols to the problem in order to bridge a concrete level to a more formal level, (iii) encouraging students to solve the real life problems by themselves, (iv) facilitating students

to interact, share and construct the solution of the problems with their peers during learning process, and (v) intertwining mathematical concepts in order to strengthen all subjects of mathematics are interrelated. The adaptation of realistic mathematics education (RME) for teaching mathematics in Indonesia have started since 2000 [10, 11]. This means that the ideas and the characteristics of RME are being adopted and adapted into the Indonesian context, later on the Indonesian people approved RME as *Pendidikan Matematika Realistik Indonesia* (PMRI) [11].

Currently, there are 20 teacher education programmes and research centres in the Indonesian universities that have been developed in order to assist teachers in implementing RME in schools. In particular, we call the local research centre of RME in Indonesia as *Pusat Penelitian dan Pengembangan Pendidikan Matematika Realistik Indonesia* (P4MRI). The members of P4MRI team, which consists of teacher and lecturers, have already conducted many activities, including conducting workshops related to RME for teachers and pre-service teachers, doing collaboration research with teachers and pre-service teachers about the implementation of RME, creating video lessons, and developing mathematics textbooks [12, 13, 10]. Both video clips and whole video lessons are very important for teachers to understand the implementation of RME in real classes. Therefore, during the workshops of RME conducted by either P4MRI of Syiah Kuala University or P4MRI in other universities in Indonesia, many teachers have admitted that they want to watch video lessons about the implementations of RME in real classes. The implementation of RME in class reflects the characteristics of RME. This study discusses teachers' understanding of the characteristics of RME from watching the provided video lessons.

Teachers' understanding of the characteristics of RME in Indonesia is conducted through teacher professional development workshops. The standards of the workshops are (i) activities in a workshop are process-oriented which can support the participants to understand PMRI ideas and are product-oriented aiming at providing materials that can be used in school, (ii) a workshop facilitates participants to experience the PMRI characteristics themselves in order to build their knowledge and skills, (iii) contents of a workshop are in line with curriculum demand and the internal and external condition of schools, and it envisions an ideal situation in order to enhance the adaptability of PMRI in schools, (iv) during a workshop, participants reflect on the relation between the activities, mathematical concepts, and PMRI theories, (v) a workshop empowers and builds the confidence of the participants to sustain the implementation of PMRI in schools [14].

Using videos in teacher professional development has been widely implemented in recent years. Video lessons benefits teachers in developing 'learning how to learn in classrooms to implement particular innovation of teaching', resulting in they became autonomous teachers [15]. The analysis of the videos is conducted gradually from a focus on the teacher's activities to the attention of the students' thinking [16]. Teachers have different interpretations while watching classroom videos [17]. To date, there is limited study investigating teachers' understanding about RME through watching the video lessons. The research question of this study is how teachers' understanding about the characteristics of RME through watching the video lesson.

2. RESEARCH METHOD

This study is a part of a larger project in investigating teachers' concern using video to implement RME in Primary Schools in Banda Aceh and Aceh Besar, Indonesia [18]. The first and the second authors, who are the members of P4MRI team of Syiah Kuala University, collaborate with primary school teachers to design lesson plans, validate the materials to experts of RME in Indonesia, record video of the implementations of RME in some schools, observe the lessons, and revise all of materials and videos. There are some topics that have been the concerns; (1) teaching decimal number, (2) spatial, and (3) circumference and area of circle. This study uses video of the implementation RME in teaching the elements of a circle and pi value for the duration of 58 minutes, which is the introduction to circumference concepts at the fifth grade. In this study, the video are analyzed by three primary school teachers (T1, T2, and T3), who were selected because they joined the preparation workshop for these topics followed by 13 teachers and are willing to use video as the guidelines for teaching in their classes. Teacher workshop in this study was conducted based on RME teacher workshop standards. The teacher professional development workshop in this study on the use of video lesson was in accordance with PMRI teacher workshop standards [19].

All of teachers are reminded about five characteristics of RME during preparation workshop. T1 and T2 have implemented RME in their classes for some topics since 2009 and T3 has implemented it since 2013. T1 has 17 years mathematics teaching experiences in the fifth and sixth grades. Meanwhile, T2 has been teaching for 24 years at the third grade and a year at the fifth grade, and T3 has been teaching for 8 years at the fourth and fifth grades.

The questionnaire for analyzing the video lessons consists of four items including; (1) assessment, (2) characteristics of realistic mathematics education, (3) standard of professional teacher, and (4) character building. Only item (2) is being discussed here. Teachers are asked to provide their comments on the connection between the learning process and the characteristics of RME. After completing questionnaire, one of the authors interviewed the teachers individually to obtain a more comprehensive information. The interviews are video recorded. The interviews was then transcribed to examine teachers' understanding about characteristics of RME. The analysis was conducted by coding teachers' comments for each characteristic of RME followed by seeking connection between teachers' comments.

3. RESULTS AND DISCUSSION

As it has been disclosed previously, the purpose of this study was to examine the teachers' understanding of the characteristics of RME from video lessons. The following description explains the understanding of the five characteristics obtained from the questionnaires and interviews.

3.1. The first characteristic: use of real life problem

All three teachers have similar views concerning the use of real life problems. T1 suggests that the characteristic of real life is fulfilled when the teacher in the video introducing elements of a circle, with the circular objects found around students' life. T1 thought that teachers have taught the students the mathematical concepts that are real and meaningful. The T2 and T3 also expressed similar ideas, that the characteristic of the real problems is met from learning process on the video when the teacher introduces the elements of a circle using objects known by the students. In addition, according to T2, using the objects that are familiar to the students could help students in identifying the elements of circle such as the diameter, radius, and circumference realistically. Here's one of an interview to T2.

R : What do you know about realistic?

T2 : Students discover when measuring, using manipulative from the students themselves

R : Yeah, so what does realistic mean? "

T2: Teacher informs only the basic concept, then the students find himself [*the measure of circumference and diameter*] with a variety of tools, such as: tape measure, ruler, and ribbon

In the video, teacher introduced the circumference and radius by demonstrate the miniature of bicycle wheel. Students identified the circumference and radius well because the wheels are familiar in students' daily life [20]. Teacher challenged student to guess which one the centre point of the circle and the diameter are. There is one student came to in front of the class to draw them on board. Then teacher asked the students to think about 'Tailor Problem', which is to find the measurement of ribbon to cover the big circular fan, without measuring the circumference of the fan. To make clear the connection of the diameter and the circumference, teacher demonstrated how to measure the circumference of cover of stopples and its diameter then asked students to predict the connection between them. To find exact ratio as pi number, teacher asked each group of students to measure length of circumference and diameter of some circular objects. They used tape measure, ruler, and ribbon to do it. So, real life problem in the video was finding some elements of a circle, constructing their meaning, and solving Tailor Problem to find the length of the circumference without measuring the circumference of the fan. These activities are common-sense for students. Real in RME as known as common-sense [21].

3.2. The second characteristic: using models or symbols to the problem in order to bridge a concrete level to a more formal level

The second characteristic of realistic approach is translated by three teachers as 'students find their own concepts of mathematics'. According to T1, the use of objects around the students helped the teacher introduces the elements of the circle. Students get opportunity to construct their own understanding. Teachers motivate students to discover mathematics concepts through the activity. This is in line with the opinion of T2, namely, students find themselves without mathematical concepts explained by the teacher. Students discover mathematical concepts through group activities. The activity encourages students to strengthen their understanding. T3 also see the teacher in the video provides the opportunities for students to solve their own problems.

In the video, students use symbols when they draw the centre point of the circle and its diameter. Then teacher guided students define the meaning of the diameter in relation to the radius. Teacher provided an activity as a bridge for students' informal model to more formal level. The activity was asking each group of the students calculate the ratio of the circumference and the diameter from some circular objects. All of 5 groups at class wrote their ratio in table. Three of five groups results as seen on Figure 1.

Ibrahim			
No	K	d	$\frac{K}{d}$
1	24 cm	8 cm	3,22 cm
2	36,4 cm	11 cm	3,33 cm
3	49,5 cm	16 cm	3,3 cm

Fathimah			
No	K	d	$\frac{K}{d}$
1	33 cm	12 cm	3,25 cm
2	19 cm	3,9 cm	10 cm
3			

Zainab			
No	K	d	$\frac{K}{d}$
1	66 cm	21 cm	3,14 cm
2	29,4 cm	7,7 cm	3,18 cm
3			

Figure 1. Student groups' answer about the ratio of K/d

Teacher asked students to check their measurements and make conclusion of the ratio of K/d. With teacher's guidance, students reached the formal level of K/d is 3,14, known as the pi value.

3.3. The third characteristic: encouraging students to solve the real life problems by themselves

The third characteristic is identified by each teacher differently. T1 explained that the teacher in the video provides the opportunity for students to construct their own answers to problems. According to T2, the teacher in the video provides the opportunity for students to freely carry out activities to measure circular objects and therefore students look creative. Students are not afraid of making mistakes because teachers do not blame the students. If students make a mistake in measuring, the teacher asks the students to be more careful and cautious. Meanwhile, according to T3, the teacher in the video provides the opportunity for students to complete a given problem using their own way.

3.4. The fourth characteristic: facilitating students to interact, share and construct the solution of the problems with their peers during learning process

In this fourth characteristic, T1 expression is slightly different from the T2 and T3. According to T1, basically students are enthusiastic during learning. The interaction between students is also going very well, but the teacher in the video is rather dominating in the early learning using a one-way interaction, where teachers provide information while students are the listeners. However, T1 acknowledges that the interaction between the students are shown when they are working in groups. This is in contrast to the opinion of T2 and T3. They argue that the interaction between the teacher and the students is already good. Teachers in the video managed to motivate students to interact in completing a given task. The explanation given at the beginning of the video by the teacher is the important part of the learning in order to help students understand the steps they are required to do.

3.5. The fifth characteristic: intertwinement

The intertwinement characteristic according to T1 is that teacher presents an ecosystem context at the beginning of a lesson that consists of circular objects. According to T3 intertwinement is associating a prerequisite knowledge with a new one. T2 did not mention this characteristic in her explanation.

In the video, student with teacher's guidance made connection between a circle and a square. One student drew the biggest square inside the circle and its diagonal to find the centre point and the diameter of the circle as illustrated in Figure 2. The other intertwinement in the video lesson is teacher encouraging students to find the results of dividing two numbers, even they are decimal number.

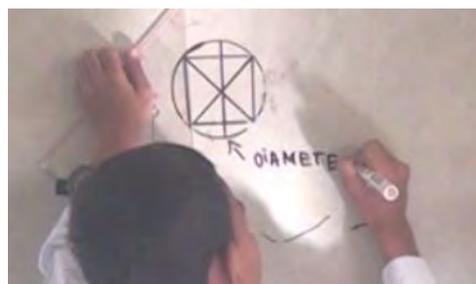


Figure 2. A student is drawing the centre point and diameter of a circle

There are three characteristics of RME that did not explained well by the teachers, they are the first, the second, and the fifth. For the first characteristic: the real life problem, the interpretation of teachers to a real life is confined to the use of concrete object around students. According to Gravemeijer [22], reality is understood as common sense experiences. While 'concrete' can be understood as either "concrete material" or as "common-sense concrete" [21]. It means students learn mathematics meaningfully because there is connection between the new knowledge and prior knowledge of students. Wiscobas project [11] explained that reality is something that can be "imagined" as reality by the students.

In this study, the teacher did not understand the meaning of reality. Reality has a broader meaning. Reality relates to the problems that students can imagine. Reality can either problems from the real world or from fantasy worlds or fairy tales, or the formal world of mathematics, provided that success can be imagined or seen real in students' minds [23].

For the second characteristic: models or symbols to the problem and bridge from a concrete level to a more formal one, does not arise from teachers as they are only mentioned the general explanation. Teachers said teacher in the video provided the opportunity for her students to solve the problems using their own way. Teachers did not explain how the teacher in the video bring her students from the informal to the more formal level of mathematics. Teachers in Indonesia are generally very familiar with the demand that to start a lesson in primary school with concrete objects. However, the next step to reach the more formal level is not an important concern for teachers. So after using concrete objects, teachers usually jump to a more formal directly, and ignore the bridge leading to the formal. Though, the bridge is very much necessary for a student. In RME this is known as a model of the real world and the model for the formal abstract [18].

Using models or symbols in solving problems to bridge the concrete level to a more formal level gets emphasis in RME. In this study, the teacher did not realize the importance of facilitating students to go through various levels of understanding, namely: from solving real problems with informal strategies and finding patterns and forming insights about how concepts and strategies are interrelated. This process inspires students to use useful models or symbols to bridge the gap between informal and more formal mathematics [24-26]. Mathematical formula or procedure which is the ultimate goal of learning and known as formal knowledge should be achieved by students gradually [9].

Teachers' understanding of the intertwinement as the characteristics of RME has not yet appeared in details. Teachers only mention that teacher in the video linked several topics which is an ecosystem as a context. According to Reference Johar [12], the intertwinement is the linkage between the mathematics concepts involved in solving real problems. In this study, the teacher was not aware of the relationship of the mathematical concepts involved in learning. The teacher has not seen the importance of the relationship between the properties or characteristics of a circle and a square [23]. This relationship should be an important mathematical process for students.

The teacher's lack of understanding of the characteristics of RME after attending the workshop using video lessons requires attention. This study suggest the future researchers to involve the teacher in designing a learning trajectory on how to teach a specific of mathematics topic before watching video lesson from other teacher. Furthermore, after the teacher watches the whole video, the researcher asks critical questions to inspire the teacher in expressing his argument about the suitability of learning route with the characteristics of the RME.

4. CONCLUSION

Teachers' understanding of the characteristics of RME is not in line with the theory of RME. The characteristics that are not well understood by the teachers are real life problems, models from concrete to formal, and the intertwinement. This is due to a lack of understanding of the meaning of 'real' in RME and teachers' lack of experience in designing learning trajectory including a real problem, a model of the real world, and a model for formal (abstract) knowledge. The implication of this study is that teachers require coaching by lecturers in watching the video in order to clarify the connection between activities and the characteristics of RME in teaching mathematics. Teachers should also involve in workshops for designing learning trajectory of mathematics lesson to bring students from the informal to the formal level gradually.

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BIOGRAPHIES OF AUTHORS

Cut Morina Zubainur is a lecturer at the University of Syiah Kuala, Aceh, Indonesia since 2002. Her education of undergraduate, graduate, and doctor program are all about Mathematics Education completed respectively in Universitas Syiah Kuala, IKIP Surabaya, and Universiti Utara Malaysia in 2015. Her research interests is about the teacher professional development in teaching mathematics including helping teacher to implement realistic mathematics education, develop classroom interaction, media, and assessment. She has some publications at national/international journals and proceedings.



Rahmah Johar is a lecturer at the University of Syiah Kuala, Aceh, Indonesia since 1998. Her education of undergraduate, graduate, and doctor program are all about Mathematics Education completed respectively in IKIP Padang, IKIP Surabaya, and Universitas Negeri Surabaya in 2006, Indonesia. Her research interests is about the teacher professional development in teaching mathematics including helping teacher to develop students' reasoning, students' character, democratic classroom, using video lesson, and implementation of technology. She is a head of Research Centre of Realistic Mathematics Education for Indonesia (P4MRI) at her university and an editor in chief of the *Jurnal Didaktik Matematika* (www.jurnal.unsyiah.ac.id/DM). She has some publications at national and international journals and proceedings.



Rahmi Hayati received his Bachelor of Mathematics Education at Universitas Syiah Kuala Banda Aceh in 2011, and obtained his Master of Mathematics Education at Universitas Syiah Kuala Banda Aceh, Indonesia, in 2016. He has worked as a lecturer at the Mathematics Education Department, Faculty of Education Mathematics and Natural Science, Universitas Al Muslim Bireuen. His research interests focus on issues in mathematics education and teachers' professional development.



M. Ikhsan is a lecturer at the University of Syiah Kuala, Aceh, Indonesia since 1989. His education of undergraduate, graduate, and doctor program are all about Mathematics Education completed respectively in Universitas Syiah Kuala, IKIP Surabaya, and Universitas Pendidikan Indonesia in 2008. His research interests is about the teacher professional development in teaching geometry, develop media, and assessment. He coaches teachers and students for preparation of mathematics olympiade in Aceh especially for geometry. He has some publications at national and international journals and proceedings.