SELF-REFLECTIONS OF GIFTED STUDENTS IN THE CONTEXT OF MATHEMATICAL PROBLEM SOLVING

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ABSTRACT

This qualitative study examines the gifted students' self-reflections about the mathematical problems in an elaborated way. The participants of the study were three gifted students in 10th grade and 10 problem solving sessions were carried out with these participants. According to findings, the gifted students did not take into account the structure of the problem and they mostly evaluated their performances based on their own behaviors such as developing their own solutions and making calculation errors. Furthermore, the researchers found out that whether the students were able to come up with a solution by themselves was considered to be the most important performance evaluation criterion. The findings suggest that the students attribute their correct solutions to their efforts. In addition, the students mostly associate the reasons behind their satisfaction with the solutions and their performances during the process.

Keywords: *Gifted Students, Self-Regulated Learning, Self-Reflection, Mathematical Problem Solving, Secondary Education*

INTRODUCTION

Values of a cultural environment have an impact on describing a gifted person (Davis & Rimm, 2004). Because of the different views on intelligence, there is not a certain description that different fields agree upon; however, traditional psychological perspectives consider a high score achieved from a standardized intelligence test as a criterion for being gifted. Not having cultural roots has often been criticized about these tests. Therefore, many researchers in the literature have begun to discuss intelligence as a broader term. They have also used several models that are mostly different from each other in explaining giftedness and they have generally related this term with ability instead of intelligence (Gagné, 2003). In the literature, although various researchers have attempted to explain giftedness, there is still no consensus on the characterization of giftedness. The reasons for gifted students' being more successful are still among the issues that attract the researches (Greene, Moos, Azevedo, & Winters, 2008). Motivation factors such as responsibility for the task, curiosity, and willingness to take risks are also considered to be some of the elements of giftedness (Wieczerkowski, Cropley, & Prado, 2000). The theory of self-regulated learning explores the role of motivational aspects in learning environments, as well. The theory plays a crucial role in explaining students' role in their learning process. Most researchers propose that the students, who mainly use self-regulative behaviors, are more successful than others (Pintrich, 2000; Zimmerman, 2000, 2001). Students who motivationally, metacognitively, and behaviorally demonstrate an active participation in their learning processes can be mostly described as self-regulated students (Zimmerman, 2001). "More specifically, self-regulated learners use specific processes that transform their preexisting abilities into task-related behavior in

diverse areas of functioning" (Zimmerman, 2013, p.137). Having the ability to direct their own learning is one of the assumptions that are attributed to gifted students (Neber & Schommer-Aikins, 2002). In their literature review, Risemberg and Zimmerman (1992) found out that the behaviors of gifted students in learning context match with the elements of self-regulated learning. These studies revealed that the gifted students mostly studied independently and individually; they preferred directing and monitoring their studies by themselves instead of having a teacher following them. It was also observed that these students had self-regulated learning elements such as high motivation and being persistent about using time while they were working on a task. These findings have recently expanded the definition of giftedness in such a way that it includes structures that are also examined in theories of self-regulated learning. Therefore, it is worth looking at these gifted students' learning process through self-regulated learning.

Various studies have been carried out to examine the self-regulated learning of gifted students. Most of these studies focus on comparing the differences between gifted students with their non-gifted counterparts. While some of these studies examined gifted students' strategy use related to self-regulated learning (Fehrenbach, 1991; Zimmerman & Martinez-Pons, 1990), the others focused on the relation between their self-regulation and academic achievement (Chan, 1996; Malpass, O'neil, & Hocevar, 1999). There are also studies that investigate self-regulated learning within the learning context (Greene et al., 2008; Scruggs & Mastropieri, 1985, 1988; Scruggs, Mastropieri, Jorgensen, & Monson, 1986). Furthermore, studies conducted with self-regulated learning training programs have been examined with gifted students and non-gifted students in the classroom (Sontag & Stoeger, 2015; Stoeger & Ziegler, 2010). Studies have shown that gifted students could use more strategies flexibly and improve more self-regulated behaviors with the training than non-gifted students do (Alexander, Carr, & Schwanenflugel, 1995; Schofield & Ashman, 1987; Schraw & Graham, 1997). They also prefer more challenging contexts and have high self-efficacy and internal motivation (Dai, Moon, & Feldhusen, 1998; Malpass et al., 1999; Schommer & Dunnell, 1997).

The recent literature review suggests that self-regulation is one of the new directions in the general gifted education (Dai, Swanson, & Cheng, 2011). Self-regulated learning shows differs significant difference between achieving gifted and underachieving gifted students (Albaili, 2003; McCoach & Siegle, 2003). It is particularly essential to understand in-depth components of self-regulated learning among gifted students. Also, since self-regulation is context-based in nature, the theory itself is in need of investigation, such as problem solving in mathematics (De Corte, Verschaffel, & Op't Eynde, 2000; Pintrich, 1999; Zimmerman, 2008). Self-reflection as one of the phases of self-regulated learning also includes important processes which are self-judgement and self-reaction (Zimmerman, 2000). These processes take place after solving problems and they have an influence on problem solvers' responses to problem solving experiences. In this phase, problem solvers evaluate how they solve the problem and make attributions to their failure and success. Attributions of problem solvers cause positive or negative self-reactions which could affect their performances for other mathematical problems. In this sense, this study could have the potential to give a new insight into the theory of self-regulated learning in the sense of participants (i.e. gifted students) and context (i.e. mathematical problem solving). Moreover, the study could extend our knowledge of self-regulated learning processes of gifted students, which is also included in many gifted education models. This study investigates the following question: "What are the self-reflections of gifted students in the context of mathematical problem solving?"

Self-Regulated Learning and Its Mechanisms

Self-regulated learning can be considered as a comprehensive structure while trying to understand how students can be active figurants during the learning process (Zeidner, Boekaerts, & Pintrich, 2000). Self-regulated learning theories from social cognitive perspectives take into account the interactions among personal processes, environmental factors, and behaviors in order to describe the actions of individuals (Bandura, 1997). The triadic cycle of self-regulation that are forethought, performance, and self-reflection are affected by these interactions (Zimmerman, 2000, 2013). Self-regulated learners encounter with tasks in the forethought phase and in order to analyze these tasks cognitively, they activate their prior content knowledge and metacognitive knowledge; set goals; and make strategic

plans. Also, motivational components such as self-efficacy, goal orientation, the perception of task difficulties, and interest / task value beliefs are included in this phase. The performance phase involves selection and adaptation of cognitive and motivational strategies for reaching self-regulated learners' goals. These learners are conscious of these self-regulated strategies and also monitor them during the performance phase. Lastly, the self-reflection phase contains these learners' judgments of their progress according to a standard and their causal attributions to their performance. These self-reflection processes affect future forethought phases of self- regulated learners (Pintrich, 2000; Zimmerman, 2013).

This study particularly investigates the self-reflections of gifted students in mathematical problem-solving process; therefore, we will enlighten the components of self-reflection phase. Self-reflection, which include self-judgment and self-reaction, could be utilized in examining and interpreting the learning experiences (Zimmerman, 2000, 2013).

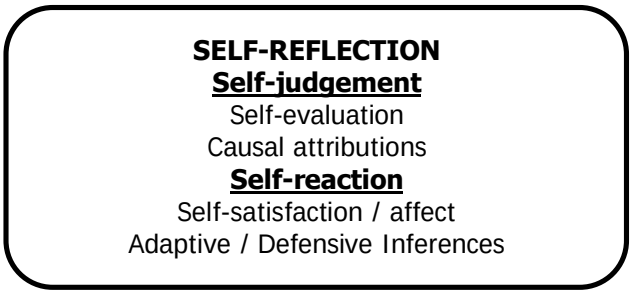

Figure 1. Sub processes of self-reflection phase (Zimmerman, 2000)

As seen in Figure 1, self- judgment consists of self-evaluation and causal attributions. Self-evaluation means comparing performance under certain criteria or a standard or an objective. For example, problem solvers can compare their problem-solving performance with that of their teachers or classmates. Causal attributions related to outcome of performance usually have connections with self-evaluation. To exemplify, poor performance in solving a mathematical problem could result from insufficient effort or the difficulty of the problem. Self-reaction has two components: self-satisfaction / affect and adaptive / defensive inferences. Self-satisfaction is about feelings that are related to performance. If the students feel that they understand a strategy or solve a mathematical problem in a learning environment, it means they are satisfied, and then they may prefer to be in this environment that includes dealing with such mathematical problems, otherwise they may avoid that environment. Adaptive and defensive inferences can arise from self-judgements (Zimmerman, 2013). For instance, reconsidering the strategy that does not work in the problem solving can be labeled as an adaptive inference. Besides, avoiding solving the problems, that are similar to those the students could not solve, is an example of defensive inferences. Within that context, we will examine gifted students' self-reflection phase of problem-solving process.

METHODOLOGY

Research Model

This study was designated as a qualitative research for thoroughly investigating the gifted students' self-reflections about the mathematical problems (Denzin & Lincoln, 1998). Case study, as one of the qualitative research designs, does not only allow for a holistic analysis of an event or a notion, but also enables a detailed description. It is considered to be useful especially for the studies focusing on understanding the processes (Merriam, 1998). Therefore, holistic multiple case study has been adopted in this study. Each situation is holistically evaluated on its own and then is compared to one another (Yin, 1994). Thus, in this study, each gifted student was evaluated as a case.

The Participants

In Turkey, the Ministry of National Education (MoNE) offered an enrichment program for gifted students in more than 100 science and art centers in 81 cities. Science and art centers support education of gifted students throughout the country (Shaughnessy & Sak, 2015). The education program of these centers consisted of five sequential stages including orientation, identification of special talents, development of individual study skills, and project-based learning (MoNE, 2007). If a student scores more than 130 on an IQ test, they can receive education on science and art centers.

The researchers selected our participants from one of these centers in a metropolitan of Turkey. The first researcher made nonsystematic observations for one semester in this center. She organized a program for gifted students in "development of individual study skills" stage for an academic year. These gifted students predominantly had abilities and interests in both science and mathematics. Three gifted students, in their 10th grade: Ahmet, Demir, and Ege (pseudonyms) were selected for this study. The researchers made observations for purposeful sampling in order to gather in-depth and rich data (Patton, 2002). The researchers selected these students for our study, because they had no difficulties in communicating with the first researcher throughout the program.

Brief information on the participants' opinions about mathematical problem (solving) is thought to be significant in providing a better understanding of the cases of the current study. Ahmet described mathematics as a constant language and stated that he likes mathematics. Ahmet saw himself as a good mathematical problem solver, he also highlighted that he really thinks on how to restate the problem and how to figure out all the possibilities for the solution. Demir reported that he enjoys solving mathematical problems. He thought that mathematics includes other disciplines and described mathematics as thinking with numbers. He said that when he was not able to see the solution of a mathematical problem, he insisted on solving that problem even if it took more than an hour. He also prefers to solve such problems and aims to be the best in mathematics classroom. Lastly, Ege reported that he likes to work with numbers and solve problems. He described mathematics as a full of fun activity and said that everything in daily life is included in mathematics. He also stated that he prefers to solve problems which are challenging and difficult to understand.

Data Collecting Process and Tools

The researchers carried out 10 problem solving sessions, which lasted for 40 minutes, with the gifted students. The research approximately took four months. The researchers selected non-routine problems from various resources (Gardiner, 1987; Krantz, 1996; Posamentier & Krulik, 1998; Posamentier & Salkind, 1988). Also, they took into account that these problems did not contain any concepts and abilities which students had not learned in their curriculum. Four experts (two professors and two PhD candidates) in the mathematical education examined the selected problems in order to decide whether the problems were suitable for the gifted students. The researchers determined these 10 problems in accordance with two pilot interviews. The problems can be found in Appendix 1.

The interviews were conducted right after they solved the problems. After the participants solved the problem with the think-aloud protocol, the students were asked nine microanalytic questions about self-evaluation, attributions, and self-satisfaction that are components of the self-reflection phase (Pintrich, 2000; Zimmerman, 2000). Microanalytic method is different from the think-aloud method in which no guidance is provided (Zimmerman, 2008). It is possible to obtain both qualitative and quantitative information by asking open-ended or close-ended context-specific microanalytic questions in this method (Cleary & Zimmerman, 2001; Kitsantas & Zimmerman, 2002). Microanalytic assessment could be defined "as an umbrella term referring to highly specific or fine-grained forms of measurement targeting behaviors, cognition, or affective processes as they occur in real time across authentic contexts" (Cleary, Callan, & Zimmerman, 2012, p. 4). The researchers finalized the questions based on two pilot interviews and the feedback received from three experts (two professors in the theory of self-regulated learning and one professor in mathematics education). These questions can be found in Appendix 2.

The Data Analysis

The impressions gathered from collecting the data were written as memos based on the self-regulation perspective (Strauss & Corbin, 1998). These memos can be considered as the beginning phase of the data analysis. The qualitative content analysis was used while analyzing the raw data (Auerbach & Silverstein, 2003). For the data analysis, constant comparative analysis was also used (Glaser & Strauss, 1967). This method functions as a systematic tool while analyzing the similar and / or different data and generating conceptual categories (Glaser & Strauss, 1967; Strauss, 1987). Furthermore, constant comparative analysis method enables the categorization which emerge from the data (Patton, 2002). Finally, in order to identify similarities and differences among cases, cross case analysis was used (Yin, 1994). The findings that emerged from the cross-case analysis is presented in the following section.

FINDINGS

The researchers adapted the names of the components of the self-reflection phase according to problem solving. Self-evaluation is referred as *the self-evaluation in the context of problem solving,* whereas; causal attributions are referred as *causal attributions in the context of problem solving;* and self-satisfaction/affect, adaptive/defensive are named as *self-reaction in the context of problem solving.*

Self-evaluation in the Context of Problem Solving

The researchers asked participants to evaluate their performance by using a scale ranging from 1 (very poor) to 5 (excellent) and we also calculated mean scores and standard deviations for each participant (Appendix 2). According to the findings, it was observed that Ahmet (M=3.5; SD=1.02), Demir (M=3.6; SD=1.28), and Ege (M=3.4; SD=0.69) mostly considered their performances as good in the context of problem solving (Table 1).

Table 1
Students' self-evaluations in the context of problem solving

		Ahmet	Demir	Ege
Criteria behind good performance	developing solutions on their own	✓	✓	✓
	explicitly giving the answer	✓	✓	
	proving	✓		
	dealing with numbers one by one	✓		
Criteria behind poor performance	having difficulty in generating a solution	✓	✓	✓
	being unsure whether their solutions are correct	✓	✓	
	frequently changing their minds		✓	
	making calculation errors	✓		
	using the longest way to solve the problem	✓		
	being unable to remember previous knowledge and / or methods			✓
	having difficulty in understanding the problem			✓
	focusing on the wrong part while solving the problem			✓

The gifted students evaluated their performances as better when *they developed their own solutions* to the problems (Ahmet, P#2; Demir, P#1, 10; Ege, P#5). Ege evaluated his performance as good, not excellent, because he was not quick enough while solving Problem 5. While solving this problem, Ege talked about himself and stated that *"when it comes to solving problems, firstly I am not quick, secondly I make a lot of mistakes, I mean even though I'm not quick I still make a lot of mistakes"* (Individual interview, December 27, 2010). It can be assumed from his statement that he had metacognitive awareness of his performance in the problem-solving process (Montague & Applegate, 1993; Sriraman, 2003). The students evaluated their performances as poorer in problems in which they had *difficulty in generating solutions* (Ahmet, P#9; Demir, P#7; Ege, P#10). Ahmet and Demir evaluated their

performances as poorer in problems in which they were *unsure whether their solutions were correct or not* (Ahmet, P#3, 7; Demir, P#3, 9). Demir's explanation regarding Problem 9 is as follows:

> "Because I could not say "Aha!" that's the main reason in my opinion. Sometimes it is like my mind is working on its own, independent from me, I'm solving it and my mind is saying "Aha, you solved it". I mean I might b
> e a schizoid [laughs]." (Individual interview, January 28, 2011).

Demir expressed that he was monitoring himself to see if he was making the operations correctly while solving the problem and sometimes, he even monitored himself without realizing it (Sriraman, 2003). However, when he monitored himself to see if he correctly solved Problem 9, he thought that he could not completely and correctly solve the problem. When Demir (Demir, P#3) *frequently changed his mind*, which increased the risk of making a mistake, and Ahmet (Ahmet, P#3) *made calculation errors* by starting the solution without adequately thinking over the problem, they stated that their performances were poor. Besides, Ahmet and Demir evaluated their performances as better in the problems to which they *explicitly gave the answer* with no hesitation (Ahmet, P#2; Demir, P#5).

Ahmet considered his performance to be poor in the problems *the solutions of which were the longest* (Ahmet, P#9). Moreover, Ahmet stated that he displayed a better performance in the problems in which he was able to *answer by proving* (Ahmet, P#6), dealt *with each number one by one* with no time limitation (Ahmet, P#5).

Different from other students, Ege, even though he was able to generate solutions to the problems in which *he could not remember his previous knowledge and / or the methods*, still evaluated his performance as poorer (Ege, P#3, 6, 7). Furthermore, Ege thought that he had a poor performance *when he had trouble with understanding the requirements* (Ege, P#6), and *when he focused on the wrong part in the problem* (Ege, P#9).

Causal Attributions in the Context of Problem Solving

Gifted students' causal attributions in the context of problem solving widely differ from each other. This variety can be especially observed in the reasons behind solving problems accurately. The students' causal attributions are indicated in Table 2.

Table 2
Students' causal attributions in the context of problem solving

			Ahmet	Demir	Ege
Reasons behind inaccurately solving the problem	External	difficult problems		✓	
	Internal	making calculation errors and being unsure of the solutions	✓		
		being unsure of the operations		✓	
		being unable to remember previous content knowledge			✓
Reasons behind for accurately solving the problem	Internal	being able to visualize the problem in their minds	✓	✓	✓
		being sure of the solutions	✓	✓	✓
		revising and justifying the solutions	✓	✓	
		benefitting from previous experiences	✓		
		avoiding negatively talking to themselves	✓		
		using formulas		✓	
		assuming that there are more than one correct answer		✓	
		transforming the given information into operations			✓
		understanding the problem			✓

Demir was the only participant who stated an external reason, which was *difficult problems* for inaccurately solving the problem (P#9). All the other reasons were classified as internal for both

inaccurately and accurately solving the problem. Ahmet identified the reasons why he inaccurately solved the problem with *making calculation mistakes and being unsure of his solution* (Ahmet, P#2, 9); Demir identified these reasons with *being unsure of his operations* (Demir, P#7, 9); and Ege identified these reasons with *being unable to remember previous content knowledge* (Ege, P#6, 7).

The gifted students accurately solved the problem as a result of *being able to visualize the problem in their minds* (Ahmet, P#4; Demir, P#2; Ege, P#4). Besides, another reason why they were able to accurately solve the problem is due to *being sure of their solutions* (Ahmet, P#8; Demir, P#5; Ege, P#3). Ahmet and Demir included *revising and justifying their solutions* in the reasons why they were able to accurately solve the problem (Ahmet, P#3, 5, 6; Demir, P#3).

Compared to Demir and Ege, Ahmet reacted differently and stated that *benefitting from his previous experiences* and motivationally *avoiding negatively talking to himself* are also some of the reasons why he was able to accurately solve the problem (Ahmet, P#1). Ahmet's explanation regarding why he was able to accurately solve Problem 1 is as follows:

> "Believing that there is a solution to the problem, in other words, not thinking of things like "I cannot solve this problem". After that, there comes experience, I mean using experiences, because if a person tries to solve every new problem by approaching it as a new problem, he can't succeed. Secondly, if there were thousands of different questions and he solved a hundred of them, he would not be in a situation where he can't solve the 101st question because all of them are similar. Even if it was new, he would still have to benefit from the problems he had solved earlier." (Individual interview, December 3, 2010).

Ahmet expressed that he solved the problem by believing that there is a solution to the problem, avoiding negatively talking to himself, and benefitting from his previous experiences. It can be deduced that Ahmet tries to motivationally regulate himself (Wolters, 1998). Furthermore, Ahmet also expressed his opinion by saying that it is necessary to benefit from previous experiences instead of solving too many problems by considering them as new problems.

Demir stated that he accurately solved the problem because he *used a formula* while solving the problem (Demir, P#8). Moreover, he also said that he accurately solved the problem by finding out only one answer in a problem which, he thought, *had more than one correct answer* (Demir, P#10).

Ege expressed that he accurately solved the problem as a result of *quickly transforming the information given in the problem into operations* (Ege, P#1). It was observed that another reason why Ege was able to solve the problem, which distinguishes him from the other gifted students, is *understanding the problem* (Ege, P#1, 2, 8).

Self-reaction in the Context of Problem Solving

In this subsection, the data of self-satisfactions of participants' problem-solving performances are presented in detail. The researchers did not elaborate gifted students' adaptive and defensive inferences because their self-judgements (e.g. self-evaluations and causal attributions) considerably depend on their own solution methods and solving processes. As a consequence, they made adaptive inferences instead of defensive ones. They constantly reviewed, altered and adjusted their methods to solve the problems in the process.

The researchers asked participants to rate their satisfaction on their performances by using a scale ranging from 1 (it did not satisfy me at all) to 5 (it highly satisfied me) and then the mean scores and standard deviations were calculated for each participant (Appendix 2). The gifted students were mostly satisfied with their solutions. However, compared to Ahmet (M=3.9; SD=1.22) and Ege (M=3.8; SD=0.74), Demir (M=3.5; SD=1.43) was seen to be less satisfied with his solutions. The students' self-reactions are as indicated in Table 3.

Table 3
Students' self-satisfactions in the context of problem solving

		Ahmet	Demir	Ege
The reasons behind being unsatisfied	having difficulty in operations or drawing	✓	✓	✓
	solving in a long time	✓	✓	✓
	being unable to remember previous content knowledge or formulas	✓	✓	✓
	being unsure of the operations		✓	
	not using all of the information in the problem		✓	
The reasons behind being satisfied	indicating why a situation will not work	✓	✓	✓
	being sure of the operations	✓		
	realizing a mistake in solutions	✓		
	justifying the answer by using various ways			✓
	obtaining the answers through equations		✓	
	generating alternative ways			✓
	developing their own methods			✓

The students evaluated solutions of the problems *in which they had difficulties in operations or drawing, and which they solved in a long time* as less satisfying (Ahmet, P#10; Demir, P#2; Ege, P#3). Similarly, they expressed that the level of satisfaction decreases when *they are unable to remember previous content knowledge or formulas* (Ahmet, P#3, 9; Demir, P#10; Ege, P#6, P#7). Furthermore, Ahmet and Ege evaluated the solutions as more satisfying *in which they were able to indicate why a situation acquired while solving the problem will not work* (Ahmet, P#3, 5; Ege, P#8). While Ahmet considered the solutions to the problems *for which he was sure of his operations,* to be more satisfying (Ahmet, P#4), Demir evaluated the solutions to the problems *for which he was unsure of his operations* (Demir, P#9).

Ahmet considered the solutions to the problems *in which he realized he was making a mistake in the solution* as more satisfying (Ahmet, P#2). Moreover, he also expressed that the solutions to the problems *in which he can justify his answer by using various ways* satisfied him more (Ahmet, P#1). Ahmet mentioned that his solution to Problem 1 highly satisfied him and added that he checked the two ways he had used while solving the problem because he found out the same answer by using both ways. He also said, *"because I double-checked both by using the first and the second way"* (Individual interview, December 3, 2010). What Ahmet meant by the first way is considering the diagonals and numbers of the polygons, and by the second way was through behaviors displayed in order to find a pattern. Afterwards, he stated that he checked the operations by using the second way, in other words, by trying to find a pattern.

Demir evaluated his solution as less satisfying when he found out the answer *by not using some of the information given in the problem* (Demir, P#4). Furthermore, Demir marked the solutions to the problems *whose answers he obtained through equations definitely* as more satisfying (Demir, P#5, 8). For example, in Problem 8, Demir discovered the relationship between number values and place values of the numbers which he represented as "ab" by an equation and he expressed that this made his solution definite.

Ege evaluated the solutions he used in problems for which he could not remember the required formula and instead *solved by generating alternative ways* as more satisfying (Ege, P#1). Ege thought his solution was more satisfying when he solved the problem *by developing his own method* (Ege, P#9). In Problem 9, Ege said that: *"most of the time, the education system gives us a formula and it does not get in detail, it uses practice. In this one I formed a formula; I generated a rule by myself and I liked it better".* (Individual interview, January 24, 2011).

In his statement, Ege expressed that the formulas in their math classes are directly given to them without any justifications. Ege also added that it is much more valuable for him to generate a rule by himself.

DISCUSSION

The study suggests that the gifted students' performance-evaluating reasons focused on the process of their problem-solving behaviors rather than the structure of the problem (i.e. problem itself). Considering the fact that the self-regulated students tended to attribute their poor performances to the lack of efforts or using inappropriate strategies (Zimmerman, 1998), this finding can be considered to be an indicator of the fact that the gifted students are also self-regulated students.

The researchers found out that whether the gifted students were able to generate a solution by themselves was the most valued criterion for them while evaluating their own problem-solving performances. The students' ability to constantly monitor themselves during the problem-solving process helped them report their self-evaluation reasons without any hesitation. Hence, this finding is in line with the literature which proposes that both the self-regulated students (Pintrich, 2000) and the gifted students (Gagné, 2003; Krutetskii, 1976) display advanced metacognitive activities. Besides, Ahmet marked his performance as poorer due to the fact that his solution took quite a long time. This suggests that Ahmet's observation on his solution is elegant and esthetical, which is considered to be a trait of gifted students (Koichu & Berman, 2005).

The findings also suggest that the gifted students mostly explained their causal attributions by claiming the effort, which is one of the internal reasons (Graham & Weiner, 1996; Weiner, 1986). The students claimed certain reasons that are specific to a certain situation such as being unable to remember the previous content knowledge, making calculation errors, and being unsure of the solutions. Furthermore, they too mentioned reasons about checking the solution, such as being sure of the solution and revising the solution. Ahmet mentioned another reason, which is benefitting from the previous experiences. Ahmet also talked about avoiding negatively talking to himself, which is a motivational strategy (Wolters, 1998).

All the participants mentioned that being able to visualize the problem in one's mind is a reason behind correctly solving the problem. Ege referred being able to transfer the problem into calculations and to understand the problem as a reason regarding correctly solving the problem. This suggests that Ege attributed it to an internal reason such as being able to visualize the problem in his mind (Dweck & Leggett, 1988; Molden & Dweck, 2000; Nokelainen, Tirri, & Merenti-Välimäki, 2007; Weiner, 1986). Demir was the only student who considered the difficulty level of the problem, which is an external reason, as one of the reasons why he could not solve the problem correctly. (Nokelainen et al., 2007).

The gifted students differ from the normal students in that the gifted students attributed their failures to their low efforts while the normal students attribute their failures to their low abilities (Chan, 1996; Dai et al., 1998; Nokelainen et al., 2007). The results showcased in this study support the findings of other studies suggesting that the gifted students mostly attribute their success or failures to making *an effort*, which changes according to the problem. However, the researchers have also seen that gifted students attribute their failures or successes to *ability*, which is also considered to be an internal reason in this study (Dweck & Leggett, 1988; Molden & Dweck, 2000; Nokelainen et al., 2007). This indicates that the students are aware of their efforts and what they are capable of doing during problem solving process. They also stated their causal attributions based on the problem.

The students mostly associated their self-satisfactions in the context of problem solving with their performances (Bandura, 1991). One of the reasons was being unable to remember the previous knowledge and formulas. The researchers observed that the students found their solutions less satisfying when they were unable to remember these knowledge and formulas. Another significant finding in this study was that Ege found his solution more satisfying when he developed his solution on his own by generating alternative ideas (Graham & Weiner, 1996; Zimmerman, 2000). Hence, it can be

claimed that when the problem challenges them and gives opportunity to improve their thinking, the level of satisfaction is positively influenced.

These results obtained from the research do share similarities with the results of other students in that they suggest that the gifted students mostly carry out evaluations based on their performances (Hekimoğlu, 2004; Koichu & Berman, 2005). The researchers observed that the gifted students were more satisfied when they overcame the challenges and when they achieved their goals (Krutetskii, 1976). Moreover, it was seen that they were satisfied with the solution only when their solutions were economic, reasonable, and elegant (Krutetskii, 1976). It is also noted that the participants did not make defensive inferences, but instead they made adaptive inferences such as reviewing their solution methods they used in the problem-solving process.

CONCLUSION

In this qualitative study, the gifted students' self-reflections in the context of problem solving was investigated. It can be concluded that the gifted students evaluated their performances better and were more satisfied when they were able to generate a solution in the self-reflection phase of the mathematical problem solving. In addition, it was revealed that they expressed reasons for their causal attributions about obtaining the solution based on the efforts they made and stated adaptive inferences for their solution methods. The study confirms the claim that gifted students have ability to direct their own problem-solving process (Neber & Schommer-Aikins, 2002) and it can be assumed that these three gifted students are self-regulated learners when we look closely to their self-reflection phase in the context of mathematical problem solving. The participants actively reflected on their problem-solving performances. In this manner, the findings of this study contribute to the literature of gifted students in terms of self-regulation.

The researchers could focus the gifted students' self-reflections in the classroom context while they are engaging with mathematical problem solving. Classroom context could have the potential to expand and elaborate the categories as revealed in this study. Anyhow, whether the self-reflection processes are influenced by the interaction among the classroom can still be analyzed. The educators working with gifted students can also adjust the mathematical problems by using a form similar to Appendix 1. In accordance with this information, educators can determine the problem-solving tendencies of students and design mathematical problems that will be used in the problem-solving context.

NOTE: This study is a part of first author's doctoral thesis.

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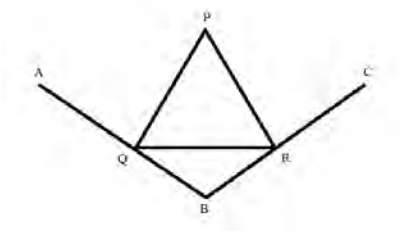
Appendix 1. The problem-solving task

1) In a room with 10 people, everyone shakes hands with everybody else exactly once. How many handshakes are there?
2) How many pairs of vertical angles are formed by 10 distinct lines, concurrent through a point?
3) If you take the digit-sum of a number, that is, you add the digits in the number, how many three-digit numbers will have a digit-sum of 10? (For example, 262 is one, since 2 + 6 + 2 = 10; 505 is another, since 5 + 0 + 5 = 10.)
4) In Figure, m<ABC = 120°, and ΔPQR is equilateral and has vertices Q and R on AB and BC, respectively. As equilateral triangle PQR changes size and moves, with Q and R remaining on the rays of <ABC, what is the path taken by point P?



5) A palindrome is a number that reads the same forward and backward, such as 747 or 1991. How many palindromes are there between 1 and 1, 000 inclusive?
6) The measure of a line segment [PC], perpendicular to hypotenuse [AC] of right ΔABC, is equal to the measure of leg [BC]. Show [BP] may be perpendicular or parallel to the bisector of <A.
7) ΔABC is isosceles with CA=CB. m<ABD=60°, m<BAE=50°, and m<C=20°. Find the measure of <EDB.
8) The number of 12 is equal to exactly four times the sum of its digits. So is 24.
   (i) Can you find a whole number which is equal to exactly twice the sum of its digits? Is your answer the only possible answer?

(ii) Can you find a whole number which is equal to exactly three the sum of its digits? Is your answer the only possible answer?
(iii) Which numbers other than 12 and 24 are equal to exactly four times the sum of its digits?

9) Every number can be written in several ways as a sum of 1's and 2's. For example, 3=2+1 and 3=1+1+1.
(i) In how many ways can the number 11 be written as a sum of 1's and 2's? In how many ways can the number 73 be written as a sum of 1's and 2's? Find a general rule and explain why it works.
(ii) We would not usually treat 3= 1+2 and 3=2+1 as different. But if we do, then there are three different ways of writing 3 as a sum of 1's and 2's. What are they? And if we count in the same way, how many different ways are there of writing 11 as a sum of 1's and 2's. Investigate!

10) Three teams play a round robin tournament. The team from New York sits out the first game. After that, the loser of any particular game sits out the following game. A total of eleven games are played. Each team won a different number of games, and New York lost the last game. What are the won-lost records for each of the three teams?

Appendix 2. The questions asked to the students in the self-reflection phase

1) Could you evaluate your performance throughout the problem-solving process? How was your performance, in your opinion? In terms of your time use and solutions? Could you evaluate your performance by considering 1 as "very poor" and 5 as "excellent"?

very poor excellent

1	2	3	4	5

2) Did the solution/method that you had developed for this problem satisfy you? Could you evaluate your performance by considering 1 as "it did not satisfy me at all", and 5 as "it highly satisfied me"?

it did not satisfy me at all it highly satisfied me

1	2	3	4	5

3) What was the hardest part about solving this problem? Did you have a hard time while solving the problem? If you did, what did you do to overcome it?
4) Do you think you accurately solved this problem? If not, why do you think you were unable to solve this problem? What do you think you need in order to accurately solve it? / If yes, what are the reasons why you were able to solve this problem, in your opinion?
5) What did you feel while solving this problem? / What kind of emotions did the solution process arouse in you?
6) What do you think of the solution/method of this problem? / How did you decide on the solution that you used while solving the problem? Was it the most effective solution, in your opinion? Why? / Is there only one solution? Do you think there are any more solutions?
7) Did your thoughts / feelings about the problem (its level of difficulty, whether you will be able to solve it or not, your emotions) change after you solved the problem? How? In what direction?
8) Did you use the methods of solution that you had thought about before starting to solve the problem while solving this problem? If you did why / If you did not, why not? / If you had changed your solution, why did you change it? How did you know you had to change it?
9) What kind of emotions did being able to / unable to solve this problem arouse in you? /what kind of emotions/ feelings did it cause?