# The Authoring of School Mathematics: Whose Story is it Anyways? 

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Romeo and Juliet died. Period.
Of course there is more than that to the tale of Romeo and Juliet, the famous star-cross'd lovers who were ultimately unable to overcome the obstacle of their feuding families. However, were it a tale told in school mathematics, a subject that traditionally privileges the final answer and not the journey along the way, "Romeo and Juliet died" would be the whole story.

Mathematics weighs heavily on students in our educational system. It is a subject with a reputation as being difficult and abstract, a solitary task meant only for those who have a natural capacity for it (Lafortune, Daniel, Pallascio, \& Sykes, 1996; Sinclair, 2008). It is perceived by many students as a series of rules imposed by an outside source, with little recognition that student thinking itself can generate mathematics. If "the only things mathematicians can be supposed to do with any certainty are scribble and think" (Rotman, 2006, p. 105), then in many mathematics classrooms most students are confined to the role of scribblers: writing, copying and calculating, rather than creating, explaining, and thinking. Yet mathematics itself is a living and creative act (Boaler, 2008), and mathematicians themselves often collaborate in their work (Burton, 2004). What, then, is holding school mathematics back? As educators, are we so conditioned to expect the act of mathematizing in school to proceed in a certain abstract, formalized way that we are neglecting other means by which mathematical learning may emerge? What if we shifted our conception of what students do in school mathematics to be an act of storytelling, where we take the time to admire how students tell their stories, take pride in how they keep their audience engaged, or, on a deeper level, the themes and greater truths they touch on in the telling?

## Mathematics and Authoring

Who authors mathematics? There is a long tradition in Western thought, stretching back to Plato and his ideal forms, of mathematics as authorless, as an eternal absolute, and that it is only through thinking and theorizing by an elite group (i.e. mathematicians) that its laws and axioms can be uncovered. The rest of us attempt to learn the rules and then apply them. Lakoff and Nuñez (2000) call this the "standard folk theory of what mathematics is for our culture" (p. 340), and they argue that its influence has had a number of negative effects:

It intimidates people, alienates them from math, maintains an elite and justifies it. It rewards incomprehensibility, and this inaccessibility perpetuates the romance. The alienation and inaccessibility contributes to the division in our society of people who can function in an increasingly technical economy and those that can't - social and economic stratification of society. (p. 341)

In reality, mathematics is itself an invention, a human process developed and refined by various societies throughout its history. Lakatos (1976), the philosopher who first set this idea out clearly (Ernest, 1998), argued for what he called "quasi-empiricism" in his Proofs and Refutations. Here, mathematics is not portrayed as a static Platonic form that is discovered, but as a process, an evolving aspect of culture. The conversation between teacher and students as
they discuss the Euler characteristic, at first seems to be a Socratic dialogue where the teacher is apprenticing students into traditional conventions of proper mathematical arguments. However, the alternative narrative provided by the footnotes in Lakatos' (1976) book undermines this interpretation, showing how "acceptable" mathematical strategies have varied during different eras of history, and pointing to an analogy between political ideologies and scientific theories (p. 49). Returning to the main storyline of the book, it becomes clear from the characters' arguments that the process of refining a mathematical proof is never-ending. There is always something else to consider. Through his characters' working, and reworking, of Euler's axioms, Lakatos illustrates how the field of mathematics evolves.

It may feel odd to consider mathematics as having authors. As Povey \& Burton (1999) write,

Human meaning-making has been expunged from the accounts of mathematics that appear in standard texts; the contents are then portrayed in classrooms as authorless, as independent of time and place and as that which learners can only come to know by reference to external authority.... because the author(s) of the narrative remain hidden, mathematics becomes a cultural form suffused with mystery and power, a discourse that mystifies the basis for cultural domination." (p. 235)

The word author comes from the Latin auctor/auctoris, meaning "one who increases, creates, fathers, founds or writes," from augere/auctum meaning "to increase," and it can be defined as "one who has created a document" or "the creator of something" (McArthur, 1992a, p. 98). The idea of author as creator has troubled some-can any author truly be considered the sole originator of a text? In his essay "The Death of the Author," Barthes (1968) argues no, noting that in early cultures narratives were delivered by shaman-like figures who acted more as conduit than creator, and that it was only the rise of positivism and capitalism that attached importance to the idea of an author being owner of a particular narrative. Ultimately, Barthes (1968) argues, there is no Author. The person physically performing the act of the writing is just a delivery mechanism for the language system that surrounds him: once the idea of Author is dead,
the writer no longer contains within himself passions, humors, sentiments, impressions, but that enormous dictionary, from which he derives a writing which can know no end or halt; life can only imitate the book, and the book itself is only a tissue of signs, a lost, infinitely remote imitation. (Barthes, 1968, p. 5)

Others are uncomfortable with the author being seen as a means for "increasing" ideas. Foucault (1984) suggests that declaring someone as an "author" is actually a way to delineate a particular set of ideas (such as identifying a piece of writing as belonging to the works of Shakespeare) so that they can be more easily managed. For example, we may try to restrict the circulation of certain ideas by identifying who their "author" is and then punishing that person. For Foucault (1984), the concept of authorship works as a "system of constraint" (p. 119).

As mentioned earlier, the idea of an author, or lack of one, can be associated with authority and power, particularly in mathematics. Povey \& Burton (1999) reframe the idea of authorship by decentering the word authority, which they describe as the traditional view of mathematical knowledge as external, fixed, and absolute, to play with the concept of author/ity in the mathematics classroom. In splitting up the word, they first foreground the presence of an
author lurking behind the scenes who negotiates this knowledge and then they open this authorship up to those in the classroom. Povey \& Burton (1999) write,

Teachers and learners... work implicitly (and, perhaps, explicitly) with an understanding that they are members of a knowledge-making community.... As such, meaning is understood as negotiated. External sources are consulted and respected, but they are also evaluated critically by the knowledge makers, those making meaning of mathematics in the classroom with whom author/ity rests. (p. 234)
In their shifting of the word authority to author/ity, Povey \& Burton (1999) unmask the authoritative, and seemingly authorless, mathematics text as the recorded interpretations of people over time. Brown (1996) suggests that when the focus of mathematics educators turns more to mathematics activities rather than to the mathematics itself, interpretation plays far greater a role-for instance, the students' understanding of a mathematical situation, and how their interpretation changes as they notice new aspects of the situation and make new connections between them. This emphasis on interpretation, Brown argues, is similar to Gadamerian hermeneutics in that the meaning of the mathematics arises from the activity and the language used to frame it. And, in that sense, it opens up the possibility of authorship to any of us who choose to engage in mathematics and communicate our interpretations to others. Mathematician Jonathan Borwein (2006) writes, "We respect authority, but value authorship deeply however much the two values are in conflict. For example, the more I recast someone else's ideas in my own words, the more I enhance my authorship while undermining the original authority of the notions" (p.3).

Povey \& Burton (1999) define authoring as "the means through which a learner acquires facility in using community validated mathematical knowledge and skills" (p. 232). I would like to push this definition further by suggesting that authoring is also an improvisational process. Improvisation is described by Bateson (2001) as an "act of creation that engages us all - the composition of our lives. Each of us has worked by improvisation, discovering the shape of our creation along the way, rather than pursuing a vision already defined" (p. 1). In mathematics education, problem posing has been defined as "the creation of questions in a mathematical context and... the reformulation, for solution, of ill structured existing problems" (Pirie, 2002, p. 929). As students work on a mathematics task, they pose problems as they go, both as a way of orienting themselves to the task and as a way of developing a solution pathway. To highlight the way in which these problems form the basis of a mathematical storyline, I will work with a simplified conception of story and storyline: although not all stories contain conflicts that are resolved (or are resolvable), and some stories may contain no conflict at all, for the purposes of this essay, I will be defining story in terms of conflicts, or problems.

In considering story as a public vehicle of mathematical expression, I will also be framing my ideas in terms of the group discourse that may take place during the work on a mathematical task-if the role of author can be opened up to all individuals, it can be opened up to groups as well. For instance, "Nicolas Bourbaki," who authored a number of mathematics texts, was actually the pseudonym used by a group of mathematicians in the early twentieth century (Mashaal, 2006). Extending the concept of collective authorship further, beyond the passing around of a manuscript between mathematicians who are employed at different institutions, or a scribe recording the ideas discussed by a group, I follow Bakhtin (1981) who suggests "The word in language is half someone else's. It becomes one's 'own' only when the
speaker populates it with his own intentions" (pp. 293-294). The discourse a collective produces cannot be parsed into individual contributions of its members, and therefore in this situation the idea of coauthors as plural, although not technically incorrect, is inadequate to describe the kind of authoring that might take place amongst small groups working collectively in a mathematics classroom.

## Mathematics and Storyline

Story is a broad term in that it is not tied to a specific format, level of truth, or purpose: As McArthur (1992b) writes:

A narrative, spoken or written, in prose or in verse, true or fictitious, related so as to inform, entertain, or instruct the listener or reader. A story has a structure that may be more or less formal, unfolds as a sequence of events and descriptions (even when devices like flashbacks alter the flow of time), and concerns one or more characters in one or more settings. (p. 987)

That it can be spoken aloud and that it is the result of a series of events make the story suitable for describing what a group creates in the course of its conversational work together. To tie this in more with mathematical discussion, it helps to reduce the story to a more basic form-its storyline. A storyline may be defined as:

The sequence or flow of events in a story: the unelaborated routine of the plot, as opposed to the theme that the plot treats. A common story line is Boy meets girl - boy loses girl - boy finds girl, and a twist in such a story line might be girl meets boy - girl loses boy - girl finds another boy. (Nash, 1992, p. 987)

More simply put, a storyline may be regarded as the linear sequence of "what happens next."
To consider what it is that "happens next," it may be helpful to note what German critic Gustave Freytag (1900) proposes in his work Technique of the Drama. Freytag (1990) pictures the structure of a five-act play as a kind of pyramid, which includes the following parts: introduction, complication, climax, resolution, and catastrophe. The "complication," or what has come to be called "rising action," is of particular interest here as it is something that is spurred on by a series of events, or conflicts, with each one triggering the next, in much the same way that a storyline works.

What may drive a story, then, is a sense that there is something that needs to be resolved. It may be a disagreement, a disconnect, an uncomfortable gap in understanding, or a conflict, but it is this something that provides an impetus to further action. William Shakespeare's (1597/1985) play, Romeo and Juliet, provides a good example of how the central conflict of a storyline can generate a number of other conflicts, which help to drive the story to its conclusion. A boy (Romeo Montague) and a girl (Juliet Capulet) meet at a feast hosted by the Capulets and fall in love; each belongs to opposite sides of a long-time feud between the Montagues and the Capulets and thus their friends and families will not approve of the match. How can they be together? They secretly marry and decide to wait for an opportune time to reveal the news to the world. However, this soon precipitates other conflicts, including the following:

- Mercutio versus Tybalt regarding Romeo's disguised and unauthorized presence at the Capulet feast;
- Romeo versus Tybalt regarding Tybalt's slaying of Mercutio;
- Romeo versus the kingdom in terms of a suitable punishment for his slaying of Tybalt;
- Juliet versus Lord Capulet regarding his wish to marry Juliet to Count Paris;
- Romeo's misinterpretation of a message about Juliet's "death;"
- Romeo versus Paris when they unexpectedly meet up at the Capulet family vault where the unconscious, but seemingly dead, Juliet lies;
- Romeo's decision to drink a poison in order to join Juliet in death; and
- Juliet's decision to use Romeo's dagger to stab herself when she finally awakens to discover the scene around her.

It is one thing to author a literary story, generating a storyline based on conflicts, but is it another to author the solution to a mathematics task? Just how original can you be in solving, for example, "the Locker Problem," a task that thousands and thousands of students have been assigned over the years and one that has a single, correct answer? ${ }^{1}$ Again, returning to Romeo and Juliet, we have an example of a "classic" storyline that recurs in Western literature. Shakespeare's central problem of the star-cross'd lovers in Romeo and Juliet is echoed in our contemporary West Side Story and even in the more recent High School Musical, and Shakespeare's play itself is a descendant of Arthur Brookes' 1562 poem The Tragicall Historye of Romeus and Juliet, which is itself a translated interpretation of one of Matteo Bandello's short stories found in Novelle from 1554 (Drabble, 1985). Yet each author has made the story his/her own by varying the storyline. While the overarching conflict is the same (young couple from opposite sides of warring worlds comes to a tragic end), it is how the smaller conflicts, or problems, are settled that makes each text unique. We can think about the storylines that result from mathematics tasks in a similar way, and in the next section, I discuss a study I conducted about collective problem posing in order to do so.

## Authoring Mathematics in the Classroom

In the study, small groups of students from a middle school located in the suburban Lower Mainland of British Columbia worked on "problem of the day" tasks assigned by their classroom teacher. The task considered here is as follows:

## The Bill Nye Fan Club Party

The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.

If the presents are opened in 5-minute intervals, starting at 1:00 pm, the last gift will be opened starting at $5: 35 \mathrm{pm}$. How many club members are there? (Armstrong, 2017)

The four groups (REGL, JJKK, DATM and NIJM) working on this particular task were audio and video-recorded and these recordings were transcribed. I then analysed the transcripts considering problems posed by each group as it worked on solving this task, 31 in total (see Figure 1). I assigned different colours to these problems, colour-coded the transcripts, and then shrank these transcripts so that the words were no longer visible and the coloured patterns formed a visual representation, a "tapestry," of each group's conversation (Figure 2). ${ }^{2}$ These
tapestries help to illustrate some of the storyline traits of the mathematical word the groups do. As I consider authorship in this study to be occurring at the level of the group, I will always refer to each group as a whole rather than to the individual people who comprise it.

| Colour | Problem posed (generalized) | JJKK | DATM | NIJM | REGL | \# |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lavender | Do we use time and divide by 5 [number of intervals]? | X | X | X | X | 4 |
| Medium blue | What about if everyone brings x gifts each? | X | X | X | X | 4 |
| Purple | Is there an extra 5 minutes? (because last gift is opened starting at $5: 35$ ) | X | X | X | X | 4 |
| Deep red | How many people are there? | X | X | X | X | 4 |
| Slate blue | What are the factors of x ? | X |  | X | X | 3 |
| Lime green | What is meant by an interval? | X |  | X | X | 3 |
| Olive green | Do all members give to everyone? |  | X | X | X | 3 |
| Goldenrod | Do they also bring gifts for themselves? |  | X | X | X | 3 |
| Orange | Does everyone bring the same amount of gifts? | [ X ] | X | X | X | 3 |
| Sky blue | How many gifts are there? |  |  | X | X | 2 |
| Brown | What if there are x people? |  |  | X | X | 2 |
| Green | How do we think outside the box? |  |  | X | X | 2 |
| Teal | Is it a square root? | X |  | X |  | 2 |
| Fuschia | Why did we get $x$ ? | X | X |  |  | 2 |
| Dark pink | How long does it take to open all the gifts? | X | X |  |  | 2 |
| Light purple | Can they take breaks in between opening gifts? | X | X |  |  | 2 |
| Pale yellow | Does it start at one o'clock? |  | X | X |  | 2 |
| Gray | What is a tournament? |  |  |  | X | 1 |
| Red | What if it's an exchange? |  |  |  | X | 1 |
| Light green | How long does it take to open one gift? |  |  |  | X | 1 |
| Forest green | Can't we just count how many people? |  |  |  | X | 1 |
| Lilac | How many gifts does each person bring? | X |  |  |  | 1 |
| Coral | How many gifts are opened in an hour? | X |  |  |  | 1 |
| Gold | Is another group's answer right? |  |  | X |  | 1 |
| Sage | Can they bring partial gifts? |  |  | X |  | 1 |
| Pink | What if someone doesn't get a gift? |  |  | X |  | 1 |
| Dark blue | How do we know if we're right? |  | X |  |  | 1 |
| Blue | What if there are x people and gifts? |  | X |  |  | 1 |
| Peach | Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings? |  | X |  |  | 1 |
| Light blue | How can we use the 24 hour clock? |  | X |  |  | 1 |
| Yellow | Can they open gifts at the same time? |  | X |  |  | 1 |

Figure 1. The Colour Coding Chart organized by numbers (Armstrong, 2017)


Figure 2. Tapestries (Armstrong, 2017)
The tapestries reveal how the mathematical stories each group authored have structures in common with a literary story. At the beginning and near the end of the tapestries, there are areas where the pattern is "thready" (a series of thin strands of colour) indicating that a variety of problems are being posed in a brief period of time. During the introduction of a literary story, an author may mention details in order to set the scene for the storyline, details that may be taken up as the plot proceeds. When beginning a mathematical task, a group may tentatively pose a number of problems for discussion, putting them "on the table," so to speak, as a way for the group not only to establish the parameters of the task, but also to negotiate a way to begin it. At
the end of a literary story, once the climax has been reached and the central problem resolved, there are often loose ends (smaller problems) that the author may wish to revisit in order to wrap up the storyline. For a mathematical task, once the group has arrived at a solution, it may choose to "check" the answer by trying to solve the task in an alternate way, or by reviewing the problems they had posed during their discussion, and thus some problems may briefly be reposed for further consideration.

Another way the tapestries reflect a literary story is the presence of the shared elements between them. What makes the storyline of Romeo and Juliet "Romeo and Juliet," regardless of who the author is, is the presence of certain features of the story: the characters (the lovers), their situation (their feuding families, and thus the social pressure that works against their relationship) and a series of events that serves to keep them apart. This may also be said of a rich mathematics task-there are certain mathematical elements that need to be addressed. The Bill Nye task is a fairly structured one in terms of which mathematical concepts are used to solve it. Students need to determine the total amount of time it takes for the gifts to be opened (as the last gift starts being opened at $5: 35 \mathrm{pm}$, the total amount of time is 280 minutes), how many five minute intervals there are ( $280 / 5=56$ intervals), and which factor pair of 56 would enable each club member attending the party to bring one gift for every other club member (i.e. no one brings a gift for themselves). In the end, eight club members each bring seven gifts $(56=8 \times 7)$ to the party. With the Bill Nye task, problems posed include the following: "Is there an extra 5 minutes?" (purple), which is used to determine the total amount of time; "Do we use time and divide by 5?" (lavender), which is used to determine the number of intervals; and "What are the factors of $x$ ?" (slate blue), which is used to determine which factor pair to use. Although the order of these posed problems is not precise-as we can see in the tapestries, groups do go back and forth between these particular posed problems and deal with them at different timesultimately, for example, the number of intervals needs to be established before the correct factor pair can be determined. Thus, lavender tends to be more prevalent in the top half of the groups' tapestries, and slate blue tends to be more prevalent in the bottom of the tapestries of the three groups who posed that particular problem.

What is probably most striking about the tapestries is how different the colour patterns are between them, despite each group being engaged in the same task and arriving at the same final answer. The problems were posed in different orders, were discussed to varying degrees (resulting in colour bands of various thicknesses), with some problems re-emerging again in the course of the conversation, while others did not. As a result, each group's solution pathway has its own style, just as authors of the various versions of Romeo and Juliet have their own styles of telling the story. For instance, JJKK tended to discuss each problem it posed at length, one at a time, and the "chunky" pattern of its tapestry reflected this. Very early in its session, NIJM identified "Do we use time and divide by 5 ?" as the main problem to pursue, posing other problems to refine its understanding but returning to "Do we use time and divide by 5?" again and again. REGL realized about halfway through their session that it was not coming up with a reasonable answer, and started working on the task over again, quickly reviewing through what it had been doing and reposing many problems over again in the process. This resulted in a thready section midway through REGL's tapestry. Finally, DATM sometimes argued about which problem to pursue next, resulting in patches of tapestry where two bands of colours alternate.

Although, there were four problems posed by all four groups, almost half the total number of problems (14 of 31) were specific to certain groups. A high proportion of the
problems posed were unique points to the potential "creativity" of mathematical storylines in terms of the richness of the different mathematical ideas that the groups explored while engaged in the task. JJKK and DATM each discussed alternate ways to determine the number of five minute intervals in the period of time given, with JJKK considering how to work with the number of five minute intervals in one hour, and DATM pondering the use of the 24 hour clock (an idea they had apparently picked up from a previous task in the study). In its session, REGL searched for a metaphor to capture the mathematical concept of combinations that was the basis of this task ( 7 gifts each by 8 club members). NIJM considered the difference between the square root of a perfect square (a natural number) and the square root of any other number (a rational number) in the process of narrowing down what possible factors might be. Some of these discussions were digressions-REGL's metaphor was not strictly necessary for solving the task, nor was DATM's 24-hour clock, but they were mathematically interesting ones. In a literary story, digressions have value - they might help to develop a character, a setting, a theme, the voice of the narrator, etc. In traditional mathematics classes, there is little room for digression. In considering the question, "if learners don't reach the right answer, have they still done the math?" the quick response is typically, "Oh, I give them part marks as long as they've shown their work." However, what this usually means is that students are awarded marks for following at least some of the expected procedures, not for exploring related mathematical topics. If the digression strays too far from the expected mathematics of the assigned task, it likely is not valued at all, even if it is mathematically sound and interesting. In the traditional mathematics classroom, there little no time to wander and think.

## Implications of Students Authoring Mathematics

The groups described above all deal with the Bill Nye task in a way that was creative and personal, having the time and the motivation to explore the meaning of the mathematics they were exploring and to author unique storylines in doing so. There are implications in connecting mathematics with storytelling. Stories serve different purposes-to entertain, to educate, to unsettle, to comfort, and so on-and many are not purposes normally associated with mathematics. Framing doing mathematics as storytelling would help to privilege the process of doing mathematics over the product that results from it (the answer), something that might make school mathematics more satisfying, more human, for our students. Despite the recent reform efforts in mathematics education in the past few decades, I still have far too many preservice teachers coming into my elementary program mathematics methods courses having had bad experiences with mathematics and having been left alienated from, and very anxious about, the subject. As students, what they learned was to regurgitate required procedures, not to understand what it was they were doing and why. Mathematics was not their story to tell. Thomas King (2003) writes, "The truth about stories is that's all we are" (p. 2). What are the stories in school mathematics, then, and when will we allow our students the author/ity to tell them?

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[^0]:    ${ }^{1}$ In the Locker Problem, there is a wall along a hallway that contains a bank of lockers. This number of lockers can vary, but let's say there are 100. A student comes along, and as she walks down the hallway, she opens every locker door. A second student comes along, and as he walks, he closes every second locker door. A third student walks down the hallway and changes the position of every third locker door: that is, if the door is open she closes it, and if it is closed she opens it. A fourth student comes along and changes the position of every fourth locker door, and so it goes. After the $100^{\text {th }}$ student walks by, which locker doors are open?
    ${ }^{2}$ For further detail about this process, please see Armstrong, 2017.

