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An analysis of the changes in geometric transformations in early childhood period

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Abstract

The purpose of this study is to analyse geometric transformations of children in the early childhood period. The study utilised a case study to design one of the qualitative research methods. Interviews were conducted with 6-, 7- and 8-year-old children, in total 24 children, who were enrolled in a private pre-school and a primary school of the Ministry of National Education. Data obtained from the interviews conducted with the children were recorded on the computer as raw data. Then, appropriate themes and codes were formed using these data, and the data were subjected to content analysis. Results of this study showed that children began to improve comprehend topological transformations as their age levels increased, and the development occurred according to age. It is recommended that further studies should investigate how topological transformation develops at earlier ages in case it is supported in the early childhood period.

Keywords: Geometric, transformation, topology, projective, early childhood period.

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1. Introduction

In recent years, there have been various reforms and updates in the Turkish Education System at every phase, ranging from pre-school to high school education. Mathematics education has also been affected by these updated programs. The new reforms aimed to form new theories and approaches, mathematics expectations, how mathematics is used, and teaching and learning processes in mathematics (MEB, 2013; NCTM, 2010). However, topologic and projective transformations, a different branch of geometry, were not included in the updated MEB-2013 pre-school and primary school curriculum (MEB, 2013). Children's geometry experience throughout primary school is only Euclid. On the other hand, studies reported that children had topologic ideas (Greenstein & Anderson 2018; Laurendeau & Pinard, 1970; Piaget & Inhelder, 1956). These findings form the base for the claim that there are topologic reasoning styles for children which can be defined mathematically, which are important and which can improve students' mathematical reasoning and research.

In topologic transformations, geometric shapes undergo transformations completely independent of their metric properties (Yomralioglu, 2000). Topology is a science of mathematics that deals with the ways the shapes in space are stretched and shrunk like a rubber. Accordingly, dimension and homomorphism are the two fundamental concepts that are studied. For instance, all the shapes that can be obtained as a result of pulling and stretching a circle and transforming it into a triangle are topologically the same. In topology, straight lines are bent in order to obtain curves, and circles are shrunk to make triangles, squares, rectangles or any other polygons, and they all have the same size (Peker & Karakus, 2013). To summarise, rather than the transformations of a completed case, topology defines cases in the process such as stretching (expansion) and shrinking.

Topology starts with the changes in children's body and with life (Dawson, 1990). To Piaget (1970), learning starts at the end of age one in the sensor motor stage. According to Piaget, topologic transformations start around the age of 5 and are completed at the age of 9 (Piaget, 1966). However, Gibson, Owsley and Johnston (1978) provided evidence for the fact that 5-month-old babies could make topologic assessments showing that they knew the difference between a transformed object or distorted/unshaped object. The finding indicating that topologic assessments start in the early childhood period is of importance in terms of conducting these kinds of studies.

Topology is utilised in different disciplines including the ways computers are connected to each other, geography (Karas & Batuk, 2005), architectural design (Tarim, 2006), digital medicine and artificial intelligence (Narli 2010). An analysis of the relationship of topology with different disciplines shows its importance in human life (Flegg, 1974). However, despite its connections with various disciplines and its place in daily life, topology is given a little place in mathematics education. In this regard, studies on topology are based on the study conducted by Piaget and Inhelder (1956). They reported that little children perceived space first in a topological way, later in a projective way and then in a Euclidean way (Baki, 2014). Following their study, various studies have been conducted in relation to topology instruction (Delice & Karaslan, 2016). Students usually do not encounter topology until university years (Dawson, 1990). Similarly, topology is given as an undergraduate course in our country; it is included neither in other levels nor in early childhood education. George (2017) reported that topologic concepts could be revealed in little children's reasoning, but topology is not included in the current K-12 education. In this regard, using Piaget's representation field theory, Van Hiele explained topologic concepts according to age by combining his views about geometric thinking levels. Results of that study showed topology was a topic appropriate for pre-school children and emphasised that the issue should be investigated.

In light of the abovementioned information, the purpose of this study is to investigate geometric transformations of children in the early childhood period according to age levels.

Sub-aims:

1. What are the changes in the topologic transformations of children in the early childhood period according to age levels?
2. What are the changes in the projective transformations of children in the early childhood period according to age levels?

2. Method

2.1. Study design

The present study utilised a qualitative design and aimed to identify geometric transformations of children in the early childhood period according to their age levels in a detailed and in-depth way. In this regard, the study adopted a qualitative design that aimed to identify a case, and it was designed based on these research techniques. The qualitative research approach was utilised through detailed and in-depth interviews with a view to learning the participants' individual perceptions, experiences and views directly, and understanding and explaining their current situation (Buyukozturk, Cakmak, Akgun, Karadeniz & Demirel, 2009).

2.2. Participants

The participants of this study were eight 6-year-old children, eight 7-year-old children, eight 8-year-old children, totally 24 (12 females and 12 males) children, who were enrolled in a private pre-school and a primary school of MNE in the centre town of Adana, who had medium socio-economic level and who demonstrated normal development. The participants were chosen using a purposeful sampling method as it enabled to obtain more in-depth data. Some codes were utilised in order not to reveal the children's identity. Hence, the participants were given codes according to the age and gender of the child interviewed. For example, for the 6-year-old girl and boy interviewed, $C_{6G}1$ and $C_{6B}1$ were coded.

2.3. Data collection tools

The participants were administered the socio-demographic form and two activity forms that aimed to reveal topologic and projective transformation. While preparing these forms, literature was first searched (Inhelder & Piaget, 1964; Piaget & Inhelder, 1956; Schultz, Colarusso & Strawderman, 1989) and forms were prepared. Prepared forms were presented to experts in mathematics and preschool field and necessary corrections were made. The pilot study was applied to 3-, 4-, 5- and 6-year-old children (*n* = 20). However, children between the ages of 3 and 5 years did not respond to this form during the application process. Taking these into consideration, the form was finalised. Thus, the main application was applied to children between the ages of 6 and 8 years. The first activity form aimed to reveal topologic transformation. This form utilised a balloon for the 'stretching face' activity that reflected 'stretching-shrinking' property. The students were asked about the changes in the balloon when it was blown, inflated or stretched as it was pulled. For instance, the students' knowledge about topologic transformations were investigated through the questions such as 'what happened to the picture on the balloon when it was stretched/shrunk, what changed, how did it happen?'. The second activity was the 'shadow activity' that reflected the perception of 'spatial contiguity' in order to reveal projective transformation. In this activity, the students were asked questions about the object and the shadow that came from a source of light. For instance, 'Are the object and its shadow the same? Are there any differences/similarities? If so, what are they?' The reasons for this situation were also asked.

2.4. Participants' data collection and analysis

In the data collection process, initially, the director of the pre-school was informed about the purpose of the study, and the necessary permissions were obtained; informed consent was obtained from the children's families. The teachers were visited in their classrooms for the children aged between 6 and 8 years. Then, the teachers were informed about the purpose of the study. As participation in the study was voluntary, they were told that they could withdraw from the study at any phase.

In the interview process, two types of activities were conducted with children in three different age groups with a view to identifying their geometric perceptions. Interviews with each child took approximately 20 minutes. The first activity was the 'stretching face' activity that aimed to reveal the property of 'stretching-shrinking'. The second activity was the 'overhead projector' activity that aimed to measure the perception of 'spatial contiguity'. While one of the researchers conducted the activity with the children, the other researcher took note of the children's answers.

Data obtained from the interviews conducted with the children were recorded on the computer as raw data. Then appropriate themes and codes were formed using these data, and the data were subjected to content analysis. Statements to be quoted directly were identified, and the ones that were chosen were presented in the findings section.

2.5. Validity and reliability of the data

Views of two experts in the field of pre-school education and an expert in mathematics education were obtained in order to identify the reliability of the analysis. Necessary corrections were made in line with the feedback received from the experts. Content analysis was performed by two researchers separately in order to identify agreement between the researchers. The two researchers first obtained the codes separately and then calculated the agreement value between these codes together and this ratio was calculated as 82% (Miles & Huberman, 1994). The codes and themes were constantly compared in terms of both content and consistency. This constant comparison process formed the themes and codes.

3. Findings

This study aimed to analyse the geometric transformations of children in the early childhood period according to their age level. Data collected from the interviews conducted with the participating children showed that the children's topologic transformations improved depending on age.

An analysis of the findings of topologic transformations showed that all the female and male children aged 6 years were aware of stretching shrinking, but they could not explain the concept completely. They explained the changes in stretching and shrinking depending on the size property of the object, small and big size of the pictures on the object/balloon, and thin or thick property of the lines in the picture. For instance, $C_{6G}1$, $C_{6B}8$ the picture became thick-thin, $C_{6G}4$ every part became smaller. It was found that the children did not know about the concept of stretching shrinking, and they provided explanations according to the changes in the shape on the balloon. For instance, $C_{6G}1$, $C_{6B}6$ it became bigger when it was inflated, the picture became thick-thin; $C_{6G}2$ it became huge as it was inflated; $C_{6G}4$, $C_{6B}7$ we pulled it, it became bigger; $C_{6G}1$ it was deflated, it became smaller; $C_{6G}3$ its eyes-mouth became smaller; $C_{6G}4$, $C_{6B}6$, $C_{6B}7$ all parts became smaller because we did not hold it.

All the children aged 7 years (four females, three males) were found to have developed the concept of stretching. The children provided explanations according to the size and colour properties of the object and started to explain how stretching happened. Only one child could not explain the change in stretching. Examples about stretching: $C_{7B}2$ its face becomes a little different when it is stretched, $C_{7G}7$ its face and head changed; it becomes bigger in stretching; $C_{7G}8$ its face became bigger; $C_{7G}9$ their sizes are different; its face changed, bigger now; $C_{7G}7$, $C_{7G}8$ its face became bigger-smaller, its face changed. Children aged 7 years were found to explain the reason for the change in the object. For instance, $C_{7B}3$ it had a big head when it was inflated, it became smaller when it was deflated; $C_{7G}8$ we pull it, it stretches; $C_{7G}9$ it becomes bigger when it's stretched. Some of the children aged 7 years were found to have developed the concept of shrinking, and two children were found to have had no information about the concept. The children provided explanations according to the size and colour properties of the pictures and started to explain how shrinking happened. Examples about shrinking: $C_{7G}8$, $C_{7G}9$ its face is becoming smaller, $C_{7G}7$, $C_{7G}12$ it becomes shorter during shrinking. Very few children aged 7 years were found to start to explain the reason for the change in the object. For instance, $C_{7B}3$ it becomes smaller when it is deflated.

All the children aged 8 years were found to have developed the concept of stretching. The children provided explanations according to the size, colour and shape of the objects. Explanations about the size property: $C_{8B}5$ we pulled it, its colour became lighter and it became even bigger; $C_{8G}6$ it became so big because we stretched it. Explanations about the colour property; $C_{8B}1$ its colour looks lighter when it is inflated. Explanations about the shape property: $C_{8}8$ the balloon looks uneven, $C_{8G}9$ the shape looks like a triangle. The children began to explain how stretching happened and the reason for the change in the object. Examples about this: $C_{8B}3$ it happens when I pull from the corner; $C_{8E}5$ we pulled, its colour became lighter and it became bigger; $C_{8G}6$ it became so big because we stretched it; $C_{8G}7$ we pulled it, it became bigger because it was stretched. Some of the children aged 8 years (four females, three males) were found to have developed the concept of shrinking, and one child was found to have no information about the concept. The children were found to have provided explanations according to the dimension property of the object. Examples about the dimension property: $C_{8G}9$ it's deflated, the balloon became smaller; $C_{8B}3$ no, it's not the same with shrinking, it became smaller. They also began to explain how shrinking happened. Explanations about the shape and colour properties: $C_{8B}1$ it became darker and smaller. Very few children aged 8 years started to explain the reason for the change in the object about shrinking. $C_{8B}3$ it became smaller, it became the same as before; $C_{8G}9$ it's deflated, the balloon became smaller, it became the same as before; $C_{8G}6$ it became smaller when it was shrunk, it became the same as before.

An analysis of the findings of the projective transformations showed that the children aged 6 years could not explain the transformations of the two- and three-dimension objects completely, but they tried to explain two- and three-dimension (spatial contiguity) objects according to their visual properties (intuitional thinking). Examples about the visual (colour-shape) properties of the two-dimension objects: $C_{6B}2$ he knows it's square (2B), $C_{6B}3$ there's a shadow on the paper, there's a picture in the block; $C_{6G}4$ I see a black track shadow; $C_{6G}5$ it has square on the paper; there are shapes and colours in the block; $C_{6B}6$ there's a square on the paper; it's not the same; shadow is big, cube is little; there are shapes in the cube; $C_{6G}7$ the shadow is big, it's actually little. Although few children could name the object and the shape they saw in the shadow of the object, they could not explain the difference between them. For instance; $C_{6B}2$ one is square, one is cube; $C_{6G}8$ cube and shadow are the same. Despite not being able to explain the shadow completely, only a few of them provided explanations about the shadow. Although they knew about the shadow, they could not explain the difference of two- and three-dimensions. $C_{6B}6$ the shadow is formed from the reflection of the person; $C_{6G}7$ one's reflection in the sunny places; $C_{6G}8$ it happens when light and darkness are combined.

The children aged 7 years were found to try to explain the transformations of two- and three-dimension objects (spatial contiguity) according to the size, visual/picture and shape properties. Examples about the size property: $C_{7B}1$, $C_{7B}3$, $C_{7G}9$ its shadow became big; examples about the picture/visual properties: $C_{7G}7$, $C_{7G}8$ there are no flowers, its tag cannot be seen; $C_{7G}8$ I can see my fingers' track on the white paper, I cannot see the bones on my hand, I cannot see the lines, I cannot see my nails. The children could name the object and the shape they saw in the shadow of the object and explain the shape according to its property. $C_{7B}3$, $C_{7G}9$ it is square and it is cube; $C_{7G}8$ it looks like rectangular. The children of this age group started to state that light was needed for the formation of shadow. For example, $C_{7B}2$ shadows happened in front of or behind people; $C_{7B}3$ because you used light; $C_{7G}9$ it's because of the light. It becomes bigger when you bring closer. The shadow here is very very big. Although they stated that a source of slight was needed for the shadow to be formed, they tried to explain three-dimensionality according to some properties of the object.

The children aged 8 years were found to explain transformations of the two- and three-dimension objects (spatial contiguity) according to the size, colour, picture and shape (3D) properties. Examples about the size property: $C_{8B}5$-$C_{8G}6$ the shadow is bigger; $C_{8G}8$ a little bigger; $C_{8G}9$ it's huge, the one on the paper is big, the one in your hand is little. Examples about the colour property: $C_{8B}1$ it has become black, the other one is red; $C_{8B}2$ they are not the same, this one is black, this one has no flower pictures; $C_{8B}3$ they do not have pictures on the shadow, the pattern on the top cannot be seen, it has no colour, it has light, there is darkness. Examples about the properties of the picture: $C_{8E}1$ lines and colours cannot be seen; $C_{8B}5$ the handles are similar, there are no flowers; $C_{8G}8$ it's a little bit bigger, the pattern on the top cannot be seen.

Examples about the shape: $C_{8B}3$ here is round, the other one is smooth, normally it is inflated, there are no pictures in the shadow. Examples about the dimension property: $C_{8B}5$ this is 3D, $C_{8B}6$ its backside (the top and the sides cannot be seen). In addition, they started to explain that the shadow was a reflection in front of the source of the light. Examples about this: $C_{8B}1$ there's a shadow now, similar flower cannot be seen, the lines and its colours cannot be seen, it became black, the other one is red; $C_{8B}2$ shadow is the thing behind the sun, it does not show light; $C_{8B}3$ shadow is the thing behind the sun; $C_{8E}5$ shadow, shadow happens when the sun reflects, it becomes even longer because it is reflected somewhere; $C_{8G}6$ shadow, the sun reflects on us when we walk, it is then reflected on the ground; $C_{8G}7$ shadow is something that happens in the sun; $C_{8G}8$-$C_{8G}9$ shadow is something that happens in the sun, there is light.

4. Discussion

This section has used two pre-existing theories with a view to explaining the children's ideas about the topologic and projective (geometric) transformations. These theories are Van Hiele's geometric thinking model and Piaget's cognitive development stages. One of the natural differences between these two theories is that, using Van Hiele's model, the emphasis is on the developmental nature of the progress in Piaget's development stages (George, 2017). On the other hand, there are similarities between the theories about how they define these phases. Moreover, Piaget's theory was utilised in order to expand Van Hiele's theory.

This study, which aimed to analyse geometric (topology and projective) transformations of children in the early childhood period according to their age level, found that stretching-shrinking in topologic transformations demonstrated changes according to age. The children aged 6 years were found to be aware of the changes in the stretching-shrinking and explain it according to the size property of the object. The children aged 7 years provided explanations according to the size and colour properties; the children aged 8 years were found to provide explanations according to the size, colour and shape properties. With age, the children were found to become more aware of the transformational changes in stretching-shrinking. The literature also indicates that this case is a developmental process, which is explained by the fact that children aged 6 years are at the pre-operational stage according to the cognitive development theory proposed by Piaget (1970). The second stage of the cognitive development, intuitional thinking (first stage is the symbolic stage), is also known as the primitive thinking stage. Children at this stage are reported to explain an object, case or phenomenon at a simple level and according to only one property. Because children aged 7 and 8 years perceive figures in a holistic physical appearance at the first visualisation stage, children aged 8 years are reported to explain the objects according to many properties in comparison to other age groups.

While the children aged 6 years could not explain how and why the change in stretching-shrinking happened, the children aged 7 years were found to be able to explain how the change in stretching-shrinking happened and started to explain the reason of the change. As to the children aged 8 years, they were found to explain both how the change happened and the reason for the change. This case is explained by the theory of conservation because they are at the pre-operational and concrete operations stage; accordingly, the children aged 6 years cannot comprehend that the reality remains the same although the appearance changes; this case is reported to change a little in children aged 7 years as they are at the concrete operation stage. At the age of 8 years, children are reported to gain conservation. Hence, the children aged 8 years were found to be able to explain the changes in stretching-shrinking with the reversibility principle.

Another important result of the study was that the children demonstrated development in the spatial contiguity theme of the projective transformations according to age. The children aged 6 years could not explain the transformations of two- and three-dimension objects completely, but they were aware of only one of the properties of the change (visualisation). The children aged 7 years tried to provide explanations with size, picture/visual and shape properties. As to the children aged 8 years, they could explain the size, colour, picture/visual, shape and dimension properties. This developmental process is indicated in the literature as well; it is explained with Piaget's (1970) preoperational and concrete operational stages of cognitive development. At the pre-operational stage, children are reported to have a simple logic level, try to provide explanations according to one property of the object and demonstrate development with age. In addition, Van Hiele's (1986) first visualisation stage coincides with Piaget's (1970) concrete operations stage. Children at this stage could explain an object according to its various properties.

In addition, children's knowledge about shadows in spatial contiguity transformations was also investigated. The children aged 6 years provided very little explanation about the shadow; children aged 7 years had knowledge about shadows, explained that the shadow was in front of or behind people, and it came from light; however, it was found that they did not know reflection. As to the children aged 8 years, they were found to know that shadow was the refection in front of the source of light. Therefore, children aged 8 years were found to distinguish two- and three-dimension objects. This case, being a developmental process, is supported in the literature as well. Piaget investigated the explanation of the shadow case in four stages. In the first stage, children aged around 5 years could not explain the source of shadow completely but thought that the shadow came either from the object or from an outer source. In the second stage, children aged 6–7 years thought that shadow was produced by only one object. However, in this stage, children could not say that the shadow was formed in front of the source of light. In the third stage, children aged around 8 years guessed the source of the shadow (Piaget, 1966).

5. Conclusion and Recommendations

This study, which aimed to analyse geometric (topology and projective) transformations of children in the early childhood period according to their age level, found that the children were more successful in stretching-shrinking transformations in topology, and their topologic and projective transformation perceptions increased depending on age. This case might result from the fact that children heard about the activities about stretching-shrinking previously, and this theme has begun to take place in their daily lives more. Dawson (1990) found that children acquired their first knowledge about the changes in their body about topology starting from early ages. Piaget and Inhelder (1956) reported that little children perceived space first in a topological way, later in a projective way and then in a Euclidean way (Baki, 2014).

A number of recommendations were made based on these results. The present study provides a consistent and comprehensive framework in order to expand Van Hiele's model and define topologic and projective intuitions and understanding of children in the early childhood period (George, 2017). This study investigated the children's topology and projective developments according to age before Van Hiele's (1986) first visualisation stage and Piaget's pre-operational stage and at the beginning of the concrete operations stage. Three different age groups were involved in the study. Future studies could include more age groups or 6- or 3-month periods could be utilised instead of age groups; this way, the development of topologic and projective transformations in children could be investigated in a detailed way. This study was conducted with children aged 6–8 years who demonstrated normal development and who had medium socio-economic level. Future studies might involve different socio-economic levels and children in different contexts.

The original Van Hiele model is composed of five learning stages, and each stage has five consecutive instructional stages. The stages are naturally defined inductively. While Piaget reported that development was biological, Van Hiele reported that the progress from one stage to another changed depending on instruction rather than biology (George, 2017). Therefore, MNE curriculum involving early childhood periods could include topology and projective transformations. In-service training on teachers' supporting children's topologic and projective transformations could also be recommended. In addition, children's developmental stages in the topologic and projective transformations could be investigated through experimental studies. In particular, courses involving the topological and projective transformations of preschool and primary teachers can be added to the undergraduate program.

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