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GEOGEBRA, A TOOL TO IMPROVE STUDENTS' VISUAL IMAGING

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Abstract: Visualisation is a necessary ability for students' mathematical education. The appearance of multiple Dynamic Geometry Software (DGS) has a stronger impact each day, and it raises many questions in the minds of researchers. In this paper, we present the findings of our exploratory case study in which a student preferring visual problem solving generally, but with a low spatial ability level, solved a spatial visualisation problem after finishing school lessons of solid geometry. The main objectives were to examine a) the solver's solution (difficulties and impediments) in the paper-and-pencil environment; b) the solver's solution (difficulties and impediments) in the combined use of the dynamic (GeoGebra applet) and paper-and-pencil (Navigation Sheet) environment, and c) whether the combined use of dynamic and paper-and-pencil environment helped the solver to confirm or correct her results and created visual images from the paper-and-pencil solution. The integration of DGS would help the solver promote the creation of correct visual images and the acquisition or the development of her spatial ability. The research findings are interpreted in terms of both the visual image and the spatial ability classification.

Key words: process of visualisation, visual image, spatial ability, dynamic geometry software, spatial visualisation problem

1. Introduction

Recently developed computer programs offer virtual manipulation with virtual objects in teaching solid geometry as an additional possibility to real manipulation with the real solids which was mainly used earlier. The integration of DGS into the teaching process can help students overcome their difficulties resulting from the deficiency of both manipulative materials and paper representations used in solid geometry lessons as well as in understanding proofs (Vallo and Ďuriš, 2013). Technology in geometry education has become relatively mainstream, yet there is still not enough research into its specific effects (Sinclair et al., 2017). Gutiérrez and Jaime (2015) point to the question: Is there any optimal teaching strategy among these three educational environments (environment with physical models, paper-and-pencil representations, and dynamic computer representations)?

In this paper, we present the findings of our exploratory case study focused on both developing the creation of correct visual images as well as acquiring and developing spatial abilities through the integration of the combined use of the dynamic and paper-and-pencil environment. The aims arise whether or not the technology-based tool representing spatial relations dynamically could be fruitful for our representative student when solving spatial visualisation problem. In literature, several research papers are focusing on the relation between the one selected environment (from the three mentioned) and the improvement of students' spatial visualisation abilities (Güven and Kosa, 2008; Baki, Kosa, and Guven, 2011; Miragliotta and Baccaglini-Frank, 2017). It has been proved that the computer environment is the most instrumental in developing students' spatial abilities, followed by the environment of manipulation materials. Also, the traditional class without auxiliary resources is the least favourable environment. Other earlier research papers (Gutiérrez and Jaime, 1993; Gutiérrez, 1996) have shown that each environment and each representation have their positive and negative impacts. Furthermore, to perform a particular activity is more suitable to use a particular environment.

To support the research carried out, we point to the theoretical elements of visualisation allowing us to analyse the proposed activity and the answers of a student involved in our case study. A "process" of visualisation is 'a mental or physical action where mental images are involved' (Gutiérrez, 1996). Mental

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images, external representations, processes of visualisation and abilities of visualisation are four basic elements of Gutiérrez's visualisation model. In this paper, we focus on visual images and students' spatial abilities reflected in solving a spatial visualisation problem. According to (Tartre, 1990), the spatial visualisation task suggests that all parts of representation are moving or altering. A visual image as a 'mental scheme depicting visual or spatial information' was identified by Presmeg (1986) and divided into five categories. The most commonly used ones in geometry are:

1. **Concrete images (pictures in mind)** – figurative mental images of real objects.
2. **Kinaesthetic images** – mental images associated with muscular activity, such as the movement of a finger, hand, head, etc.
3. **Dynamic (moving) images** - mental images in which we imagine the visualised object (or some of its elements) moving. Unlike the kinaesthetic images, there is no physical movement but only visualised in mind.

Secondly, following the paper's objectives, we are concerned with the abilities of visualisation. More specifically, we follow up spatial abilities, which Linn and Petersen (1985) generally refer to 'skill in representing, transforming, generating, and recalling symbolic, non-linguistic information.' As stated in (Gutiérrez, 1996), a student should use several visual abilities. Based on Gutiérrez's classification, the solutions of research participants reflect the following abilities:

- **Figure-ground perception –** the student is able to identify a specific figure by isolating it out of a complex background. This ability is used when the object is composed of several parts or there are several overlapping figures.
- **Perceptual constancy -** the student is able to: a) recognise that some properties of a real object or a mental image are not dependent on colour, position or size; b) remain unconfused when an object/picture is perceived from different viewpoints.
- **Mental rotation -** the student is able to: a) create a dynamic mental image, and b) visualise a moving configuration.
- **Perception of spatial positions or recognition of positions in space -** the student is able to relate a position of a mental image/object/picture to oneself.
- **Perception of spatial relationships -** the student is able to correctly identify the properties of relations of several images, objects or pictures among them or within themselves. For example, identify that the objects are rotated, perpendicular, etc. It is closely related to the previous perception.

However these abilities are very subtle to see them in quantitative research, we conducted an exploratory case study, to observe them deeply in one particular case. Prior to the case study description, we briefly present the results of our previous research.

2. Preliminary study

In April 2018, pedagogical research was carried out with 93 university students from the first bachelor study program Teaching of Academic Subjects. The main objective was to examine whether iconic external representations of 3D objects used in geometry lessons affect students in solving spatial visualisation tasks. Participants solved a non-standard Cube Layer Problem based on visualisation of hidden spatial objects (the main idea of the problem is similar to the following one in section 3.1). Students' solutions were analysed in relation to a) the type of external representations used and, b) the group in what they were assigned by using the Mathematical Processing Instrument (later MPI) (Presmeg, 1985). This valid and reliable research instrument measures the preference of using visual methods when solving mathematical problems. To explain, Presmeg (1986) defines the Mathematical Visuality (later MV) as follows: "*A person's mathematical visuality is the extent to which that person prefers to use visual methods when attempting mathematical problems which may be solved by both visual and nonvisual methods*." Based on the MV score students were divided into three groups:

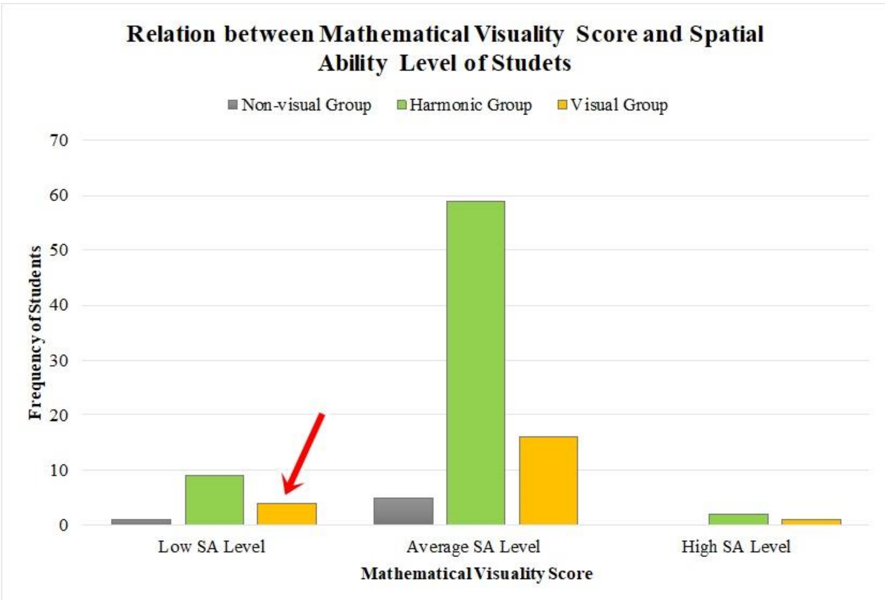
1. **Visual group -** the students who prefer using visual methods (methods that include visual imagery, with or without diagrams, as an essential part of the solution) in solving mathematical problems that can be solved by either visual or non-visual methods. We named these students as 'visual group students'.

2. **Harmonic group** - the students who do not prefer either visual or non-visual methods in mathematical problem solving. The method depends on the task. We named these students as 'harmonic group students'.

3. **Non-visual group** – the students who do not prefer using visual methods in solving mathematical problems that can be solved by either visual or non-visual methods. We named these students as 'non-visual group students'.

In summary, many common characteristics were observed among the solutions of individual students in both the first and the second classification. The students were not very successful in solving a non-standard spatial visualisation problem when using a visual method was required. Most of the students relied much more on drawn external representations than manipulation with their visual images. The detailed research outcomes can be found in (Vágová, 2018).

Currently, starting and inspired by the research mentioned above, we attend to observe and evaluate spatial geometry teaching on 87 students at three selected grammar schools. Moreover, we are interested in figuring out and testing a set of activities based on the combined use of physical and digital resources to promote the development of students' spatial abilities. The participants' MV score (by MPI) before starting, and the spatial ability level (by standardized Spatial Ability Test) after finishing space geometry lessons, were tested and divided into 9 individual classes (see Graph 1). Reliability and construct validity of the Spatial Ability Test were tested and judged to be satisfactory by the Slovak National Institute for Certified Educational Measurements.



Graph 1. *Classification of students in relation to the mathematical visuality score and spatial ability level*

One of the main research objectives is to examine the students' strategies in solving spatial visualisation and spatial orientation problems requiring visual methods in relation to the group they had been assigned to according to the above-mentioned research instruments.

In the next paragraph, we present a case study on both the paper-and-pencil and the combined dynamic and paper-and-pencil solutions of one visual group student with a low spatial ability level (see the red arrow in Graph 1) who solved our non-standard spatial visualisation problem, 'Look Inside' (see section 3.1). We intentionally picked up a student from this particular class because of a non-standard combination of MV score and spatial ability level. Our goal was to see in detail the problem solving steps of a student who, on the one hand, prefers visual methods in solving a problem but, on the other hand, has a low spatial ability level. Concretely, our goals were to examine how this student would be able to solve the problem in a) the paper-and-pencil environment first, and then in b) the paper-and-pencil environment in combination with the dynamic geometry one, and c) whether the combined use of dynamic and paper-and-pencil environment helped the solver to confirm or correct results and created visual images from the paper-and-pencil solution. The participant was not informed of her MV score and spatial ability level before the problem was solved. We assumed that the knowledge of the results could affect her method of solution. The research findings are interpreted in terms of the Presmeg's visual imagery classification and Gutiérrez's classification of spatial abilities.

3. Exploratory case study

3.1. The problem description

The solution of the problem named Look Inside (later LI) was constructed in GeoGebra, the final version can be retrieved here: https://www.geogebra.org/m/pfsjk6vz (see Figure 1).

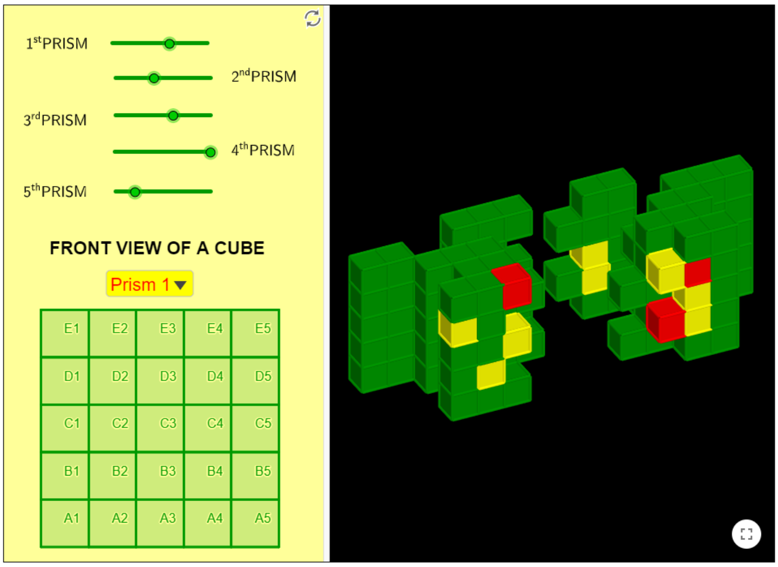


Figure 1. *Illustration of the last version of the GeoGebra applet available on* https://www.geogebra.org/m/pfsjk6vz

There are three possibilities for how the correct representation leading to the solution of LI problem can be visualised. Users can highlight the correct unit cubes (see Figure 2), hide the incorrect unit cubes (see Figure 3) or combine highlighting and hiding these objects.

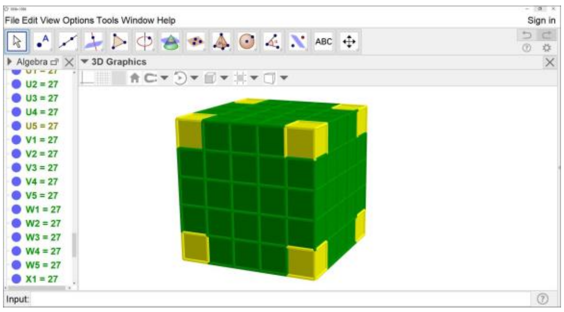


Figure 2. *Illustration of solution e) – option 1*

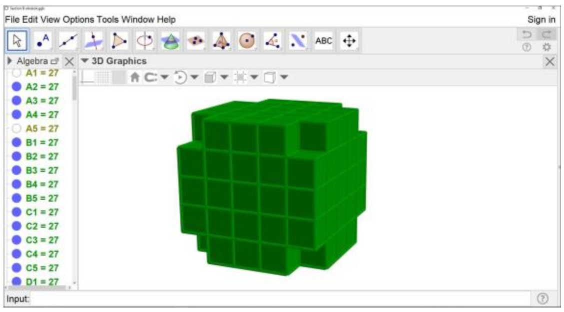


Figure 3. *Illustration of solution e)- option 2*

Look Inside

Imagine an unlimited number of small unit cubes (all the same size 1×1×1). From these unit cubes, you start to build bigger and bigger cubes in such a way that the initial cube will be wrapped into other unit cubes. This "unit cube wrap" can be called a layer. Then imagine the built cube C of the size 5×5×5 unit cubes and try to respond to the following questions:

a) *How many layers of cube C do you have to unwrap to get to one single unit cube?* (2 layers)

b) *How many unit cubes does each layer have?* (26 and 98 unit cubes)

c) *How many unit cubes are hidden in cube C that cannot be seen at all?* (27 unit cubes)

d) *How many unit cubes of the visible layer touch the faces of unit cubes of the previous layer?* (54 unit cubes)

e) *Remove the unit cubes from cube C that have just three touching faces with the other unit cubes. How many unit cubes remain in the visible layer?* (90 unit cubes)

The solution is written in brackets at the end of each question. The problem is not difficult in terms of mathematical operations. The secret of its solution lies in the ability to visualise the configuration of 3D object positions and manipulate these objects in mind.

3. 2. Research process and objectives

The selected student with a low spatial ability level solved the LI problem twice in a row:

- At first, she solved the problem without any physical models or plane representations (paper or computer) of cube C. She could use "only" her visual images with or without a graphical record of these images. The primary objective was to examine the solver's strategy in solving this visualisation problem after finishing the spatial geometry lessons. At school, students used either one type of plane representations of geometric solids or transparent physical models of a cube.
- Secondly, the student solved this problem with:
- Navigation Sheet (later NS), which is a paper representation of the front view of each vertical slice (prism of size 5×1×5) of cube C. Each unit cube is marked with a single alphabetical letter in combination with a number (see Figure 4).

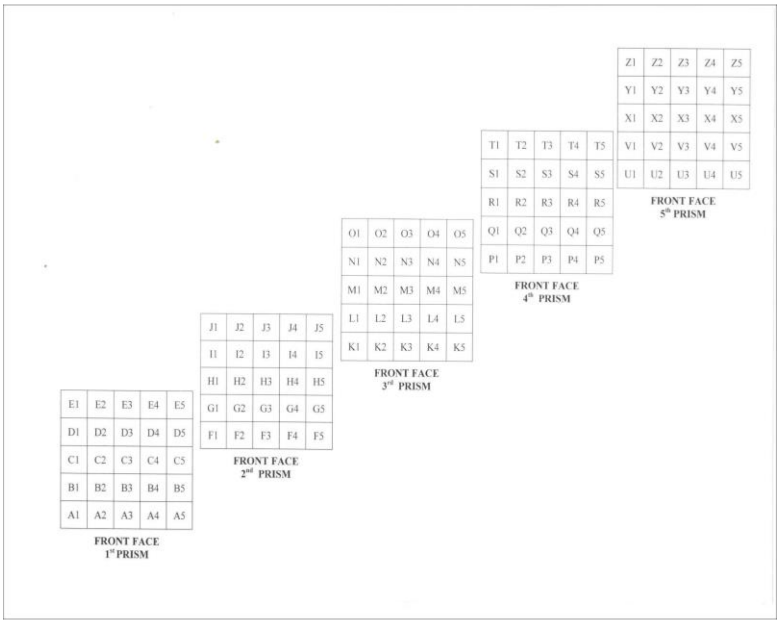


Figure 4. *Illustration of Navigation Sheet*

- GeoGebra applet - dynamic computer representation of cube C made of unit cubes in GeoGebra (see Figure 5).

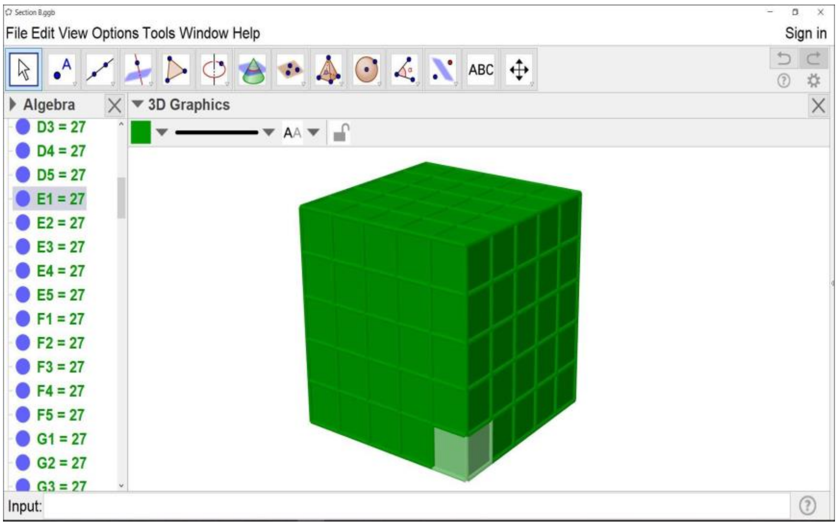


Figure 5. *Illustration of the GeoGebra applet used by the participant*

The principle of the solution method is elementary. Each unit cube has its name (see Figure 4 and Figure 5) shown on the left side of the GeoGebra screen (see Figure 5), thanks to which the solver could easily hide or change the colour of the unit cubes (initially all unit cubes were green). The student had to work mainly with a dynamic representation of cube C, and the Navigation Sheet helped her manipulate the unit cubes on the computer screen. However, in regard to the answering of question a), she initially had to try to answer the question without navigation. We consider it essential for the solver to try to manipulate the cube C herself, identify an object and its unit cubes when it is moved or when the object is seen from different viewpoints. In other words, to absorb the situation and to look inside the cube to find the correct answer. The main goal was to examine how the student would be able to solve the LI in two different environments (paper-and-pencil and dynamic). Also, to confirm or correct her previous solution and, promote the creation of correct visual images and the acquisition or the development of her spatial abilities.

3. 3. The student's solution in the paper-and-pencil environment

In responding to each question of the problem, Look Inside, the student preferred to start with a drawing of cube C. Question a) was the only exception, where she determined the number of layers by relying on her visual image in particular. The student's paper representation of cube C, which was drawn after answering the question, served only to confirm her inference (see Figure 6). As soon as she read the problem, she said: 'Hmm … wait for a second, please … [She closed her eyes and moved her hands around in space.] Yes! … Hmm…. Yes! [She opened her eyes.] I know exactly how these cubes are configured. But still, I would rather draw it to be sure. [She drew a cube with the individual unit cubes marked.] So, yes. There are - one, two, three, and four - layers.' Finally, she added, 'By the way, I have to say that I deal with almost every task by drawing it. It is my starting point for solving mathematics.'

Notably, the participant did not try to draw a configuration of 3D object positions (see Figure 5). She preferred to draw a usual representation (the right front view) of the cube where the individual unit cubes were marked. This visualisation process involved the use of *kinaesthetic* and *dynamic images,* but the student had difficulties in using them. The mistake she made reflected the lack of *mental rotation* (incorrect unwrapping of cube C) and *perception of spatial positions and relationships* (incorrect image of the inner configuration of cube C).

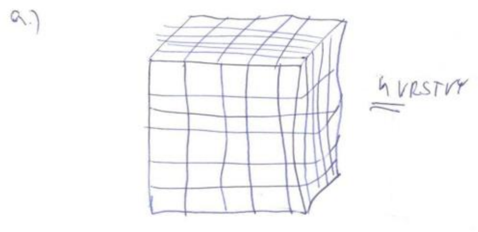


Figure 6. *Illustration of the student's solution: question a)*

In Figure 7, there are shown the student's visual images and calculations when answering questions b), c), d) and e). Importantly, the most significant difference between drawings in these questions and the one in question a) is that individual unit cubes are not identified in any way. The solver used only the *concrete picture*. She said, 'Now, I am going to draw it without marking those unit cubes to make it clearer. That is how I can see the inside of the cube better.' As we mentioned above, at the beginning of answering these questions, she drew a situation as a starting point for thinking about the solutions. The desired unit cubes were calculated mainly by using drawings that helped the student not to make a mistake. In other words, she needed external support for mental manipulation of each desired visual image.

In reference to the analysis of drawings and results of calculations, the visual group student with a low spatial ability level did not score any points out of the maximum 6. At the same time, ironically, the principle of the problem was understood correctly, and numerical calculations were correct in questions c) and d). The solver's failure in the rest of the problem was caused by the mistakenly appointed number of the layers at the beginning of the solution. Additionally, in the second question, instead of calculating the number of layer unit cubes, she calculated the volume of individual cubes.

We assume that the student would have been able to calculate the right number of unit cubes if she could have looked inside the cube and seen the configuration of 3D object positions (with external support). However, the initial unwrapping of cube C to one single unit cube was not correct. We believe that the participant experienced difficulties when the solution required a different mental manipulation (unwrapping) of visual objects (unit cubes) as she had used in solid geometry lessons. The mistakes of these solutions reflected the lack of the abilities of *figure-ground perception*, *perceptual constancy*, *mental rotation*, and *perception of spatial positions and relationships* were more evident. Each of them was associated with the incorrect identification of layers and the incorrect calculation of the cubes in the individual layers. Also, the solver did not attempt to draw cube C from a 1×1×1 cube by wrapping it into other unit cubes in any of the questions. She automatically drew a 5×5×5 cube in each question and then put smaller and smaller cubes into it.

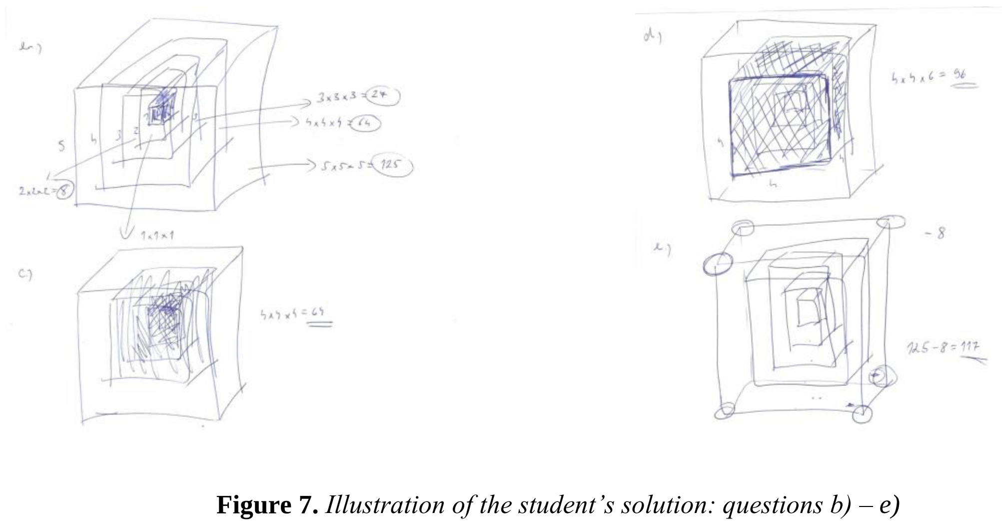


Figure 7. *Illustration of the student's solution: questions b) – e)*

We noticed that the visual group student with a low spatial ability level was overly fixated on her external support when responding to the questions. In other words, the *concrete pictures* that had been stored in her mind formed an essential part (either mental or also drawn on paper) of many of her solutions to most mathematical tasks. With this in mind, when the problem required the use of a visual representation that had not yet been adopted, she was not able to solve the problem successfully.

3. 4. The student’s solution in the combined use of the dynamic and paper-and-pencil environment

When solving the problem in the combined use of the dynamic and paper-and-pencil environment, the student used both the GeoGebra applet and the paper representation of cube C on Navigation Sheet.

3.4.1. Questions a) and b)

The initial instructions were explained to the solver before the start of the second solution. In the beginning, she could work (move, rotate, enlarge, hide, etc.) only with a dynamic model of cube C or its unit cubes on the computer screen. This manipulation without any external support should help the student to become familiar with the use of our GeoGebra applet, understand the principle of working with objects and look at a cube dynamic model in several possible ways.

At first, the visual group student made several rotations one after the other, and then she hid individual unit cubes starting with cube A1. Depending on where the unit cube, A1, was hidden, she rotated cube C to the position shown in Figure 8. When the whole front face was invisible, she said, ‘Hmm … this is incorrect, isn’t it? There is a small number of unit cubes inside. Why are there just 3 and not 4 small cubes?’

In this excerpt from the answer to question a), we can see that the student realised the incorrect mental image used in the first solution (*change of mental picture*). She added, ‘That means I have everything wrong! [She closed her eyes and made some hand gestures.] There are not four, but only three or two layers. [The student opened her eyes, and she was moving her finger on the computer screen pointing to cube C.] So, this is the beginning, the centre cube [she pointed to the central cube (the red cube in Figure 8) which was still green at that moment] that was wrapped into these small cubes [she pointed to the next cubes that are yellow in Figure 8, but they were still green at that moment], and then these cubes were wrapped into these unit cubes [she pointed to the green cubes in Figure 8]. And, that’s all, there are only two layers.’ As a consequence, the student recognised her mistake when manipulating the dynamic cube and determined the correct number of its layers. Importantly, she changed her perspective; now she built cube C wrapping the initial unit cube into the unit cube wraps layer by layer.

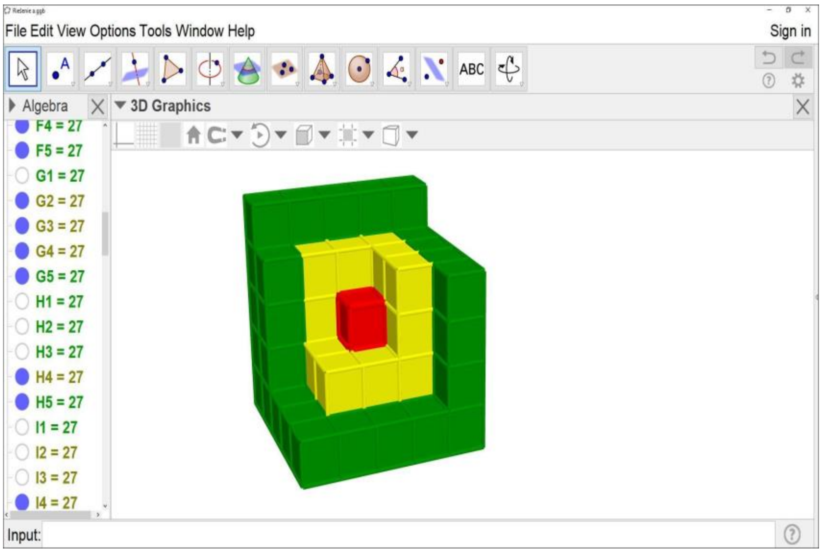


Figure 8. *Illustration of the dynamic part of the solution: a) and b)*

We can see that the student created a new visual image for the same situation. At the same time, the development of *concrete* and *dynamic images* was observed. Thereafter, the NGS sheet along with markers were offered to her as additional tools. Under those circumstances, the student coloured the unit cubes of individual layers on the NGS (see Figure 9), and then these unit cubes were also colour-coded on the dynamic representation (see Figure 8). This situation confirms the student's ability to work with two different types of cube representation. The primary objective of the combined use of a combination of dynamic and paper-and-pencil environments was to promote the development of the student's abilities, the deficiency of which was demonstrated in the pen and paper paper-and-pencil solution (*mental rotation, the perception of spatial positions and relationships*).

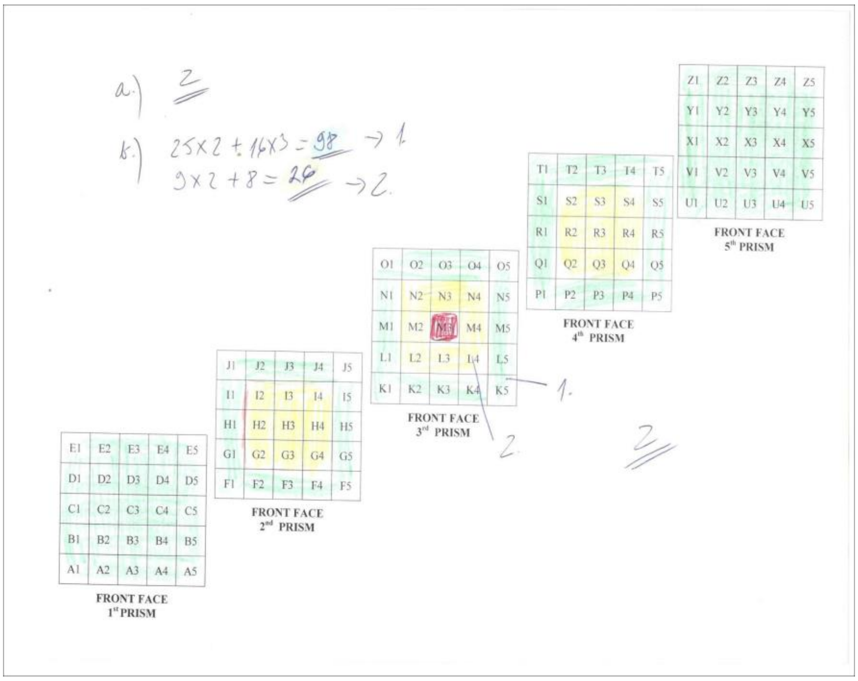


Figure 9. *Illustration of the paper-and-pencil part of the solution: a) and b)*

We noticed that the visual group student was able to interconnect these two environments and respond to both questions a) and b) correctly. In answering the second question, the number of unit cubes was first calculated in the dynamic environment and then using the NGS plane representation.

3.4.2. Question c)

Question c) was more accessible for the student compared to the first two questions. As in previous cases, the desired unit cubes were coloured and crossed off on NS (see Figure 11), and then the missing (invisible) yellow cubes from the previous solution were shown (see Figure 10). The student attempted to use the *dynamic image* and was freed from drawing a concrete picture.

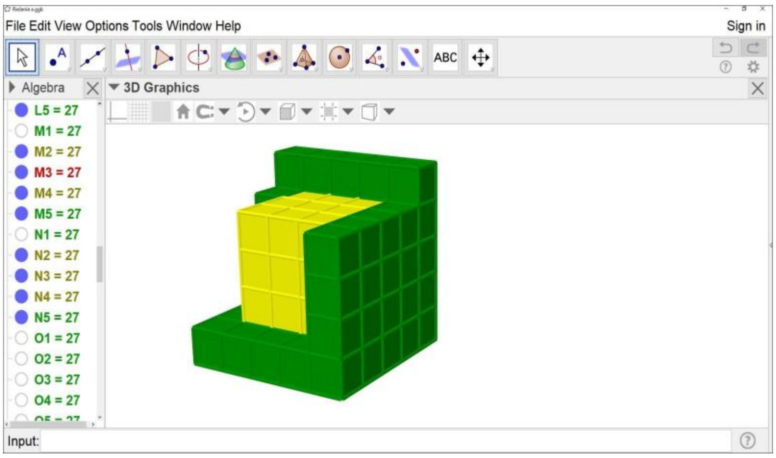


Figure 10. *Illustration of the dynamic part of the solution: c)*

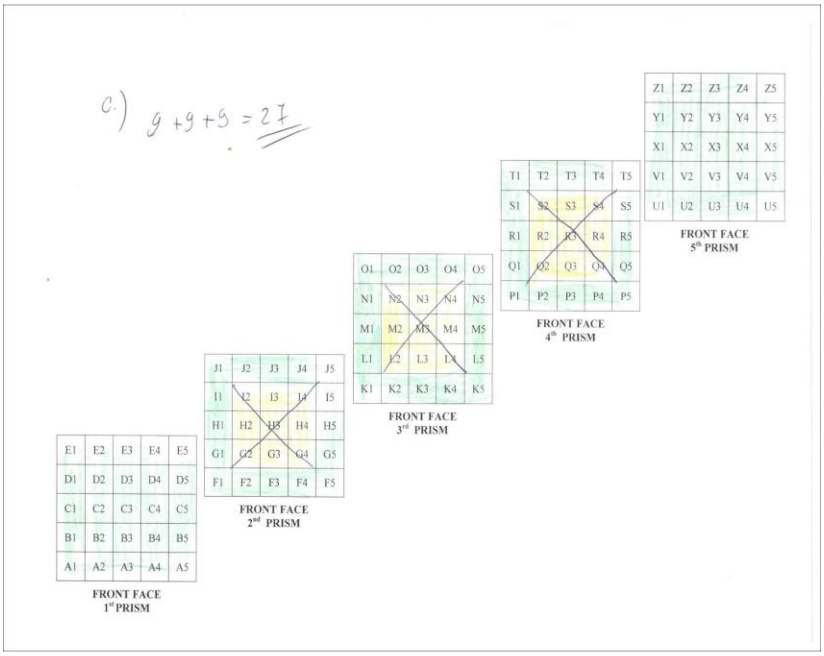


Figure 11. *Illustration of the paper-and-pencil part of the solution: c)*

Returning to the paper-and-pencil solution, the incorrect answer was caused only by the mistakenly appointed number of layers related to question c). In Figure 11, it can be seen that she was able to interconnect two different environments and calculate the correct number of unit cubes again. The primary goal of this solution was to develop the student's abilities of *figure-ground perception, perceptual constancy* and *perception of spatial positions and relationships*.

3.4.3. Questions d) and e)

Compared to the previous inquiry, question d) was the most difficult for our participant. Although she understood that the green cubes (see Figure 10) having one touching face with the yellow cubes (see Figure 11) had to be calculated, she had difficulties in colouring them on the paper.

The lack of the abilities of *perceptual constancy*, *mental rotation* and *perception of spatial positions and relationships* was the most apparent in solving this task. The most significant difficulties were associated with the colouring of the cubes in the 2nd, 3rd and 4th prisms on NS. The breakthrough occurred when the student attached the NS to the computer screen and compared the two representations. The student said, 'There are so many blocks on each face. So, this [she pointed to the front face of the 3rd, 4th, and 5th prism] must be done in 9 unit cubes.' The promotion of the development of the abilities mentioned and the *dynamic image* was the main objective here.

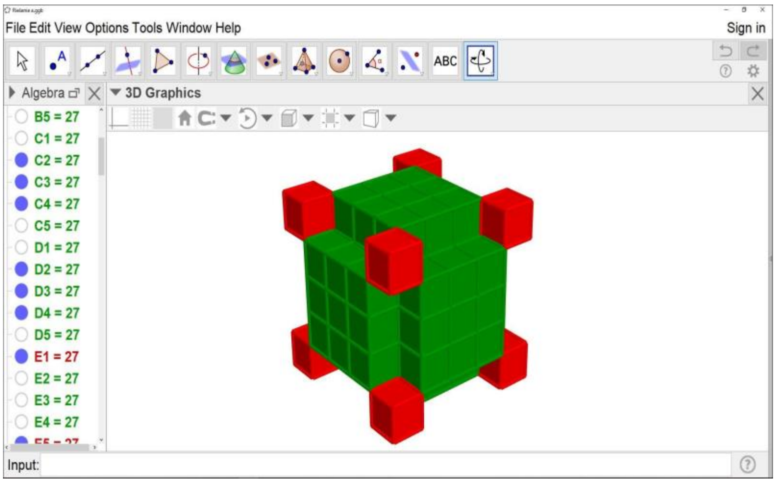


Figure 12. *Illustration of the dynamic part of the solution: d) and e)*

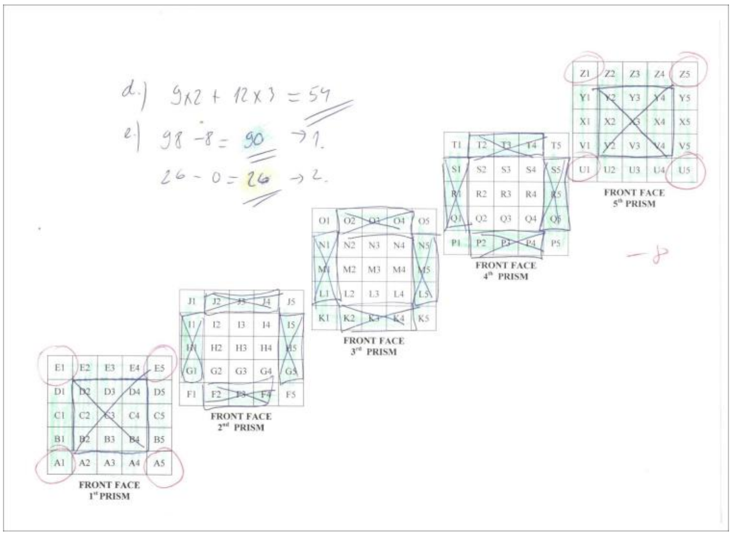


Figure 13. *Illustration of the paper-and-pencil part of the solution: d) and e)*

Finally, question e) was answered very quickly by the solver. The unit cubes with just three touching (invisible) faces were circled with a red marker on the NS (see Figure 13) and also colour-coded in the dynamic representation (see Figure 12). The student was able to interconnect the dynamic and the paper representation very quickly and determine the correct number of desired unit cubes. We can notice that the *dynamic image* was involved and used efficiently in answering the last question of the problem.

4. Discussion

The detailed analysis of our student's problem solving steps shows that the use of the *concrete image* was the most evident in the paper-and-pencil solution (see Sol. 1 in Table 1). We found out that the solver experienced difficulties and impediments when the solution required a different mental manipulation of visual objects compared to that which was used in solid geometry lessons. These difficulties were especially observed when she was creating dynamic or kinaesthetic images and applying them to solve the problem correctly. We assume that the origin of the difficulties was the lack of her *ability of figure-ground perception* (identifying the outer and the inner layer of cube C, determining the number of unit cubes in each layer or the number of touching unit cubes), *perceptual constancy* (looking on the "unwrapped" cube C), *mental rotation* (unwrapping cube C, removing the unit cubes from the visible layer of cube C), *perception of spatial positions* (placement of the unit cubes inside cube C) and *perception of spatial relationships* (identifying unit cubes from the question d)). She did not score any points out of the maximum 6.

Table 1. *Using specific images in individual questions*

	'Look Inside' questions									
	a)		**b)**		**c)**		**d)**		**e)**	
	Sol.1	**Sol.2**	**Sol.1**	**Sol.2**	**Sol.1**	**Sol.2**	**Sol.1**	**Sol.2**	**Sol.1**	**Sol.2**
Concrete Image	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
Kinaesthetic Image	yes	yes	-	yes	-	yes	-	yes	-	yes
Dynamic	yes	yes	-	yes	-	yes	-	yes	-	yes
Correct answer	*no*	*yes*	*no*	*yes*	*no*	*yes*	*no*	yes	*no*	yes

On the other hand, in the combined use of the dynamic and paper-and-pencil environment, the student demonstrated the ability to interconnect these two environments to solve the problem correctly. Afterwards, she checked and corrected her answers from the previous solution and scored 6 points out of the maximum 6 (see Sol. 2 in Table 1). The participant recognised her mistakes while manipulating the dynamic cube and determined the correct number of layers and its unit cubes. In other words, the solver was able to interconnect the GeoGebra applet (the dynamic environment) and Navigation Sheet (the paper-and-pencil environment) to confirm or correct her results and visual images. Based on the solution's analysis can be assumed that the combined use of the dynamic and paper-and-pencil environments has contributed to the support of both the creation of correct visual images and development or improvement of her spatial skills. Thanks to the application of GeoGebra, the solver could manipulate 3-dimensional objects, look inside them, unwrap and wrap cube C, change the colour of individual unit cubes, and correct the incorrect visual images in particular. The use of technology-based material was fruitful for our representative student and offered her a possibility to solve the problem from a new, different perspective. Our research project is still in progress but the findings encourage us to go forward and develop a set of activities based on the combined use of physical and digital resources (virtual objects). Consequently, in cooperation with a grammar school teachers, we intend to carry out the pilot verification of the designed learning material with 11th grade students to promote the creation of correct visual images and the acquisition or the development of their spatial abilities.

5. Conclusion

This paper reported an exploratory case study on a visual group student with a low spatial ability level who solved 'Look Inside' spatial visualisation problem after finishing school lessons of solid geometry. The student solved the problem sequentially in two different environments, in the paper-and-pencil environment, and then in the combined use of the dynamic and paper-and-pencil environment. The main objectives were to examine the student's strategy in problem-solving with respect to these environments. We found out that the combined use of dynamic and paper-and-pencil environments (using Navigation Sheet) helped the student to confirm or correct her results and images from the first solution. The integration of DGS could help the solver promote the creation of correct visual images and the acquisition or the development of her spatial abilities (see Table 1). The research findings were interpreted in terms of both the visual image and spatial ability classifications (see Sections 3.3. and 3.4.).

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