



PROSPECTIVE MATHEMATICS TEACHERS' COGNITIVE COMPETENCIES ON REALISTIC MATHEMATICS EDUCATION

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Abstract

Realistic Mathematics Education (RME) is based on the idea that mathematics is a human activity; and its main principle is to ensure the transition from informal knowledge to formal knowledge through contextual problems. This study aims at revealing how RME is configured in the minds of prospective mathematics teachers and their cognitive competency in that sense. For that purpose, at the end of the process, in which the approaches used in mathematical education including RME are examined and interpreted, 32 prospective teachers were asked various open-ended questions. Moreover, they were expected to pose contextual problems that could be used in RME. After analysing the obtained data via qualitative research techniques, it is seen that the majority of the prospective teachers possesses theoretical knowledge on RME. However, it is also observed that their ability to present its differences and similarities with other approaches and to pose contextual problems suitable to RME has been decreased.

Keywords: Realistic Mathematics Education, Prospective Mathematics Teacher, Cognitive Competency, Contextual Problem

Abstrak

Pendidikan Matematika Realistik (PMR) didasarkan pada gagasan bahwa matematika adalah aktivitas manusia; dan prinsip utamanya adalah memastikan transisi dari pengetahuan informal ke pengetahuan formal melalui masalah kontekstual. Penelitian ini bertujuan untuk mengungkapkan bagaimana PMR dikonfigurasi dalam pikiran calon guru matematika dan kompetensi kognitif mereka dalam pengertian itu. Untuk tujuan itu, pada akhir proses, 32 calon guru ditanyai berbagai pertanyaan terbuka, yang mana pendekatan yang digunakan dalam pendidikan matematika termasuk PMR diujikan dan ditafsirkan. Selain itu, mereka diharapkan untuk dapat mengemukakan masalah kontekstual yang dapat digunakan dalam pendekatan PMR. Setelah menganalisis data yang diperoleh melalui teknik penelitian kualitatif; terlihat bahwa mayoritas calon guru memiliki pengetahuan teoritis tentang PMR. Namun, pengamatan lebih jauh terkait kemampuan mereka untuk menyajikan perbedaan dan persamaan antara pendekatan PMR dengan pendekatan yang lain dan pengajuan masalah kontekstual yang sesuai dengan PMR, telah berkurang.

Kata kunci: Pendidikan Matematika Realistik, Calon Guru Matematika, Kompetensi Kognitif, Masalah Kontekstual

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The studies conducted in mathematical education, which is highly impacted by theoretical and methodological framework of psychology (Sriraman & English, 2010), have paved the way for the innovative ideas on learning including new learning outcomes, new types of learning processes, and new instructional methods that are both wanted by society and currently stressed on in psychological and educational theory (Simons, Van der Linden, & Duffy, 2000; Yackel, Gravemeijer, & Sfard, 2011). These ideas occur in a manner that is totally different from behavioral approach studies (Aubrey & Riley, 2016) which traditional education methods are based on. And their approaches can be fundamentally categorized as cognitive approaches and constructivism. Cognitive theories focus on

the conceptualization of one's learning process and emphasize how information is received, organized, stored and retrieved in one's mind (Davis, 1990; English, 1995; Ertmer & Newby, 1993; Jonassen, 1991; OECD, 2003). Despite being viewed as a branch of cognitivism as a result of expressing learning as a cognitive activity (Ertmer & Newby, 1993), constructivism is based on the idea that the mind filters the input received from life in order to create its own reality (Jonassen, 1991a) and the information is not simply received but actively constructed by the person himself/herself (Driscoll, 2005).

The different perspectives in education led to radical changes in objectives and nature of mathematics as well as many other areas. This change in mathematics education particularly emphasizes that mathematics learning can occur only when students discover things through actual experience and structured problem-solving procedures and finally by means of interaction process among students and/or teachers (Kwon, 2002). In addition to this, many countries aiming at raising productive and innovative individuals in the 21st century made radical changes in their curriculum with the purpose of attaching much more importance to the quality of their mathematics education. Essentially, various countries such as UK, USA, Singapore, Finland and Australia believe that mathematics enables the improvement of thinking capacity as well as its use in real life in a critical, creative and logical manner. These countries have begun to pursue the goal of raising individuals with problem solving, reasoning, and connecting skills, as well as productive disposition and conceptual comprehension (Cai & Howson, 2013; Stacey, 2005). It is believed that among these skills, the problem solving is the key skill in the 21st century (Barell, 2010). Moreover, it is asserted that the teaching process can be implemented for students through case-based or problem-based learning methods by presenting authentic problem situations called as real-world problem (Barell, 2010; Barrows, 1986). Conceptual comprehension, on the other hand, is a process that takes place in coordination with such skills (Wu, 1999) and involves the understanding of concept, relation and operation (National Research Council, 2001).

This new perspective shares many common points with the theoretical perspective of Realistic Mathematics Education (RME) (Bray & Tangney, 2016; Kwon, 2002) as discussed by Freudenthal (1973). RME focuses on mathematization that is actualized through the re-invention of formal mathematics. In RME, the student is informally guided by the teacher in a class-interaction, thus is encouraged to utilize self-developed models in order to solve and interpret the experientially real contextual problems (Dawkins, 2015; Gravemeijer, 1994, 1999; Treffers, 1991). In this regard, it is believed that RME contributes to the formation of the targeted skills such as problem solving, reasoning and connecting; as well as to conceptual comprehension (Borko & Putnam, 1996; Hadi, 2002).

Teachers' Pedagogical Knowledge (PK), which includes knowledge the teachers have on curriculum and teaching methodology as well as on how to teach them (Lianghuo, 2014), and Pedagogical Content Knowledge (PCK) that is referred to as "special amalgam of content and

pedagogy" (Shulman, 1987), are important determinants of instructional quality that impact students' learning gains and motivational development (Baumert & Kunter, 2013). And also they have vital importance in gaining the targeted skills (Borko & Putnam, 1996). This situation causes many results, which could affect teacher education and causes major changes in prospective teachers' knowledge (Battista, 1994; Putnam & Borko, 2000; Szydlik, Szydlik, & Benson, 2003). However, teachers' efforts to change their PK and practices are usually not reflected on their tendencies at skill-based applications in the targeted curriculum (Anderson, White, & Wong, 2012). Although, it is difficult to evaluate teachers' PCK (Beswick & Goos, 2012), understanding the pedagogical theories underlying the radical changes about mathematics education and detecting effective teaching strategies are significant in terms of being able to professionally assessing teachers (Sowder, 2007). Furthermore, besides prospective teachers' knowledge of general and specific approaches; what is essentially important is whether they comprehend when, where, how, and why they should use these approaches, rather than their variety (Feiman-Nemser, 2001; Tsamir, 2008). However, many prospective teachers are not ready to use them the way they should (Herman & Gomez, 2009). Therefore, it is believed that it is important to know to what extent prospective teachers have cognitive competences about these approaches, in order to be able to choose the approach that will be used depending on place and time and to implement them in a manner that is suitable for the purpose.

In a general sense, the study aims at identifying prospective teachers' cognitive competency regarding the approaches before moving onto practice from theory. In this regard, the study mainly focuses on RME in terms of the approach and the answer to the question of "How are the cognitive competencies of prospective mathematics teachers related to RME?" has been sought. Thus, the sub-problems of "How do prospective mathematics teachers explain RME and its implementation? How do they interpret the similarities and differences with other approaches? How a contextual problem do they pose in compliance with the theoretical structure of RME?" have been discussed.

In the below-mentioned theoretical framework, firstly the definition and primary elements of RME have been explained, and information related to its implementation has been presented. Subsequently, the criteria used in determining the cognitive competency and the levels of cognitive competency have also been explained.

THEORETICAL FRAMEWORK

Realistic Mathematics Education (RME)

RME, which is based on the idea that mathematics is a human activity (Freudenthal, 1973) and the idea that student achieves formal mathematics knowledge by using his/her informal knowledge by means of re-inventing under the guidance of a teacher (Treffers, 1991) has a significant place in the studies conducted in the field of mathematical education (e.g. Barnes, 2004; Beswick, 2011; Bray & Tangney, 2016; Makonye, 2014; Rasmussen & King, 2000; Streefland, 1991). In this approach, the informal knowledge in real life is transformed into formal knowledge after being abstracted and is

again associated with real life as a result of mathematical implementations (De Lange, 1996). Transformation is believed to be achieved by contextual problems that are experientially real to the students (Gravemeijer, 1999). The process of conceptual mathematization in RME has been given in Figure 1. In this process, mathematical concepts start to develop from the real world and ends with the reflection of the solution back to the real world.

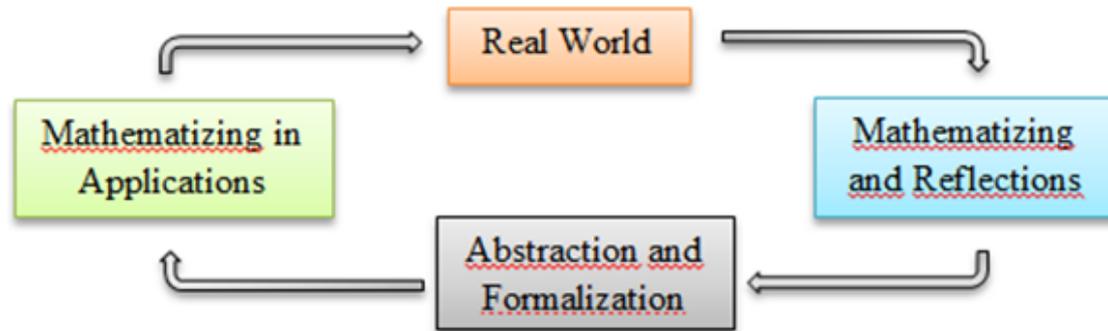


Figure 1. Conceptual mathematization (De Lange, 1999)

The contextual problems used in RME are mathematical problems presented in the real life situations that children are familiar with, through stories that are fictionalized from the real world. These problems can be a word problem, a game, a drawing, a newspaper clipping, a graph or the combinations of such elements. At the same time, a pure mathematical problem can also be a contextual problem. However, the main point here is to what extent the problem would fulfill the criteria of being experientially real or authentic; and thus would provide a concrete orientation towards a new concept/skill; and would also allow utilization of prior knowledge (De Corte, 1995; Doorman, Drijvers, Dekker, van den Heuvel-Panhuizen, de Lange, & Wijers; 2007; Gravemeijer, 1999).

Gravemeijer (1994, 2001) emphasizes the necessity for three main elements while designing education in RME, which includes the following:

1. Guided reinvention through progressive mathematization
2. Didactical phenomenology
3. Self-developed or emergent models

Guided reinvention is based on configuring and organizing problems in order to discover rules by revealing mathematical factors in a problem. This research, which is conducted with a strong intuitional component, is considered to be the discovery or reinvention of mathematical conception (De Lange, 1987; Freudenthal, 1973, 1991). In this process, the teacher should design the roadmap to enable students to learn correctly and should provide the students with the opportunity to experience a process that is similar to the discovery process of mathematicians (Gravemeijer, 1994; 2001). By doing so, students have the opportunity to obtain knowledge by themselves (Freudenthal, 1991;

Gravemeijer, 1999; Yackel & Cobb, 1996) and they mathematize the contextual problems by solving them (Treffers, 1987; 1991).

Guided reinvention through progressive mathematization can be considered as a two-stage process: horizontal and vertical mathematization. Horizontal mathematization is where one uses informal strategies by schematization in order to define and solve contextual problems, in other words transforming real life problems into mathematical problems. Vertical mathematization is to abstract the conception in the world of symbols and solve the problem by adopting different models or find the relevant algorithm by using mathematical language in the light of informal strategies (Freudenthal, 1991; Gravemeijer, 1994; Treffers, 1987, 1991; Van den Heuvel-Panhuizen, 2003). In Figure 2, horizontal and vertical mathematization is described.

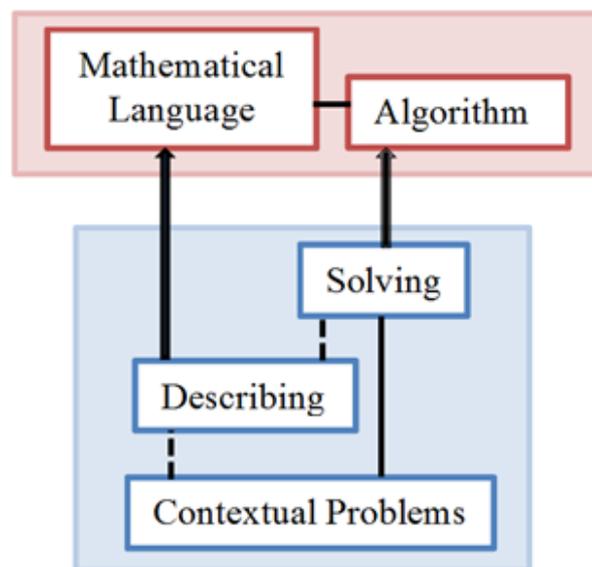


Figure 2. Horizontal and vertical mathematizations (adapted from Gravemeijer, (1994))

Didactical phenomenology requires working with phenomenon that are meaningful to students in the process of learning mathematics, can be organized by students, are stimulating for the learning process and meet four functions including concept formation, model formation, applicability and practice (Gravemeijer, 1994, 2001; Treffers & Goffree, 1985).

Self-developed or emergent model bridges informal knowledge of students with formal knowledge while solving a problem. At the beginning, the student develops a model, which gradually becomes a dynamic and holistic model compatible with his or her own mathematical thinking after generalizing and formalizing processes (Gravemeijer, 2001; Treffers, 1991). Thereby, at the end of this process, which is named as the transformation from 'model-of' to 'model-for', the student obtains a model that enables him/her to achieve mathematical reasoning (Gravemeijer, 1999).

Cognitive Competency

Cognition refers to the variables with respect to the kind and quantity of information, and the

classification of relations among the variables of information (Kraiger, Ford, & Salas, 1993). Cognitive domain in learning contains learnings, in which person's mental sides are in the foreground (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956; Bloom, 1994). Cognitive competency is a psychological construction which cannot be directly observed but can be inferred from the behavior or performance of an individual on content-relevant tasks (Wang, 1990). Researchers have asserted various taxonomies in order to evaluate this kind of learnings. Some of these taxonomies are Bloom's Taxonomy (Bloom, et al. 1956), Revised Bloom Taxonomy (Anderson & Krathwohl, 2001), Barrett's Taxonomy (Barrett, 1976), Pearson-Johnson Taxonomy (Pearson & Johnson, 1978), Webb's Depth of Knowledge Levels (Webb, 1997, 1999) and Program for International Student Assessment (PISA)'s Competency Levels (OECD, 2003). Bloom's Taxonomy among such taxonomies, is the first and most widely accepted classification (Granello, 1995) in the subject of cognitive abilities and educational objectives used in education. In a similar way, PISA is implemented and reported in many countries and regarded as a new approach in national and international evaluation (Sadler & Zeidler, 2009).

Categories are classified in Bloom Taxonomy from simple to complex and from concrete to abstract acting as prerequisite for one another: Knowledge refers to a person's acts of remembering such as recognizing, expressing when asked, or repeating the characteristics of any object or event from his/her memory; Comprehension means interpreting, assimilating, and expressing the obtained targets without losing their meanings at knowledge level; Application, is when a person implements knowledge by solving the problem in a new situation by making use of his/her learning at knowledge and comprehension level; Analysis refers to cognitively differentiating among the items of a pattern or knowledge in terms of their relationships and organizations; Synthesis means bringing together and creating a whole of the items in such a way that they would bare characteristics such as innovation, originality, and creativity based on certain relationships and rules. Evaluation is when a person decides whether the products created are competent enough by stating justifications (Bloom, et al. 1956; Bloom, 1994; Krathwohl, Bloom, & Bertram, 1973; Krathwohl, 2002).

In PISA, OECD (2003) has determined three levels for detecting the competency levels of students in order to define their cognitive activities. These levels include reproduction, connection, and reflection. In reproduction level, already known contents, previously used knowledge, standard algorithms, and elementary formula are used and basic operations are conducted. In connection level, less commonly known contents are interpreted and explained; systems, representation of which are different, are obtained through association; and necessary strategies are chosen and used for extraordinary problem solving. In reflection level, in which comprehension is required; reflection, creativity, and knowledge necessary for solving complex problems are associated; observed results are generalized and justified; and abstraction is carried out.

The association between Bloom's Taxonomy and PISA's Competency Levels is indicated in Figure 3.

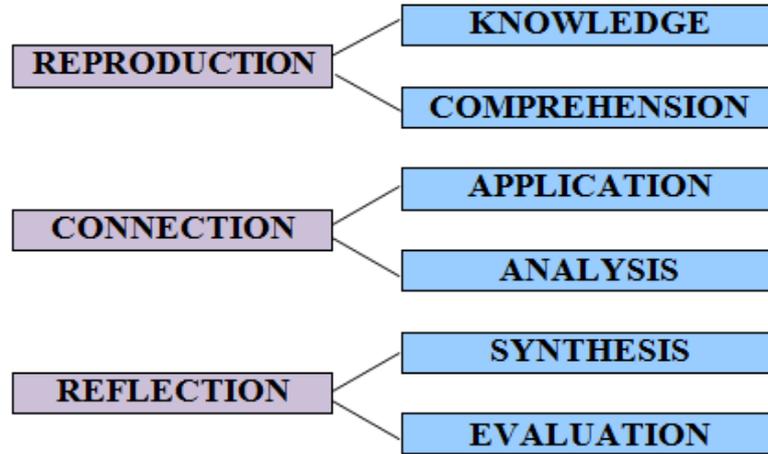


Figure 3. The relations of cognitive competency (OECD, 2003; Bloom, et al. 1956)

METHOD

This research is a qualitative study that has been conducted with the purpose of researching the cognitive competencies of prospective mathematics teachers related to the approaches used in mathematics education. Research design is a case study, which has been evaluated as a case of RME.

This study has been conducted in the faculty of education of a state university in the Black Sea region of Turkey with 32 prospective mathematics teachers, 20 of which are female and 12 of which are male. These prospective mathematics teachers are all senior students studying their fourth year of their 5-year education plan. The majority of these prospective mathematics teachers have successfully completed fundamental education courses such as introduction to education, educational psychology, guidance, theories and approaches of learning and teaching, curriculum development and instruction besides pure mathematical courses. All participants were informed about the process of the research. They volunteered to attend the research and gave the researcher the permission to use the data acquired from their interpretations and their posed problems in the manuscript.

The prospective mathematics teachers had been taking Methods of Teaching Mathematics lesson during the time this study was conducted. In the lesson instructed by the researcher and lasted for four hours a week, some approaches about mathematical education (e.g. Ausubel's meaningful learning approach (Ausubel, 1963), Freudenthal's RME approach (Freudenthal, 1973), Bruner's discovery learning approach (Bruner, 1961) have been elaborately examined and their implementation in teaching has been interpreted and discussed. In this research, it has been aimed to reveal cognitive status of prospective teachers about RME at the end of their 12-hour experience.

In this study, two sessions that were suggested by Selter (1997, 2001) and grounded on the study of Zulkardi (2002) about how preservice teachers have developed the RME learning environment, were taken into consideration: understanding the new approach by providing a theoretical overview and by actually doing mathematics-the mathematical component, designing

instructional materials-the didactical component. Thus, cognitive competencies of prospective teachers in the theoretical sense were focused on and the components of the sessions were discussed as “overview of RME theory, doing mathematics and designing contextual problems” (Zulkardi, 2002).

During ‘overview of RME theory’ session, firstly, the prospective teachers were provided with theoretical information on RME, and then they conducted activities about the approach. Afterwards, the implementation process and effectiveness of the activities were discussed. The prospective teachers stated their comments on the elements of the transition of RME application from real life to mathematics such as the characteristics of contextual problems applicable to horizontal mathematization, the modelings created during its solution, and roles of teacher and students. Then, they continued to discuss about vertical mathematization. During doing mathematics session, prospective teachers were treated as learners while the researcher performed as a teacher and it was aimed that they learn how to teach using with RME. In Appendix, there are some examples of the contextual problems used in the applied activities during these sessions (Altun, 2011; Fauzan, 2002; Feijs, 2005; Wubbels, Korthagen, & Broekman, 1997). During the second session, for instance, Feijs’s (2005) Grand Canyon Problem (problem 4 in appendix) developed in line with the construction of RME learning environment was implemented with prospective teachers. The prospective teachers formed 3- person student groups in real classroom environment and two of the prospective teachers from the group sat on one desk, whereas the other one sat on the desk that is parallel to the other desk. On the paper, which was hung down these parallel desks in order to create an imaginary river in the gap between the two desks, the points where the vision lines of these two people have been drawn with the help of a third person and canyon tables activity have been implemented. In the practice process, discussions were held on how they identify situations that can be seen or not seen by a person and how they perceive the vision lines from these situations. After the implementation, the situation of treating steepness of vision line as phenomena was examined according to the didactical phenomenology criterion where the main idea is familiarity and appeal for them. At this point, the mathematical content, which requires the ratio of a right triangle formed by an angle, was associated with the concept of steepness, and the state of abstraction resulting in the concept of tangent was also discussed. In addition, the self-developed or emergent models’ criterion of the approach was also discussed, considering the models that they can suggest for solution of the problem.

During designing contextual problem session, it was aimed to relate the context and the concept in a learning environment. In this manner, prospective teachers could learn how to design contextual problems to use in RME. So, they were asked to pose a contextual problem, for which they were given an adequate period of time and which would enable them to conduct horizontal mathematization.

After these sessions, they were asked some open-ended questions about RME and the problems they posed; moreover, they were expected to write down their opinions about these questions.

The questions asked to the prospective teachers are as follows:

1. What is RME? Please explain.
2. How is RME applied? How is this approach different from the other approaches you have learned? Please explain.
3. Pose a (contextual) problem that you could apply in RME, and explain for which conception's mathematization, this problem will be used. Please, evaluate the approach, on which the problem you posed will be used, by considering different approaches.

The written answers received from the prospective teachers and their contextual problems were analyzed, through the descriptive analysis method so as to determine their cognitive competencies about RME. In descriptive analysis, primarily a framework is established for analysis; the data are processed in accordance with thematic framework; findings are described and interpreted (Patton, 1990). Due to that reason; PISA Competency Clusters (OECD, 2003) and Cognitive Domain Taxonomy (Bloom, et al. 1956), which are displayed in Figure 3 below in order to reveal the cognitive competencies of prospective mathematics teachers, were used as the foundation for the data analysis. In this process, the relevant categories and data related to sub-categories that belong to each category have been processed and common themes were determined (Creswell, 1998; Patton, 1990). In the process of analyzing the data, the data collected from the first part of the question asked to the prospective teachers have been processed in the reproduction-knowledge category; the data collected from the second part of the first question have been processed in the reproduction-comprehension category; the data obtained from the first part of the second question have been processed in the connection-application category; the data collected from the second part of the second question have been processed in the connection-analysis category; the data retrieved from the first part of the third question have been processed in the reflection-synthesis category; and the data collected from the second part of the third question have been processed in the reflection-evaluation category. Furthermore, the sub-categories within each category have been composed as adequate ones in and deficient (or incorrect) ones in the related category.

In order to ensure the reliability of the study, two researchers, who have completed their PhD in the field of mathematics education and are experts in qualitative studies, firstly individually analyzed the data then it was discussed among the members until they reached a consensus on overall categories, sub-categories and themes. So, the use of multiple experts, as well as the use of their evaluations has led to conformability of the data. The credibility has been increased through the data obtained from the contextual problems consisting of the written answers given by the participants to the open-ended questions. For transferability, in order to help applying the findings to the other contexts, the description of the context was delivered in a clear and detailed manner. In other words,

the participants, the approaches examined and the activities conducted in the classroom were elaborated. Besides, in order to ensure that the results can be conveyed into similar media, the obtained findings have been supported with the quotations and detailed descriptions have been made (Berg, 2001; Lincoln & Guba, 1985; Yıldırım & Şimşek, 2005).

RESULTS AND DISCUSSION

The contextual problems and answers given by 32 prospective teachers participating in the study were examined and evaluated; furthermore, common themes were created based on their cognitive competencies. The categories and sub-categories created for determining common themes are displayed in Table 1. Moreover, frequencies of categories and sub-categories are also demonstrated in that table. The findings on cognitive competencies of the prospective teachers have been explained within each category by directly quoting them.

Table 1. Frequencies and percentages of categories and sub-categories of prospective teachers' competencies

Category	Sub-category	Frequency	%
Reproduction	Having knowledge about RME	28	87.5
	Deficient or incorrect knowledge about RME	4	12.5
	Making explanation or interpretation about RME	27	84.375
	Deficient or incorrect comprehension about RME	5	15.625
Connection	Applying knowledge about RME	26	81.25
	Deficient or incorrect application about RME	6	18.75
	Analyzing knowledge about RME	18	56.25
	Deficient or incorrect analysis of RME	14	43.75
Reflection	Posing an original (or an unfamiliar) contextual problem	11	34.375
	Deficient or incorrect posing about contextual problem	21	65.625
	Making valuation by passing a judgment	5	15.625
	No answer or deficient evaluation	27	84.375

Reproduction

Knowledge

In the first part of the 1st question, prospective teachers were asked to answer to define RME approach. Thereby, it was aimed to reveal the level of knowledge of the prospective teachers. The 28 (%-87.5) of 32 prospective teachers have correctly answered to the question 'what is RME?'. An

example from the answers given by the participants is stated below:

“We come across with mathematics in many parts of our everyday lives and we have to do the math. RME starts with this idea. In RME, a problem about real life are presented to students before teaching them the subject, and students solve this problem by making use of their previous knowledge through their own models. Thus, the student achieves a mathematical expression” (3rd PT).

It was observed that 4 (12.5%) of the participants have deficiently or incorrectly defined RME. There were no participants, who did not answer to the question. One example from one of the prospective teachers, who deficiently answered to the question, is as follows:

“Enabling them comprehend mathematics by giving them examples from life. How can we make life easier by using mathematics in our life. This is based on this” (13th PT).

The prospective teacher, who deficiently answered the question, mentioned the association of the approach with real life and said that mathematics could be learned through real life examples. However, the fact that she did not mention contextual problems, which are considered to be necessary for the transition from real life to mathematics and is the focus of the approach, demonstrates that the prospective teacher does not have adequate knowledge about the approach.

The answer received from one of the prospective teachers, who incorrectly answered the question, is as follows:

“RME is an approach that could help us solve the problems we may face in life. It is teaching students a subject by giving examples of the events we may experience in our lives about the subject” (31th PT).

Even though the prospective teacher talks about real life problems, his/her '...teaching students a subject by giving examples of events from life...' statement indicates that he/she has inaccurate knowledge on the roles of the student and teacher and also on the application of the approach.

Recalling the knowledge is vital in terms of making learning meaningful and solving a problem, thereby this knowledge can be used in more complex assignments (Anderson & Krathwohl, 2001). Moreover, Ayer (1936) discusses that when something is defined, it should contain the definition itself or its synonymous or expressions with equivalent meanings. Examining the answers received by the prospective teachers; majority of the prospective teachers have knowledge on RME.

Comprehension

In the 1st question, which was asked in order to understand whether the prospective teachers comprehended RME or not, they were expected to explain the approach. Their explanations suggest that 27 (84.375%) of the participants have correctly comprehended RME. One of the teachers, who correctly comprehended RME, explained it as the following:

“The mathematical conception, which is intended to be taught to the student, is taught through a problem that could be experienced in real life. The student tries to solve this problem, which is created by preparing the necessary setting, through his/her own knowledge and modelings. In a sense, how mathematicians discover mathematics in the events, which they come across with in real life, is experienced by the student. They are introduced to mathematical knowledge by solving the problem. Thus, this problem should be compatible with real life and should enable students reach mathematical conceptions” (24th PT).

The explanations suggest that 5 (15.625%) of the prospective teachers didn't comprehend RME. One of these prospective teachers (16th PT) could not adequately comprehend it even though he/she correctly defined it. The other four prospective teachers are in the category of those who incorrectly or deficiently answered the question at the knowledge level. The prospective teacher, who could not comprehend the conception, explained it as the following:

“In this approach, real life problems are asked to the student. We solve the problem and teach them the subject by saying this problem is its application in mathematics” (16th PT).

The statements of the prospective teacher, suggest that it should be begin with real life problems. According to the approach, the problem should be solved through a model by the student himself/herself by discussing it with his/her group friends. However, his/her statements about the teachers' solving the problem for students and conducting the mathematization indicate that the prospective teacher incorrectly comprehended the approach.

If we consider explaining as conveying the meaning to the other person and interpretation as realizing broad understandings (OECD, 2009); it is possible to say that both actions refer to comprehension. Thus, the answers given via explanations or interpretations indicate that the majority of the prospective teachers could significantly comprehend the approach.

Connection

Application

Examining the answers received by the prospective teachers for the first part of the 2nd question about the application of RME, 26 (81.25%) of them correctly answered to the question about the application of this approach. One of the prospective teachers explains the application of approach as follows:

“Firstly, a real life problem, which could enable student to reach a conception at the end, is given to the student. The student tries to actively solve this problem through the model he/she desires by discussing. The teacher helps them as a guide. At the end, the

student reaches mathematics through the model he/she created and is introduced to the desired conceptions. This part, in other words transition from real life to mathematics is called horizontal mathematization. Afterwards, the operations conducted within mathematics are called vertical mathematization. For example, everybody solved the snake problem we applied in classroom through their own desired models and discussion; and everybody was introduced to geometrical progression without even realizing. While doing so, we discussed and made associations in the classroom; and the example was modelled through expressions such as exponential numbers or common factors. This part was horizontal mathematization. Then we continued with vertical mathematization through conducting operations about geometrical progression. What is important in the approach is that students create their own models, discover mathematics through teachers' guidance, and obtain knowledge” (18th PT).

The above-quoted prospective teacher pointed out the three main factors, which are required in RME applications and stated by Gravemeijer (1994, 2001) including reinvention through progressive mathematization, didactical phenomenology, and self-developed or emergent models. The explanations suggest that he/she correctly evaluated the RME applications conducted in classroom; and coordinated between the approach and its application.

It was observed that 6 (18.75%) of prospective teachers incorrectly evaluated the application of RME. 5 of these prospective teachers fall under the category of those who incorrectly comprehended the approach; while 1 of them is in the category of those who correctly comprehended the approach. The prospective teacher, who correctly comprehended the approach but made incorrect statements about its application, made the following statement:

“The student is asked real life problems. Afterwards, the student tries to solve this. For example, in the problem about Canyon, various lines were drawn in order to see the river. From what perspective these lines should be coming and the distance between the lines and the river were discussed and tackled. We thought of their ratios and associated them with tangent conception. In conclusion, the teacher makes associations in order to enable the student to solve the problem; and the student solves the problem by bringing them together” (30th PT).

In the process of transition from real life to mathematics which is called mathematization, full process of transmitting to mathematical model from original problem situation is referred as modelling (Blum & Niss, 1991). However, in RME, modelling should be conducted by the student (Gravemeijer, 2001; Treffers, 1991). The statements made by the prospective teacher, which expressed that associating should be done by the teacher while solving the problem, suggest that

he/she is unable (or incorrectly) coordinate between the approach and its in-class application.

Analysis

In the second question asked to prospective teachers, they were asked to compare RME to other approaches besides discussing about its application. 18 (56.25%) of the prospective teachers correctly analyzed the approach and evaluated its similarities and differences with other approaches. Some of the comments they have made are as follows:

“The student reaches at a mathematical conclusion through his/her own knowledge by himself/herself. Since student is in the center of learning and the teacher guides; it is similar to discovery learning. However, in discovery learning, it is not necessary to begin with real life problems. In other words, a student can learn through discovery in a setting, which could enable generalizing a conception, and conduct applications after discovering the conceptions. In fact, in general, this application and constructivist approaches are quite different from traditional education. All of them are focused on the process and student-centered. The students themselves make sense of it. The main difference in RME is the environment is a stimulant and real life problems are what we begin with” (3rd PT).

Their explanations suggest that 14 (43.75%) of the prospective teachers are inadequate at analyzing the approach, thus their comparisons of approach to other approaches are sometimes incorrect and sometimes deficient. 6 of these prospective teachers are those, who already incorrectly applied the approach, and other 8 are among those who correctly applied it. One of the students, whose comparisons of other approaches are deficiently, explains it as:

“I think there is not a huge difference between them. All of them are student-centered. Some of them create a problem while some of them do the same through an activity instead of a problem. In conclusion, they all end the same” (14th PT).

Constructivism, which deals with how knowledge emerges and is based on cognitive psychology, takes the relationships among complex problem solving and cognitive structures and behaviours as basis (Noddings, 1990) and is based on the idea that knowledge is not directly taken from the teacher but structured actively by the learner (Lesh, Doerr, Carmona, & Hjalmarson, 2003; Von Glasersfeld, 1987). RME fundamentally share similarities with various learning and teaching theories of constructivism, which can be considered to be a theory of knowledge. In discovery learning, which is founded by Bruner (1961) and is one of the constructivist approaches, the learner is not informed of target knowledge or the conception, and learning setting is prepared by ensuring proper circumstances (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011). Learning occurs through the assumptions made by organizing patterns and examples, which are based on conceptualizations and generalizations from simple to complex; and conducting researches in an intuitive and systematic

manner (Jacobsen, Eggen, & Kauchak, 1993) furthermore, in this process teacher's duty is to guide students (Hammer, 1997; Svinicki, 1998). In RME, even though the student is not informed of target knowledge; basic differences in main principles and learning setting play a major role in differentiating approaches from one another. Primarily, basic differences in RME are based on the idea that human knowledge structures the knowledge and mathematical intuitions and procedures are invented and not discovered (Freudenthal, 1973). Thus, even though 'guided reinvention', which is one of the main principles of RME, stresses on guidance in the process; it is still different from the other approaches from that perspective. Moreover, the meaning of guidance in RME refers to the facilitative role of teacher for reinvention while enabling scaffolding instead of making explanations to the students (Hamzah & Bustang, 2014).

Essentially, RME and social constructivism, which focuses on the impact of the setting, are concerned with whether student is active; creativity; problem solving; reality of contexts; mathematical reality; and emergence of mathematical objects. However, in RME it is necessary to not only motivate students with everyday life contexts; but also to associate with experimentally real contexts and use them as the starting point for progressive mathematization (Gravemeijer, 2001).

Examining the views of the prospective teachers on differences of RME approach from other approaches; the fact that more than half of them mentioned these basic differences suggests that they can analyze at connection-building level in terms of cognitive competency.

Reflection

Synthesis

The prospective teachers were asked to pose a contextual problem (4th question) that they could use in RME, in order elaborately examine the cognitive status of the prospective teachers about RME. Thereby, above the theoretical and practical knowledge they have about the approach; it was aimed to find out how they bring this knowledge together in a different manner and integrate it with associations. Examining the problem they posed, 11 (34.35%) of the prospective teachers produced a contextual problem, in which they could apply the approach; whereas 21 (65.625%) of them did not pose such a problem.

Some of the contextual problems are stated below: When these problems are examined, it becomes visible that the students can solve these problems and mathematize new conceptions through the models they created.

“Ayşe will meet her friends and go to a movie this weekend. However, she cannot decide what to wear. When she looks at her closet she sees alternatively blouses, skirts, pairs of shoes, and socks with different colors. How many different outfit combinations can she create by doing these pieces in her closet?” Conception: Counting rule-factorial (2nd PT).

“Uncle Ahmet wants to cover his rectangular garage with square tiles. He measures the size of his garage and goes to buy the tiles from a construction market. However, Uncle Ahmet believes that the less number of tiles, the less it will cost him. For that job, what dimension of tiles Uncle Ahmet should buy?” Conception: The greatest common factor (6th PT).

“A medical aid helicopter taking off from the emergency aid base brings aid to those injured in a traffic accident. If the pilot knows the distance from emergency aid base to the hospital, the distance from the scene of accident to the hospital and the angle between them, how can he find the distance between the emergency aid base and the scene of accident?” Conception: Cosine Law (12th PT).

The two problems stated below, which were posed on the mathematization of proportion conception by the prospective teachers, are inappropriate for the approach. Although these problems exist in real life, the modeling that will be conducted in the process of solution will not help the mathematization of any conception. Moreover, it is believed that these problems can be an exercise question, which could be applied on taught proportion-ratio conception.

“Ezgi's watch loses time 5 minutes every hour. At 10.00 o'clock Ezgi agreed to meet her friend Mehmet at 17.00. What time Ezgi should be there according to her time, in order to meet Mehmet punctually at the time they agreed to meet?” (26th PT).

“6 workers worked at a construction and completed the building in 20 days. These workers want to make a planning for their next job. According to this, if 6 workers work together again; in how many days can they finish 5 buildings?” (8th PT).

Similarly, even though the problem stated below can exist in real life; it cannot be qualified as a contextual problem.

“How many pieces can we cut the cake by 5 moves?” (29th PT).

It is believed that the below-mentioned problem cannot be evaluated as a contextual problem, which could serve as a real life problem and help teaching a conception; it rather aims at achieving a mathematical relation by making students generalize through different cases.

“Four children play with one hoop each. Later, the children pile hoops over one another and start to examine their junction points. What is the maximum number of junction points for these hoops?” (23th PT).

Posing a problem is a significant component for problem solving and good mathematical problems can be posed by good mathematics teachers (Kilpatrick, 1987). The person in the process of posing a problem is actively engaged in challenging situations that involve them in exploring,

questioning, constructing, and refining mathematical ideas and relationships (English, 2003). In light of this view, it is believed that most of the prospective teachers have carried out the activities mentioned in the process of problem posing. However, a problem should point out the types of realistic thinking, which characterizes out-of-classroom problem solving, in order to be realistic (Verschaffel, Greer, & De Corte, 2000). Also, it should bring out a variety of mathematical interpretations and solution strategies which serve as a basis for progression to a more formal and sophisticated mathematics, and should support students' mathematisation process (Widjaja, 2013). For example, even though a problem such as "Do the medians of a triangle intersect at a single point" is concrete; it is still far from everyday life problems, thus cannot be considered realistic. Hence, all contextual problems are not realistic. In order to evaluate a problem as realistic, it has to be real or experienced by the person in an interesting manner (Wubbels, Korthagen, & Broekman, 1997). Similarly, if a person wants to pose a problem about a farm, using sentences such as 'imagine a cow in a sphere form' will not make the problem realistic (Greer, 1997).

Taking into account of all of these ideas, it is observed that more than half of the problems posed by the prospective teachers are not qualified enough to serve for RME and only 34% of them fit for the purpose. Thus, it can be accepted that only the students in this percentage have cognitively reached the synthesis stage at reflection-construction level.

Evaluation

In the 4th question asked to the prospective teachers, they were expected to evaluate the application of the problem they posed and interpret it according to the other approaches. The 5 (15.625%) of the prospective teachers, who posed a relevant problem, have carried out an evaluation; whereas the other 27 (84.375%) of them failed to either pose a relevant problem or to carry out an evaluation.

The evaluation made by a prospective teacher, who is believed to have posed a relevant problem to RME is as follows:

"I aimed to enable the students to achieve the greatest common factor in my problem. The teacher could try to make student comprehend the conception through explaining its meaning. In other words, if it were given through presentation, the student would be provided with prepared knowledge. Furthermore, since the student would not be active; he/she may be bored and mathematics would be far from real life for him/her. I believe that the things a person does or achieves on his/her own is always more valuable and not forgotten. Therefore, for a student to solve a problem, a real problem, and to meet a new conception will be more meaningful for him/her. We could also give this knowledge through discovering numbers and the correlations among factors, without using a problem. This may be effective, too. However, I believe that it will be more permanent for students to reach at knowledge through real life events, discussion and creating their own models" (6th PT).

According to Hiebert and Carpenter (1992), understanding is defined as making connections between ideas, facts or procedures, and occurs through recognizing relationships between pieces of knowledge. Consolidated knowledge enables the use of this knowledge in various cases in a proper and confident manner (Wubbels, Korthagen, & Broekman; 1997). Also, if knowledge is consolidated; the evaluations about this knowledge will be conducted in a meaningful manner. In that sense, it can be concluded that the evaluations of prospective teachers through the problems they posed are insufficient.

CONCLUSION

Enabling students to learn mathematics in a meaningful manner and to acquire various targeted skills during the process can be only made possible by placing education of mathematics teachers in the center. Therefore, it is of vital importance for prospective teachers to be able to prepare appropriate learning environments and to correctly plan the roadmaps for that purpose (Brousseau, 1987; Richards, 1991). In order to achieve that, the prospective teachers must have enough knowledge on the key principles and fundamental philosophy of the approach that they plan on using while preparing such environments, moreover, they must also be able to implement and interpret instructional activities (Gravemeijer & Cobb, 2006). In this study, where prospective teachers' cognitive structures about the approaches have been examined, RME has been the focused approach and prospective teachers' cognitive structures about this approach have been evaluated. The study suggests that the majority of the prospective teachers have theoretical knowledge about RME and can generally interpret that knowledge, however, their ability to associate that knowledge with other approaches falls by half. Moreover, the prospective teachers' skills fell even shorter when they were asked to pose a contextual problem suitable with RME, which was asked of them in order to see their high cognitive competencies. Even though it is necessary to place mathematical connections in the relevant social situations for achieving meaningful learning in mathematics; creating authentic activities is naturally quite difficult (Ainley, Pratt, & Hansen, 2006) and also designing lesson materials, especially finding real life examples that match with the mathematics concepts to be taught is difficult (Zulkadri, 2002). Also, preparing a learning environment that allows for reinvention process, is quite complicated (Gravemeijer, 2008). Furthermore, some of the reasons why these problems emerged may be due to the fact that they do not have adequate contextual knowledge about the nature of some of the conceptions they were supposed to learn; or they could not detach from the constructions in the learning settings, in which they were raised. Also, as Widjaja (2008) mentioned, their prior knowledge about the concept and the nature of their knowledge may cause this result. For that reason, prospective teachers should be trained in a way that reflects what is expected from their teachings (Gravemeijer, 2008).

Since this study aims to reveal the cognitive competency of prospective teachers on RME; it is believed that the obtained results suggest ideas on the construction about RME only in the minds of

prospective teachers. Hence, reflections of prospective teachers on RME can be conducted through a phenomenological study so as to obtain more elaborated knowledge. Moreover, in the school experience and school practice stages; the application processes of prospective teachers on RME can be examined and in the light of the retrieved results, necessary adaptations can be conducted for real classroom situations.

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APPENDIX

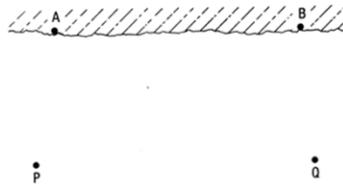
Problem 1



When the sea snake in the picture becomes 1 month old, a black ring emerges around its body. Each month, a yellow ring emerges in the middle of the black ring; and thereby, two black and one yellow rings appear. In the following months, this continues in the same way. In other words, each black ring is cut in the middle with a yellow ring. How many rings does a 12 month old sea snake have? How can we find the age of a sea snake according to its number of rings; or the number of black and yellow rings of a sea snake of a certain age? (Altun, 2011).

Comment: It was aimed to reach geometrical sequence conception through this problem.

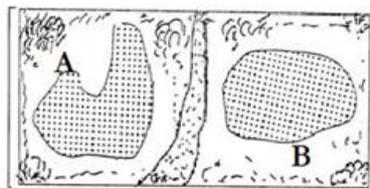
Problem 2



Ship P is going in a straight line toward point B on the shore, at a constant speed. Ship Q is going in the direction of A at double the speed of ship P. How close do the two ships get? (Wubbels, Korthagen, & Broekman, 1997).

Comment: It was aimed to reach vector conception through this problem.

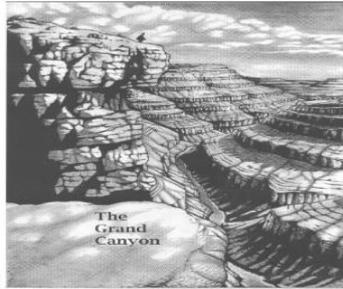
Problem 3



The figure below shows two rice fields separated by a road. Both rice fields are planted with the same rice and they are given the same fertilizer. The dots on the figure represent rice clusters. Which rice field produces more rice? (Fauzan, 2002).

Comment: It was aimed to reach reallocation-congruent area conception through this problem.

Problem 4



This is a photograph of a hiker on the rim of the Grand Canyon looking down trying to see the Colorado River at the bottom of the canyon. Can the hiker see the river below? From which points and perspective should the hiker look in order to see the river? (Feijs, 2005).

Comment: It was aimed to discuss tangent conception through this problem.

