Narrative-first approach: Teaching mathematics through picture story books


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The four pillars of student engagement, teacher engagement, breadth of mathematics and depth of mathematics are used to explain the benefits of a *narrative-first approach* for supporting the integration of mathematics and children's literature.

The benefits of using children's storybooks to support mathematics instruction in primary schools is well established. Muir et al. (2017) note that children's literature can be used to engage students in the topic or lesson, contextualise important mathematical concepts and promote mathematical reasoning. Typically, however, attempts to use children's storybooks begin with identifying a relevant aspect of the curriculum, and then selecting books that connect to the mathematical area in focus. Sometimes these children's books have been purposefully written to explore particular mathematical ideas (e.g., *The Greedy Triangle*, by Marilyn Burns). Other times the mathematical idea is explored explicitly in the book, even though the book itself may have been written for a broader purpose (e.g., *The Doorbell Rang*, by Pat Hutchins). There are many examples of teacher educators and classroom teachers publishing articles in journals such as *Teaching Children Mathematics* and *Australian Primary Mathematics Classroom* sharing lessons and units of work in which they have harnessed children's storybooks that possess these explicit links to specific mathematical concepts (e.g., Malinsky & McJunkin, 2008; Padula, 2004; Taber & Canonica, 2008).

An alternative means of exploiting the benefits of children's storybooks when teaching primary mathematics is to be led by the story, rather than by the curriculum. We refer to this as a narrative-first approach, and contrast it with a curriculum-first approach (Russo & Russo, 2017e). The narrative-first approach involves first identifying rich narratives, for example, favourite picture storybooks or novels, mapping out the key components of the story (e.g., the characters, the plot), and then developing rich problem-solving tasks that connect to these key components. Curriculum links are then made retrospectively.

We believe the benefits of the narrative-first approach for supporting the integration of mathematics with

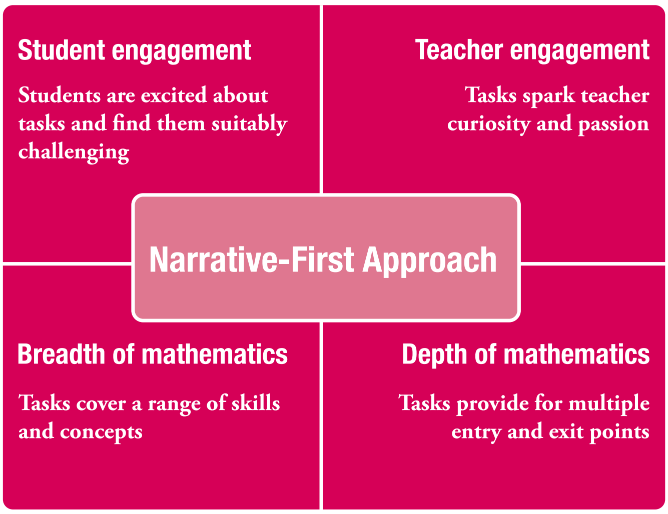

Figure 1. The four pillars of a narrative-first approach.

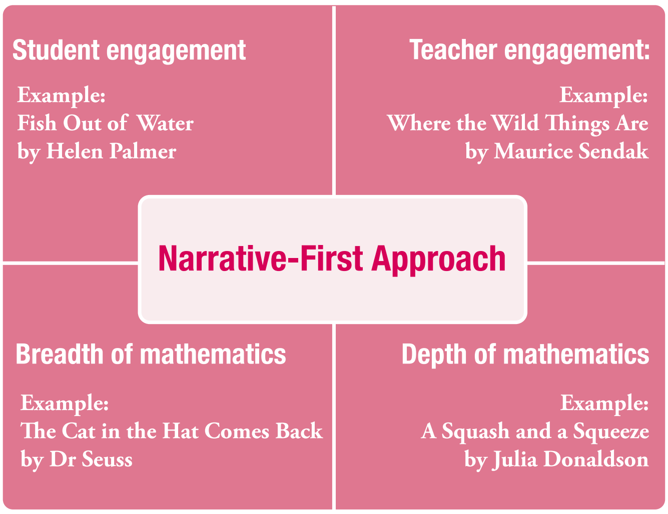

Figure 2. Illustrating the narrative-first approach—examples of titles.

children's literature are considerable. In addition to the aforementioned benefits which apply more generally to using children's literature to support teaching mathematics (e.g., contextualizing mathematical ideas, engaging students), we believe this narrative-first approach has some additional advantages, that can be framed around four 'pillars' (see Figure 1).

The narrative-first approach can be effective for engaging students and teachers, and for differentiating mathematical experiences (i.e., teaching for depth), whilst simultaneously covering a range of important mathematical concepts (i.e., teaching for breadth).The remainder of the paper will provide examples of problem-solving activities, that have all been delivered in a primary school classroom, from four diverse children's storybooks. For emphasis, each example is used to illustrate one of the four pillars of the narrative-first approach (see Figure 2). However, we would argue that any one of these examples is built around all four of these pillars.

Student engagement: Fish Out of Water

Student engagement is paramount to the narrative-first approach. Through using the story as a 'hook', students are highly motivated to engage with the associated mathematical tasks. In addition, the rich fictional world created by the narrative serves as an authentic imaginative space through which students can engage in meaningful and relevant mathematics. In our view, for mathematics to be authentic, problems do not have to be built around a real-world context, so as long as the fictional world created by the author is 'alive' for the students.

The text *Fish Out of Water* forms the basis of an investigation using the narrative-first approach that was undertaken in a Year 5/6 class, with a high level of student engagement in the task (Russo & Russo, 2017e).

The text tells the story of a boy who buys a pet goldfish and, disobeying the instructions of Mr. Carp the pet shop owner, overfeeds his new pet. The fish grows and grows until it needs to be transported into a swimming pool. When Mr. Carp finally returns he saves the day and turns the fish back to normal.

To begin, the text was read to a class, who were then asked to consider a mathematical problem: the *Fish Out of Water* investigation below.

Following the lesson, students were invited to complete a reflection exploring both what they enjoyed about the activity and their own learning. As we have discussed in a previous paper (Russo & Russo, 2017e), students responded very positively to the lesson. Whilst some students emphasised the benefits of the narrative 'hook' for engaging them in the corresponding problem-solving task, other students valued the narrative because it supported them to make sense of exponential growth as a concept. Two exemplar quotes from students are provided below:

> I thought it was a really good idea starting a maths lesson with a story because it opened up my mind in a way and made me want to know what this had to do with the lesson. So I was feeling intrigued and sort of open. The lesson was really fun and interesting as well.

> It was fun to read the story at the start and then go into maths because it helped explain some things… it helped me make sense of it and understand it better.

Student reflections on their learning emphasized two major themes: a developing appreciation for the power of exponential growth and understanding the

Fish Out of Water investigation

At the beginning of the story, the little fish, Otto, was only 5cm long but by the end of the story, Otto was so big he was just over the length of a 50m Olympic size swimming pool! If Otto doubled in length every ten minutes, how long did it take Otto to grow to this size?

Extension

If Mr Carp had not have dived into the pool and saved the day, Otto would have kept on growing and growing. How long would it have taken Otto to grow so big he wouldn't fit into Albert Park Lake? How long until he would have outgrown the length of the Yarra river? How long until he would have outgrown the Pacific Ocean? How long until he would have outgrown the earth? How long until Otto was so long, he would have stretched all the way to the moon? What about the sun? Pluto? Alpha Centauri? The known universe?

relationship between different units of measurement. Again, two illustrative student quotes are provided:

> I learnt about how doubling again and again can makes things big in such a short amount of time.

> I learnt about exponential growth which, I think, is when something gets bigger and the size doubles each time… and I got to practice converting m-cm-km.

Teacher engagement: Where the Wild Things Are

Many teachers feel restricted covering curriculum content and this can be an impediment to developing engaging tasks. We believe the narrative-first approach allows for greater flexibility and creativity when planning maths activities as teachers can primarily focus on developing rich, authentic problem-solving tasks. This allows for the focus to be on the four proficiencies (understanding, fluency, problem solving and reasoning) when planning tasks, rather than simply the content strands.

The narrative-first approach also facilitates teacher engagement as teachers can choose texts that they personally enjoy and are excited to share with students. This can be further enhanced by a teacher's personal connection to a text, which can motivate the development of the associated task. For example, one of the first author's favourite books as a child was Maurice Sendak's *Where the Wild Things Are.* This classic picture storybook begins with the protagonist Max being sent to bed without supper. His room transforms into a mysterious world and he travels to the Land of the Wild Things. Here he becomes king of many strange creatures, only to travel back to his room where his supper is waiting for him, still hot.

Part of the author's fascination with this text was the way in which so much time seemed to pass on Max's adventures, only for him to return home apparently minutes later. This narrative opened up the concept of alternative realities and the idea the time can be relative; ideas that continue to fascinate into adulthood. The author's excitement has been harnessed and translated into a mathematical investigation exploring these concepts, which was undertaken with a Year 5/6 class (Russo, 2018).

After reading the book, the class explored the sections of text that specifically state how much time Max spent travelling to and from the Land of the Wild Things. This could be done as a class, leading to the investigation outlined below, or as a stand-alone mathematical investigation.

> And an ocean tumbled by with a private
> boat for Max
> and he sailed off through night and day
> and in and out of weeks
> and almost over a year
> to where the wild things are.

The class concluded that it took 365 days ('almost over a year') to get to the Land of The Wild Things.

> But Max stepped into his private boat
> and waved goodbye
> and sailed back over a year
> and in and out of weeks
> and through a day.

The class concluded that this period was equal to one year plus two weeks plus one day, plus five days staying at the Land of The Wild Things or 385 days. This makes a total of 750 days (365 + 385) that had passed in the Land of the Wild Things.

Students then considered "How much time do you think passed at home?", looking at this section of the text:

> Where he found his supper waiting for him
> and it was still hot.

Student A: Well his dinner was still hot, so it couldn't have been all that long.

Student B: I reckon about five minutes, no longer than that.

It was agreed that five minutes had passed in the 'real world'.

Where the Wild Things Are investigation

Using this information, how much time passes in the Land of the Wild Things, compared with one minute in the real world?

Extension

Max went to sleep at 9pm, after eating his supper, and woke up at 6.30am. He then went straight back to the Land of the Wild Things. How much time has passed there?

The next time Max returned it was four years later, at age 12. How much time had passed now?

Number and algebra (Level 6)				Measurement and geometry (Level 6)	
Number and place value	Fractions and decimals		Patterns and algebra	Using units of measurement	
Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers and make estimates for these computations. (VCMNA209)	Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies. (VCMNA213)	Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers. (VCMNA214)	Continue and create sequences involving whole numbers, fractions and decimals. Describe the rules used to create the sequence. (VCMNA219)	Convert between common metric units of length, mass and capacity. (VCMMG223)	Solve problems involving the comparison of lengths and areas using appropriate units. (VCMMG224)

Figure 3. Content descriptors from the Victorian Curriculum linked to The Cat in the Hat Comes Back investigation (VCAA, 2017).

Breadth of mathematics: The Cat in the Hat Comes Back

In our experience, creating rich applied tasks and investigations using the narrative-first approach results in numerous and various links to relevant curriculum documents. After reading the book *The Cat in the Hat Comes Back* to his Year 5/6 class, the first author had students engage with an investigation that involved a range of mathematical ideas. This story is about the return of the Cat in the Hat to the home of Sally and her brother, who proceeds to make another horrendous mess and then pulls out a series of small cats (Little Cats A to Z) to help him clean up.

The Cat In the Hat Comes Back investigation

The Cat in the Hat is 1.6m tall and his hat is another 65cm high.
Little Cat A is $\frac{1}{5}$ the size of the Cat in the Hat.
Little Cat B is $\frac{1}{2}$ the size of Little Cat A.
Little Cat C is $\frac{1}{2}$ the size of Little Cat B,
and on it goes.

How tall is Cat C? What about Cat E?
Can you think of things in real life the same size as these cats?

Extension

What is the size of Cat H?
Can you even see Cat Z?!
Are there any things in real life the same size as these cats?
Do you think all these little cats would really fit inside the Cat in the Hat's hat?

The curriculum links for this task are somewhat dependent on the strategies used by individual students. However all students recognised the initial need to determine the size of Little Cat A and, in order to do this efficiently, their intuition was to convert 1.6 metres to 160 centimetres to make the problem more workable (VCMMG223; VCAA, 2017). Students were then required to explore fractions of an amount (VCMNA213; VCAA, 2017), i.e., "If The Cat in The Hat is 160cm and Little Cat A is one fifth his size, then what is one fifth of 160?" "If Little Cat B is half the size of little Cat A, what is one half of 32?"

As students worked through the extension, they needed to create a sequence involving fractions or decimals based on continuous halving (VCMNA219; VCAA, 2017). Some students used multiplication of decimals as a strategy for solving this problem, which is within Level 7 of the curriculum (VCMNA244; VCAA, 2017), i.e., If Little Cat G is 0.5cm, then Little Cat F is 0.5cm × 0.5cm. At this point, many students elected to use digital technologies (i.e., calculators) for the required division problems (VCMNA209; VCAA, 2017) and some even had a go at using an algebraic expression to find a more efficient solution, which is within Level 7 or beyond (VCMNA252; VCAA, 2017). In order to solve the ultimate question, students were required to add a series of decimals (VCMNA214; VCAA, 2017) and then compare this answer with the original size of the hat (VCMMG224; VCAA, 2017).

Table 1 outlines the breadth of curriculum links connected to this rich task when taught in a 5/6 classroom. For the purpose of this table, all content descriptors are from the Victorian Curriculum—Level 6, although it should be noted that many connections could also be made to the Level 5 and Level 7 curriculum.

Depth of mathematics: A Squash and a Squeeze

In general, when developing a problem-solving task using the narrative-first approach, we aim to ensure that the task is suitable for a variety of ability levels and grade levels. The following investigation was developed around the text *A Squash and a Squeeze*. This is a story about a little old lady who feels her house is too small, so a wise old man advises her to bring her animals into the house, which eventually results in the lady realising her house is relatively big once the animals are removed. The investigation was taught with both a Year 2 class and a Year 5/6 class. Whilst all Year 2 students attempted the initial investigation, around half the students subsequently attempted the extension (Extension A). By contrast, all Year 5/6 students attempted both the initial investigation and extension (Extension A), with around three-quarters of the class also engaging with the additional extension (Extension B).

Initial investigation: A Squash and a Squeeze investigation

The little old lady's cottage was exactly 60 square metres in area. Sharing her house with a hen, goat, pig and cow did not leave a lot of space for the old lady! To live comfortably:

- A hen needs exactly 5 square metres of space;
- A pig needs exactly 15 square metres of space;
- A goat needs at least 10 square metres of space, but no more than 15 square metres;
- A cow needs at least 18 square metres of space, but no more than 25 square metres.

When she was living with all four animals, how much space might have been left for the old lady? Show as many different possibilities as you can. What is the least amount of space the old lady would have to herself? What is the most amount of space?

The initial investigation focuses on additive thinking, in particular the part-whole idea, which is a central concept in early primary school mathematics. Reflecting their developmental phase, many Year 2 students attempted to represent their thinking using either concrete materials, such as *Unifix* (see Figure 4) or the square grid on the back of a 100s chart to represent the area of the house (see Figure 5).

Figure 4. A student represented her thinking using Unifix and associated number sentences.

Figure 5. Another student used a grid to represent the house, and the initial of the animal to represent how much space it is occupying.

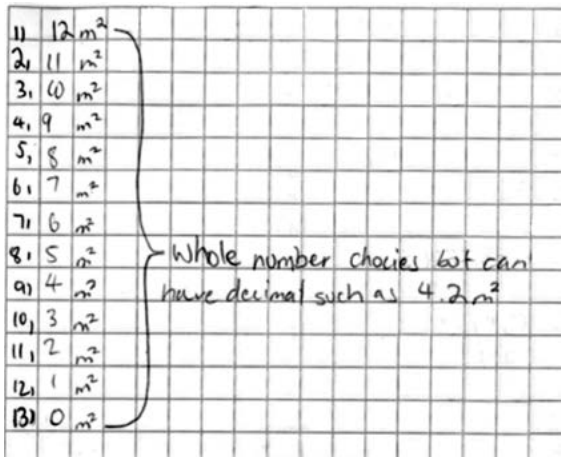

Figure 6. A Year 5/6 student systematically identifies all whole-number solutions to the Squash and a Squeeze investigation.

All Year 5/6 students were able to solve this part of the problem without manipulatives. Furthermore, some students understood that by using decimals there would be many more solutions, with some proposing infinite possibilities (see Figures 6 and 7).

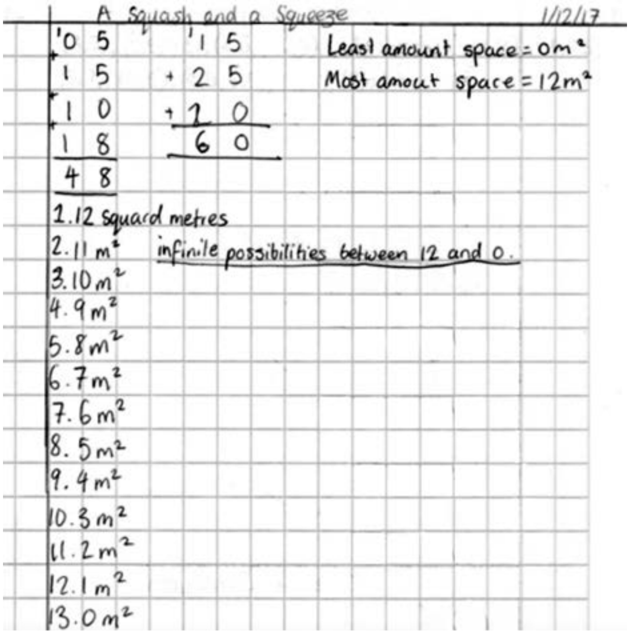

Figure 7. This student has recognised that there are "infinite possibilities between 12 and 0" due to possible decimal amounts.

Extension A: More part-whole and opportunities for multiplicative thinking

The average Australian house is 240 square metres in area. Given the above requirements for space, what are some different combinations of hens, goats, pigs and cows that could live comfortably in the average Australian house?

The extended investigation provides further opportunities to explore the part-whole idea (with larger numbers), whilst also providing opportunities for multiplicative thinking (e.g., students can 'scale up' their answers from the initial investigation). It was common for Year 2 students who attempted the extended investigation to build on their grid representation employed for the initial problem (see Figure 8).

The focus of Extension B is to invite students to contrast additive thinking with multiplicative thinking and, in particular, apply proportional reasoning.

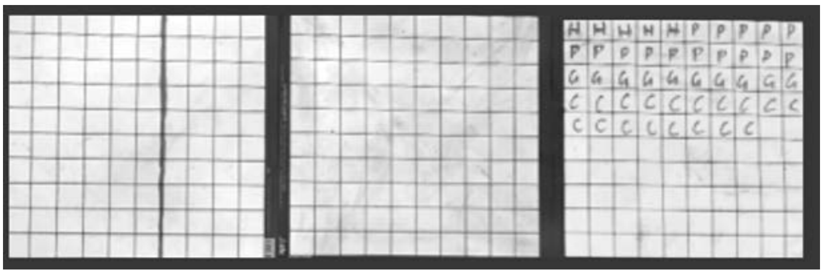

Figure 8. A student built on a grid representation to attempt the extended investigation.

Extension B: Contrasting additive thinking with proportional reasoning

After doing the initial investigation and Extension A, Josh and Jill were discussing the *Squash and the Squeeze* problem.

Josh: Why didn't the little old lady just buy herself a bigger house to begin with? If the average Australian house is 240 square metres, she would have had a lot more space to herself.

Jill: I actually think she is better off having lived with the animals, and then getting rid of them, than just buying a bigger house to begin with. Her little house without the animals feels bigger than moving to a bigger house would have felt at the beginning of the story.

Josh: But buying a bigger house would have given her way more extra space! Instead of having 60 square metres, should would have had 240 square metres all to herself!.

Jill: But her little house still felt so much bigger! With all the animals living there, she had no more than 12 square metres of space. When all the animals moved out, she had 60 square metres all to herself! This change would feel even bigger than if she had of just bought a bigger house to begin with!

Who do you agree with, Josh or Jill? Can both of them be right? Can you use mathematics to prove that Josh's reasoning is correct? Can you use mathematics to prove that Jill's reasoning is correct?

Some Year 5/6 students were limited to additive thinking when tackling this problem and therefore determined that "Josh is correct" because a difference of $180m^2$ is larger than a difference of $48m^2$. Others made the comparison between additive and multiplicative thinking, although some of these students required additional prompting questions from the teacher to get to this point. Several Year 5/6 students used proportional reasoning to conclude that both Josh and Jill could be correct, depending on how you interpreted the problem (see Figure 9).

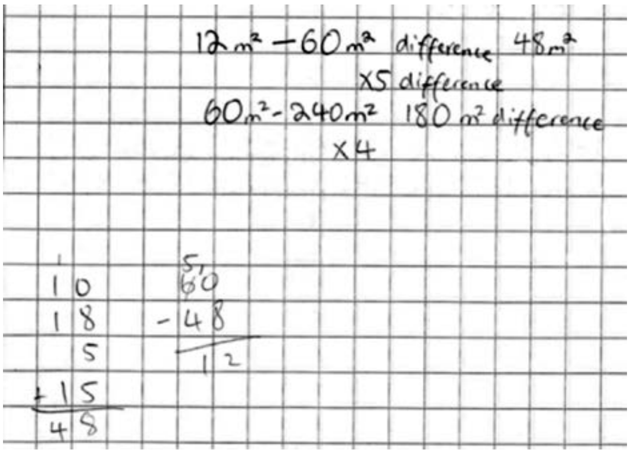

Figure 9. A student demonstrated both multiplicative (top right) and additive reasoning (bottom left) and in discussion compared these to determine both Josh and Jill could be correct.

Concluding thoughts

It is common for children's literature to be used in connection with mathematical learning, but often the maths is superficially linked to the text or a text is chosen for its mathematical focus. This article outlined a narrative-first approach to lesson planning, whereby key ideas, themes, and characters from well-known children's stories are reconstructed through a mathematical lens.

We have attempted to demonstrate how the narrative-first approach can simultaneously engage-teachers and students and energise the mathematics classroom, whilst allowing a range of mathematical skills and concepts to be covered across a variety of ability levels. For other examples of our attempts to employ this approach, see Russo & Russo (2017a, 2017b, 2017c, 2017d., 2018). If you'd like to find out more about this approach or any of the example lessons, please feel free to email the authors.

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