Mathematical manipulatives for misers

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The author showcases a number of readily available and affordable resources that may be used in mathematics classrooms to provide students with concrete learning experiences. Resources, alone, do not provide educational experiences unless they are accompanied by appropriate teaching to ensure rich and connected learning experiences.

Introduction

A recent survey has revealed that Australian teachers spend over $500 a year on items such as stationery, with 10% of teachers spending over $2000 (Australian Associated Press, 2017). The purpose of this article is to showcase a number of readily available and affordable resources that may serve to enhance the teaching and learning of mathematics at the primary level. The intention is to provide an illustrative rather than exhaustive list, and resources have been included that address specific aspects of number, measurement, geometry, and probability. Priority has been given to the use of physical manipulatives over online resources, since the potential benefits of ICT to support the teaching of mathematics in primary schools has been well-established elsewhere (Serow, Callingham, & Muir, 2016).

Effective teachers use manipulatives to "provide concrete experiences that help children make sense of mathematics and build their mathematical thinking" (Reys et al., 2016, p. 45). When used effectively, mathematical manipulatives have the "potential to lead to an awareness and development of concepts and ideas linked with mathematics" (Swan & Marshall, 2010, p. 14). While manipulatives provide individuals with sensory experiences, they do not in themselves impart mathematical understanding. Appropriate discussion and teaching is needed in order to make the links to mathematics explicit.

The effective use of manipulatives allows mathematical ideas to be illustrated in multiple ways. Students construct knowledge as they make connections between the models and the mathematical ideas (Reys et al., 2016). According to Haylock and Manning (2014) students' understanding of mathematical concepts develops as they make connections between concrete experiences, pictures, language and symbols. Teachers can assist by introducing and modelling the use of appropriate mathematical language. The teacher should also bring to the fore salient points that might otherwise go unnoticed.

Cost and quality are not always synonymous in the educational market. While the value of an educational resource is highly subjective, good resources need not be sophisticated or expensive (Drews & Hansen, 2007). The resources discussed here may be utilised in various mathematical contexts and may be adapted to suit a variety of student learning needs.

The Fraction Formula Game
($30 from Educational Insights)

This game is designed for two to four players and is recommended for students in Year 3 (or ages 8+). The set contains four measuring cylinders and 52 coloured fraction pieces consisting of halves (brown), thirds (grey), quarters (yellow), fifths (green), sixths (orange), eighths (red), tenths (blue) and twelfths (purple). The use of cylindrical pieces provides a useful alternative to the linear and regional fraction models.

The aim of the game is to create a fraction that is the closest to 1 without exceeding it. Each player begins with an empty measuring cylinder. Players take turns drawing a fraction card from the top of the deck and placing the matching fraction piece in their cylinder. On each turn, a player may choose whether or not to take a card. If a player takes a card, they must place the matching fraction piece in their cylinder. If a player chooses not to take a card, he or she says "hold" and play passes to the next player. Once all players have said "hold", the round is over.

It is likely that the enterprising teacher might wish to modify the rules or gameplay to suit a wide range of student ages and abilities. For example, gameplay could be enhanced by introducing wild cards that skip the next player, reverse the direction of play or require the next player to draw two cards. Note that the game is not recommended for students below three years of age since it does contain some small pieces.

Instructions are also included for using game pieces to introduce and reinforce fraction concepts. For example, the game pieces can be used to demonstrate fraction equivalences such as $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}$ and $\frac{1}{3}=\frac{2}{6}=\frac{4}{12}$. The game can also be readily extended to include improper fractions such as $1\frac{1}{2}$ and $1\frac{1}{3}$.

The Fraction Formula Game may be useful in addressing the following aspects of the *Australian Curriculum: Mathematics*, in addition to cross-curricular links with the science learning area.

Table 1: Mathematics curriculum links for the Fraction Formula Game.

Level	Content descriptor	Description
Year 1	ACMNA016	Recognise and describe one-half as one of two equal parts of a whole.
Year 2	ACMNA033	Recognise and interpret common uses of halves, quarters and eighths of shapes and collections.
Year 3	ACMNA058	Model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole.
Year 4	ACMNA077	Investigate equivalent fractions used in contexts.
Year 4	ACMNA078	Count by quarters, halves and thirds, including with mixed numerals.
Year 4	ACMMG084	Use scaled instruments to measure and compare lengths, masses, capacities and temperatures.

The Fraction Formula Game may be purchased online at https://www.educationalinsights.com/product/fraction+formula--8482-+game.do

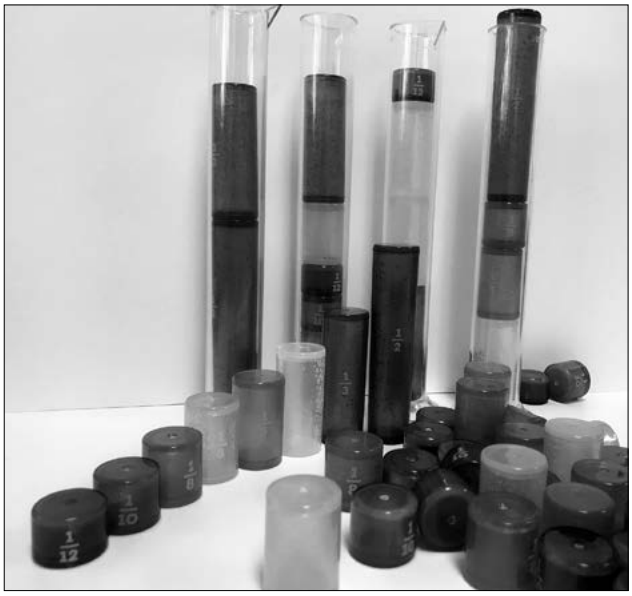

Figure 1. Fraction Formula Game.

Magnetic geometry sets
($10–25 from Kmart)

Magnetic tiles are an excellent resource for exploring concepts in plane and solid geometry. Each set of tiles contains a variety of polygonal shapes including (but not limited to) triangles, squares and hexagons. Each edge of a polygonal piece contains a magnet which is free to realign so that it always attracts the magnet in the edge of another piece. Thus linking the pieces together and taking them apart does not require a large amount of strength or dexterity. While the pieces clip together readily, the magnetic force is not sufficient to create a pinching hazard. As toys designed for infants, the shapes are durable and the pieces are too large to present a choking hazard.

The larger sets of magnetic geometric tiles represent excellent value at less than $1 per tile. Objects that can be constructed using a single large set of magnetic geometric tiles include, but are not limited to: triangular prisms, rectangular prisms, rhomboidal prisms, regular hexagonal prisms, tetrahedrons, square-based pyramids, and octahedrons. Some sets also contain components necessary to make wheeled 'vehicles' of various designs and complexity. Kmart's *Magnetic Carousel Tile Set* can be viewed or purchased at http://www.kmart.com.au/product/44-piece-magnetic-tiles/1253319

Various mathematical ideas can be introduced or explored using the magnetic tiles. Since the pieces come in various colours, it is possible to create and explore various patterns and sequences involving colours and shapes. Hexagonal and triangular tiles can be used to explore fractional relationships in a similar manner

to pattern blocks. Pieces can also be arranged to explore the relationship between angles on a line, angles in a circle, and exterior angles to polygons.

Figure 2. Magnetic geometric tile sets ($10–25 from Kmart)

In the classroom, the magnetic geometric tiles are useful for showing the relationship between two-dimensional nets and three-dimensional objects. When equilateral triangles are placed on each edge of a central triangle, the pieces snap together to form a tetrahedron when gentle pressure is applied. Similarly, an open cube snaps together when four squares are placed around a central square and the net is lifted up by the centre square. Magnetic shapes allow students to explore the relationship between faces, vertices and edges of various polyhedra without requiring the same degree of dexterity as manipulatives such as polydrons or geoshapes. Students can also explore the number of faces that meet at an edge and the number of edges that meet at a point.

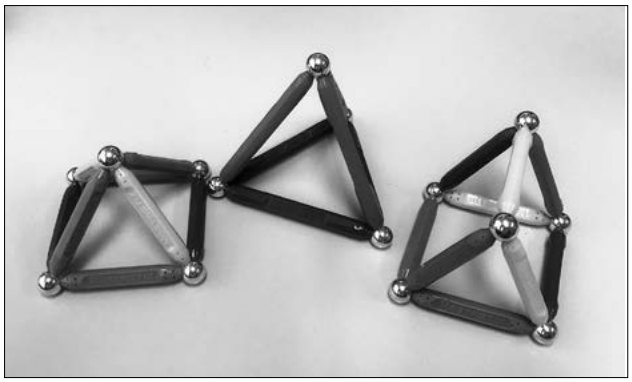

Figure 3. Magtastix Construction Kit.

Magtastix are an alternative resource that consists of plastic rods (in two sizes) and metal bearings. The rods and bearings allow children to explore the ideas of edges and vertices, respectively (see Figure 3). However, care should be taken since the loose metal bearings present a potential choking hazard for young children; *Magtastix* are not suitable for children under three years of age and it is recommended that older children use them under adult supervision. *Magtastix* are available in a 40 piece set which contains 17 short rods, 6 long rods and 17 bearings ($25) or a 20 piece set ($15). *Magtastix* sets may be viewed or purchased online at http://www.kmart.com.au/product/40-pieces-magtastix/1553803

Magnetic construction kits can be used to address the following content descriptions from the *Australian Curriculum: Mathematics.*

Table 2: Mathematics curriculum links for magnetic construction kits

Level	Content descriptor	Description
Year 1	ACMMG022	Recognise and classify familiar two-dimensional shapes and three-dimensional objects using obvious features.
Year 2	ACMMG043	Describe the features of three-dimensional objects.
Year 3	ACMMG063	Make models of three-dimensional objects and describe key features.
Year 5	ACMMG111	Connect three-dimensional objects with their nets and other two-dimensional representations.
Year 6	ACMMG140	Construct simple prisms and pyramids.

Hamburger Kit
($8 from Kmart)

This kit contains everything you need to construct a hamburger with all the trimmings: a bun, tomato, lettuce, cheese, meat, pickles, mustard and tomato sauce (see Figure 4). Students can explore the possible combinations that can be achieved by adding or removing ingredients. For teachers wishing to provide a hands-free demonstration, a small amount of velcro can be placed on the back of each ingredient.

Figure 4. Felt Hamburger Kit.

Table 3: Mathematics curriculum links for Hamburger Kit.

Level	Content descriptor	Description
Year 4	ACMSP093	Identify everyday events where one cannot happen if the other happens.
Year 5	ACMSP116	List outcomes of chance experiments involving equally likely outcomes and represent probabilities of those outcomes using fractions.
Year 6	ACMSP144	Describe probabilities using fractions, decimals and percentages.
Year 7	ACMSP167	Construct sample spaces for single-step experiments with equally likely outcomes.

Fraction shapes and counting boards
($5–$10 from Toys With Pizzazz)

Toys with Pizzazz are a small family business at Carrara Market in Queensland. They import a wide range of wooden educational toys direct from manufacturers. These include sets of fraction circles, squares, and triangles (each comprising coloured shapes divided into halves, thirds, and quarters). The wooden counting board (see Figure 5) has multiple representations of the counting numbers including domino-like tiles, numerals, pegs of the corresponding height, and coloured rings.

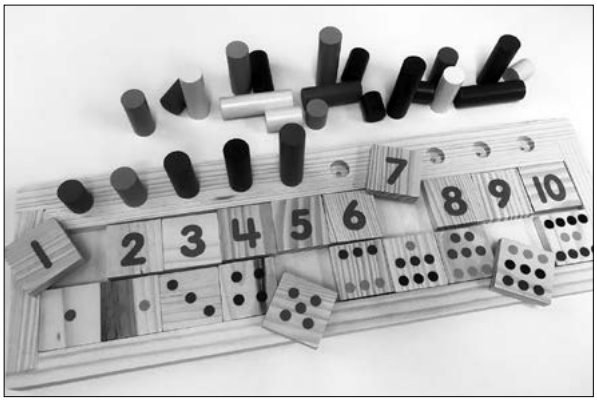

Figure 5. Counting board from Toys with Pizzazz.

Toys with Pizzazz can be contacted through their Facebook page or +61 (0) 404 045 657.

Conclusion

Few teachers will be surprised by the results of the recent survey that revealed that they spend a significant amount of their own money on educational resources (Australian Associated Press, 2017).

Table 4: Mathematics curriculum links for counting board.

Level	Content descriptor	Description
F	ACMNA001	Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point.
F	ACMNA002	Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond.
F	ACMNA003	Subitise small collections of objects.
F	ACMNA289	Compare, order and make correspondences between collections, initially to 20, and explain reasoning.
Year 1	ACMNA013	Recognise, model, read, write and order numbers to at least 100.

After spending a great deal of time searching for mathematical manipulatives that offer the best value for money, all of the resources discussed here have been incorporated into aspects of my teaching, both in schools and in teacher-education settings. In all cases, they have been received with interest by students, and led to useful discussions and learning opportunities. There is considerable research that suggests students benefit from experiencing mathematical concepts in multiple representations and contexts (Haylock, 2006). I have no hesitation in recommending the resources discussed here to teachers who wish to build their collection of hands-on resources to support the teaching and learning of mathematics in a fun and engaging manner.

References

Australian Associated Press. (2017). Public school teachers pay for own classroom supplies, survey shows. *The Guardian.* Retrieved from https://www.theguardian.com/australia-news/2017/apr/02/public-school-teachers-pay-for-own-classroom-supplies-survey-shows?CMP=share_btn_link

Drews, D., & Hansen, A. (Eds.). (2007). *Using resources to support mathematical thinking.* Exeter, UK: Learning Matters Ltd.

Haylock, D. (2006). *Mathematics explained for primary teachers* (3rd ed.). London, UK: SAGE Publications.

Haylock, D., & Manning, R. (2014). *Mathematics explained for primary teachers* (5th ed.). London: SAGE Publications.

Reys, R., Lindquist, M., Lambdin, D., Smith, N., Rogers, A., Cooke, A., . . . Bennett, S. (2016). *Helping children learn mathematics* (2nd ed.). Milton, QLD: Wiley.

Serow, P., Callingham, R., & Muir, T. (2016). *Primary mathematics: Capitalising on ICT for today and tomorrow* (2nd ed.). Cambridge: Cambridge University Press.

Swan, P., & Marshall, L. (2010). Revisiting mathematics manipulative materials. *Australian Primary Mathematics Classroom, 15(*2), 13–19.