Promoting flexible and adaptive use of data representations

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Reporting on one learning activity, as part of a larger research project, demonstrates how Year 2 students carry out a class survey and then transfer the data collected in tabular form to a different representation of their choice.

Introduction

This article encourages approaches that develop children's understandings of data representations. Research suggests (English & Sriraman, 2010) that young children need increased exposure to opportunities to experience complex and meaningful learning activities that enable them to develop their reasoning-with-data skills. As described by Curcio (2010), these meaningful activities with data also provide the scope for children to engage with different levels of data comprehension including understanding, interpreting and predicting, which in turn enable flexible and adaptive use of data representations. In a data-rich world, this develops in children the necessary critical reasoning skills to engage with data that is part of their everyday lives.

In particular, when children develope flexibility in their approaches to data representation and understanding, the same data can be represented in different ways. Flexible and adaptive use of representations is part of mathematics competence which should be directly addressed in classrooms (Heinze, Star, & Vershaffel, 2009).

Representations (e.g., drawings, tables and graphs) are both tools for problem solving and a means of explaining or communicating mathematical processes (Heinze et al., 2009). English (2006) has suggested it is "imperative that students be given experiences that encourage them to interpret mathematical situations in different ways and to communicate their understandings of these situations meaningfully to their peers" (p. 303). Australian research (English, 2012; Mulligan, 2015) has shown that Year 1 and Year 2 children are able to actively engage in statistical inquiry. Both researchers reported increasingly advanced representations and statistical thinking, particularly when the topic was authentic and meaningful to the children. Conducting a class survey, for example, is an effective way of enabling these experiences while drawing on authentic real-world situations (English, 2013).

Research context

This article is an analysis of one learning activity that was completed as part of a larger research project. Seven Year 2 teachers and their classes conducted a series of learning activities that focused on data and different representations, across science, technology, engineering and mathematics contexts. All of the learning activities were designed to develop student understanding of various ways to represent data using a range of familiar contexts. This article focuses on the mathematics learning activity and reports research data from three participating teachers (co-authors) and 81 children who completed the mathematics activity in the seven classrooms. In this learning activity, children either individually or in pairs, surveyed their fellow classmates about their favourite chocolate. Conducted over two lessons, this activity drew on suggested components of Year 2 mathematics in the *Australian Curriculum: Mathematics*, specifically, the statistics and probability content descriptions about data representation and interpretation (Australian Curriculum, Assessment and Reporting Authority, 2018) that include activities that:

- Identify a question of interest based on one categorical variable. Gather data relevant to the question;
- Collect, check and classify data; and
- Create displays of data using lists, tables and picture graphs and interpret them.

The activity promoted data visualisation, interpretation, representation, and enabled children to 'generate and develop their own mathematical ideas or concepts' and therefore 'learning mathematics via problem solving' (English, 2006, p. 304). Similarly, Pfannkuch and Wild (2004) identify the importance of representations supporting the development of understanding. In their model of statistical thinking, Pfannkuch and Wild (2004) use the term "transnumeration" as the process of "changing representations to engender understanding" (p.18). When representing their survey data, the students were required to consider and compare how the alternative visual representations would best display their data. Creating opportunities for children to work with different representations of the same data, enables a flexible and adaptive disposition to develop (Heinze et al., 2009).

This article reports on two elements from this learning activity. The first element was the types of data representations produced when asked to present their survey data in a different way (transnumeration). Work samples were analysed and used as an indicator of flexibility with data representations, that is, shifting from a tabular format to an alternative representation. The second element was the teacher experiences of the activity, with specific reflections from three participating teachers. In this article, the student work samples and individual teacher observations while implementing the activity, and a focus group interview reflecting on the learning activity, form the basis of the discussion about the learning activity. The key reflections included engagement, flexibility in choosing new representations, and understanding data representations, as well as challenges and future directions for teacher practice.

Activity sequence

The children were given a blank tabular template (see Figure 1) and asked to develop a question that would guide their data collection. They were also asked to decide on the chocolate categories they would use, with most using a combination of brand names (for example, 'Kit-Kat') and general chocolate type (for example, 'milk chocolate').

Figure 1 is an example of how children organised their survey data, predominantly using a variety of symbols to identify the chocolate preference of each participant. These included lines, ticks, crosses, and the occasional picture. Some children also totalled the columns based on chocolate category (see Figure 1), but many did not document totals.

Figure 1. Example of tabular survey data collected.

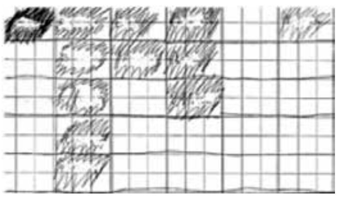
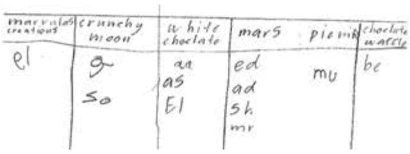
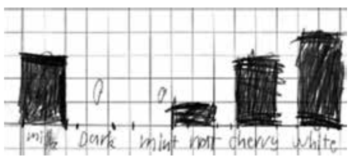
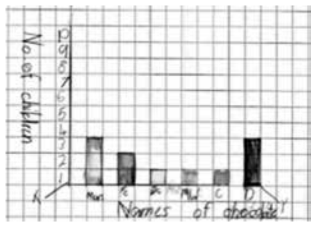
In the first part of the activity, following data collection, each classroom teacher facilitated a discussion about the findings. The children were asked to review their survey findings and make statements about what they noticed about chocolate eating habits, and also about the representation of these data. The second part of the activity was designed to promote flexibility with representations, and move beyond the tabular template. Children were asked: "Can you represent these data in a different way?" As part of the transnumeration of their data, children were offered the options of using blank paper and graph paper. After completing their new representation, children were asked about their choices, how they organised their information and how the interpretation of their second representation compared to their first.

An analysis of the data representations

Using student work samples, the new data representations were analysed by the researchers using a purposefully-developed coding frame. Figure 2 identifies an excerpt of the coding frame that is relevant to this article, including the number of children who created this type of representation. The children's graphic representations were coded into three broad groupings: elementary pre-representations that offered no clear organisation of the survey data (see pale-blue section in Figure 2); representations using appropriate techniques but included

Elementary pre-data representation	Intermediate data representation	Established data representation
Visual No discernible pattern (n = 4)	**Graph-like: Pictogram** Some correct elements but no key (n = 7)	**Graph: Pictogram** Many correct elements and a key (n = 5)
Tabular No discernible pattern (n = 15)	**Graph-like: Bar/Pie** Some correct elements but no scale (n = 23)	**Graph: Bar/Pie** Many correct elements including a scale (axis) (n = 27)

Figure 2. Type of data representation.

Elementary pre-data representation	Intermediate data representation	Established data representation
 A: Visual	 C: Graph-like: Pictogram	 E: Graph Pictogram*
 B: Tabular	 D: Graph-like: Bar/Pie	 F: Graph: Bar/Pie #

* Student response in E included a key, whereas the student response in C did not include a key.
# Student response in F included a scale for the y-axis, whereas the student response in D did not include a scale for the y-axis.

Figure 3. Examples of student responses for each of the data representations.

inconsistencies and/or inaccuracies (see medium-blue section in Figure 2); and representations that accurately represented the survey data (see dark-blue section in Figure 2). Of the seven-class cohort (n=81):

- 24% of children produced items at an elementary pre-data representation level;
- 36% of children at an intermediate data representation level; and
- 40% of children at an established data representation level.

Examples of each type of data representation are presented in Figure 3. The most common representation selected by children across the participants was a bar graph, with 34% of the participants accurately representing their data in a bar graph (see Figure 3F) and 27% constructing a bar-graph-like representation with some inconsistencies or inaccuracies (see Figure 3D). In total, 15% of the participants represented their data in pictograms of some form (Figures 3C and 3E), while 19% reproduced a tabular style graph (Figure 3B), similar to the original template. The remainder opted for a visual representation (Figure 3A).

Teachers' experiences of the activity

The three teachers—Annette, Madeline and Rayelene—shared their experiences of the activity during an interview with Aspa and Sonia. They discussed in detail overall engagement with the activity and their experiences of children's approaches to the representation of data, focusing on the flexible and adaptive use of data representations. The challenges they faced in relation to implementing the activity were discussed, as well as some ideas for future applications and modifications.

Engagement

The teachers identified the importance of relevant authentic contexts to classroom activities in engaging students in their learning. One teacher stated: "Kids love surveys … they love surveying. [They] get their clipboards out and off they go; they feel very important." As acknowledged in other research (English, 2013; English & Watters, 2005), teachers in this project saw the value of developing activities with 'real-world' experiences that were meaningful to the children. Teachers suggested, "it gives them a purpose", enables "ownership", provides a "context" for their learning, and "gets them excited" when the topic is "relatable to them". Certainly, the idea of chocolate appealed to many of the children, and one teacher indicated:

> They wanted to know the outcome, especially with the favourite chocolates … what the favourite chocolate in the classroom was. Having that end goal and understanding that at the end of the class they could actually have a discussion around what was the most popular, really engaged them.

Indeed, this class had a discussion about their findings, and it was an opportunity for children to make connections to one another through similarities and differences of opinion (which meets the Personal and Social Capability within the F–2 curriculum). The teachers capitalised on student engagement by providing other real-world examples of how market surveys are used by businesses to make decisions. One teacher shared, "I used the tuck shop as an example. [The tuck shop] wants to get only five brands of chocolate and we're helping them pick the best five". "So, we do try to find real-life situations where they're collecting data and analysing it for a real purpose". In addition, it also enables children to feel that their opinions are valued and that their data collection is meaningful. This is an important role of a survey as it provides young children with an opportunity to have their say about matters that concern them (Baroutsis, McGregor & Mills 2016).

Flexible and adaptive use of data representations

The teachers discussed the value of providing children with opportunities to represent their data. This approach enabled children to demonstrate their ability to switch to a new representation and use this new representation to communicate different mathematical processes with data. These processes included beginning processes (Irons, 2007), such as sorting, ordering and comparing, and, in some cases, more advanced comparisons that involved computations with values within the representations. The notion of flexible and adaptive use of data representations is discussed with a focus on children choosing the new representations. Of interest here is the children's decision-making about choosing a new data representation. Decisions included those relating to the use of resources, such as a ruler or colour pencils, or the more abstract decisions about the use of scales or dimension and other graphic elements to identify discrete units. The children were offered two formats for representing their data: plain paper and graph paper. Just over half of the children selected graph paper (56%), with the remaining 44% selecting plain paper. Interestingly, some children who selected plain paper attempted to draw their own grids to support their representation.

Of the children who drew bar graphs (n=50), 30% selected plain unruled paper while 70% selected graph paper. Similarly, of the children who drew pictograms (n=12), 58% selected plain paper, with the remainder selecting graph paper. While having learnt that bar graphs are drawn on graph paper, some children found the activity to be a new situation, separate to their prior learning. For example:

> The majority of my class chose the plain paper to start with ... I tried to guide and say "you know, graph paper's great, we've got the little boxes already." But a lot of them struggled with the sizing of the graph paper. They liked the plain paper. Then as we progressed through the task, a couple of them said, "Wait, I need to change my mind" and they took a piece of graph paper. I think that was a big shift that they hadn't predicted or hadn't planned where they wanted to take the data yet. They kind of just jumped in and plain paper's something that they see every day … and then as they started to lay it out and represent the data, they did take the graph paper.

While the selection of paper was only one aspect of decision-making, it was one that influenced children's representations.

Understanding data representations and interpretations using current research

The teachers indicated that when undertaking the tabular representation task, children tended to interpret data at an individual level, for example, 'Sophia likes milk chocolate'. This was perhaps prompted by the tabular template that was provided for data collection (see Figure 1) as it enabled children to list the individual names of the people they surveyed. Konold, Higgins, Russell, and Khalil (2015) identify this approach as a focus on 'case values', that is, a value such as 'milk chocolate' is associated with the individual case, 'Sophia'. However, when children were asked to re-represent their data, there was a shift towards interpreting data as 'classifiers' (Konold et al., 2015). This involves combining the individual cases into groups or clusters, for example, 'seven people like milk chocolate'. In Figure 3, examples A and E both represent data as classifiers; however, example D does this with a greater number of bar graph elements (e.g. scale and axis labels). Like English (2012), and as shown in Figures 1 and 3, we found that there was a large variation in children's approaches to representing data. As described by Heinze et al. (2009), this is likely indicative of individual differences in mathematics competencies. Interestingly, Mulligan (2015) reported that as students produced more representations of the same data, there was evidence of different types of representations and, in some cases, greater abstraction. However, in a comparison between the first representation using the table template, and the subsequent representation (n=81), 36% provided totals based on category groupings in the table template format whereas 81% did in the re-representation. We are interpreting this increase in the percentage of children using category groupings and totals as a possible indicator of their progression towards 'classifiers'.

Challenges and future directions

While the teachers appreciated the value and scope for encouraging the flexible and adaptive use of data representations as an important part of mathematics classroom activities, they identified two potential challenges, in the form of time and curriculum expectations. These two factors are linked; while the curriculum provides some scope for developing Year 2 students' skills in data representation, this is often tempered by the lack of time available to promote both flexibility and interpretation with data. These challenges and possible future directions are discussed in the next two sections: time and opportunities to engage with data interpretation and curriculum expectations and children's prior experiences.

1. Time to engage with data interpretation

When describing how the children communicated their data interpretations, the teachers suggested there was limited opportunity for children to engage in comparisons between or within their representations. While there was evidence of 'case value' and 'classifier' interpretations, time constraints may have limited further interpretation with the different data representations. Konold et al. (2015) describe a further 'aggregate' lens where children identify trends in data. In the current activities, for example, children did not make statements such as, ' more than half the class like milk chocolate'. This, in part, may be due to the participating children being in Year 2, however, it is important to have activities that have scope for advanced thinking. English (2012) suggests that such activities present children with opportunities for generating 'informal inferences' such as identifying variation, or making predictions. Teachers found that the children tended to interpret the graph rather than move beyond the data seen in the representation. For example, one teacher stated: "Our questions are more interpreting the graph. Where we should move on and frame questions around inference and prediction". The teachers suggested that reading beyond data they saw in front of them was somewhat difficult for Year 2, suggesting children tend to not "think out of the box as it would seem—to make an inference about why. You'll ask them why and they'll respond with 'I don't know'. Because they like it, or they don't like it." The careful design of the task context and strategic teacher questioning could be used to specifically facilitate children to identify trends ('aggregate' lens), as well as read beyond the data towards inference and prediction. This is something that warrants further consideration and investigation.

In terms of future directions, teachers agreed that moving beyond the data towards 'informal inference' is an important skill for children to develop (Makar & Rubin, 2009). Discussions with meaningful data are seen as essential, as children were eager to see the findings in relation to the most popular chocolate in their class and who else in the class had similar tastes in chocolate. To allow time for further interpretation and discussions with different representations of data, one teacher suggested the use of age-appropriate software to reduce the time taken to create new representations, and spend more time encouraging further data interpretation.

2. Curriculum expectations and prior experience

In the current activity, curriculum expectations and children's prior experiences posed some challenges to the open-ended nature of the 'represent in a different way' task. Specifically, the teachers were mindful that the curriculum progression was prescribed as, "grade 1 they learn about pictograms and grade 2 it is bar graphs" (Australian Curriculum, Assessment and Reporting Authority, 2018). Furthermore, the teachers commented that in previous survey tasks, children would typically create their own table for data collection and would use tally marks. The students were unaccustomed to the tabular template and number of categories within the variable. One teacher indicated that the sheer size of the table was "overwhelming" for the children. Previously, children had collected data using 'three categories and four students to collect on'; however, this activity comprised six categories of chocolate and 12 respondents. Some teachers suggested that future data representation lessons should include larger data sets. Then scaffolding, such as using the familiar tally marks, could be used as an intermediate phase to help children understand the requirements of the task through explicit instructions, and help young students progress their understanding of various representations.

Conclusion

This article has focused on a Year 2 activity to promote flexibility in data representation and interpretation. When representing survey data in a different way, approximately three-quarters of the 81 children attempted a pictogram or bar graph, which aligns with *Australian Curriculum: Mathematics* content area descriptors, respectively. The participating teachers acknowledged that the promotion of 'representing the data in a different way' did result in shifts in levels of data interpretation of the children in their class. This was evidenced in work samples, with the second representation demonstrating a greater proportion of students using totals for the category groupings, indicating evidence of the more advanced interpretations. For teachers, this implies that allowing time for children to flexibly and adaptively engage with data representations in meaningful contexts is an important mechanism to support more advanced data interpretation and communication of mathematical ideas, but this may mean going beyond what is in the content area descriptions.

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