Enabling mathematics: Curriculum design to support transfer

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Introduction

Enabling education has existed in Australia for over 40 years (Pitman et al., 2016). It is a significant entry pathway for students who have not been able to access university education through the traditional school pathways. Such programs routinely include mathematics subjects as part of the core curriculum.

However, teaching and learning mathematics for students in these programs is not without its challenges. Students typically arrive with diverse knowledge and experiences (Viskic & Petocz, 2006) and often present with high levels of anxiety, negative views, low self-efficacy and only a simple procedural understanding of mathematics (Klinger, 2006; 2008; Carmichael & Taylor, 2005; Taylor & Galligan, 2011; Givvin, Stigler & Thompson, 2011; Gordon & Nicolas, 2013; Benken et al., 2015). Yet the application of mathematical skills and knowledge is essential for success in their chosen undergraduate degree programs (Wandel et al., 2015) whether it is in a traditional science or other courses such as nursing, allied health, education, business, tourism, the arts and social sciences.

To prepare students for the rigours of university study that involve the application of quantitative concepts, students must be able to apply the skills and knowledge they learn at the enabling level at the undergraduate level and beyond. The ability to transfer knowledge and skills from one context to another is the essence of preparing students for mathematics encountered in undergraduate studies. Yet there is a paucity of literature on the curricula of enabling programs. In Australia and New Zealand changes have been evolving over a number of years. Taylor and Mohr (2001) designed and evaluated a program built around a numeracy framework including some problem solving, Galligan (2004) designed a curriculum for International students which focused on mathematical communication skills and problem solving. In New Zealand Miller-Reilly (2007) described three different approaches to teaching enabling mathematics 'in context' through non-routine problem solving. The

need of enabling mathematics students to apply mathematics learnt to degree level studies (Galligan & Hobohm, 2015) means that problem solving together with skill development should be at the heart of an enabling curriculum.

In a mathematical context, it is to be expected that individuals will at times struggle to apply their mathematical knowledge and skills when solving problems in unfamiliar settings, particularly when writing about the mathematics itself. The struggle itself requires students to put in effort to make sense of the mathematics, particularly when solutions and the processes required to obtain them are not immediately apparent (Hiebert & Grouws, 2007, p. 387). Such struggle may contribute towards 'math anxiety'. However, this struggle can be valuable and lead to positive outcomes (Boaler, 2016). Ideally by putting in the aforementioned effort, students make connections between mathematical problems and the concepts and procedures required to develop solutions. While teachers can optimise learning conditions to the best of their ability, mathematical connections ultimately must be made by the students themselves and effort is required by the students to achieve this.

This paper presents a curriculum framework for an Australian enabling mathematics subject which aims to incorporate many of the above ideas. It reports on the implementation and effectiveness of the program through students' performance in undergraduate programs and teachers' and students' perceptions of its success and challenges.

Curriculum framework for enabling mathematics: The theoretical

Although a number of curricula frameworks for mathematics exist in the school sector (e.g., www.australiancurriculum.edu.au/mathematics/curriculum), no curricula framework exists for enabling programs. The Singapore Ministry of Education Framework (2001, p. 5) is a framework with problem solving at its heart. Stacey (2005, p. 345) described the value of this framework in some detail while Anderson (2009, p. 3) compared curriculum frameworks from four countries and reported that the Singaporean implementation of the framework included assessment of non-routine problems, making it more viable. Thus the Singaporean framework most strongly represents the centrality of problem solving to mathematical development. Using this structure, the framework proposed for enabling mathematics places problem solving as the principal goal and organizer of the curriculum. The surrounding factors contribute to problem solving, rather than operating as equal partners. The contributors include attitudes, skills, concepts, processes and metacognition as displayed in Figure 1.

If the curriculum is driven by problem solving, then it takes more than rhetoric to implement it (Anderson, 2009). It thus follows that problem solving must occur within every facet of student activity, including direct instruction

(content delivery), experiential learning activities (individual and interactive), learning support and most importantly high stakes assessment.

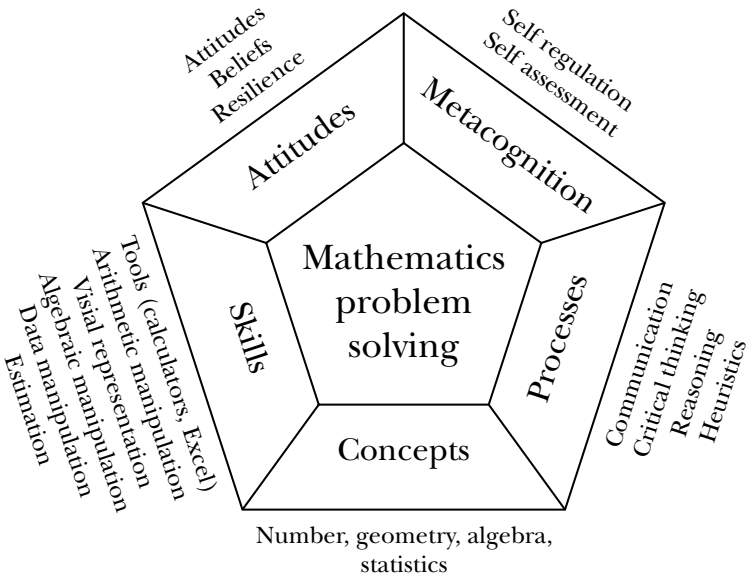

Figure 1. Curriculum framework for enabling mathematics (modified from Singapore Mathematics Curriculum Framework (2001, p. 5).

Curriculum framework for enabling mathematics: The practical

Southern Cross University is a regional university in northern NSW, Australia, which enrols approximately 16 000 students annually. The Southern Cross Preparing for Success Program (PSP) is a 13-week enabling program which enrols approximately 1000 students each year and has been operating for 10 years. Students undertake four subjects related to learning management, communication, mathematics and an elective. The program is studied full-time or part time at one of three campuses or online. Tutorials have approximately 25 students. The program's principal goal is to equip students with the capabilities, skills and knowledge to succeed in university undergraduate study.

The mathematics subject within PSP is called Applying Quantitative Concepts and enacts its title through the application of non-routine problem solving. The subject has been a core subject since 2010 and was designed around problem solving. Today the subject routinely enrols 300–400 students each session with 40% of students studying online. The curriculum is delivered through instructive learning resources (12 written online topics and videos, on-campus and online classes), learning activities (on-campus and online classes), learning support (individual and small group consultations, online discussion board) and assessment.

The first learning module encourages students to explore their beliefs and attitudes to learning mathematics. It initially draws on the work of the

UK-based National Numeracy Challenge (www.nationalnumeracy.org.uk/national-numeracy-challenge) to get students to question the roles of value, effort and ability in returning to study mathematics. This is followed by an exploration of the usefulness of making mistakes in the learning process based on the work of Boaler (2016) which in turn promotes a 'growth mind-set'. Other subjects within PSP aim to develop a student's growth mind-set and this is reinforced in this early mathematics topic. These early concepts are strengthened throughout the subject by a success-focused, 'can-do' approach that teachers advance in classes and student discussions. These tactics are essential to commence the challenging non-routine problems students will face in the rest of the subject and in their future undergraduate studies.

The remaining learning modules follow a defined pattern. They commence and close with a real-world problem set to explore problem solving within the context of particular mathematics concepts and skills. Skill development is addressed through online numeracy resources. In Weeks 3 and 10 of the 13 week session, students spend intensive time investigating more extensive non-routine problems. These group-based investigations are enhanced through video clips, the use of discussion boards and online exemplars.

Although problem solving occurs in most components of the curriculum, assessment in particular drives the students' focus. In this program 80% of assessment time is devoted to non-routine problem solving and embedded skills (through two high stakes assessments worth 35% and 45% submitted in weeks 6 and 13 respectively). Twenty percent (20%) of students' assessment time is allocated to standalone mathematical skills (accrued continuously from week 2 through the completion of four online quizzes). A typical problem solving question is displayed in Figure 2. Further examples of assessment topics include:

- impact of an increasing Goods and Services Tax (GST);
- using a microscope;
- investigating the age when children start to speak;
- effectiveness of suburban speed limits;
- blood alcohol levels and driving.

Students are advised from the beginning that:

> Problem solving is always challenging. It is not just about using a formula but means that you have to think carefully about what assumptions you have made and how you might solve the problem. There is usually no one right answer or no one right method, as each of you will bring different ideas and experiences to the problem solution (online audio recording provided to students in Week 1).

To help develop critical thinking skills and construct a logical response, students prepare a written solution using three section headings. Students commence their response with a 'getting started' section, where they analyse

A newly appointed apprentice wants to purchase a new mobile phone plan. His annual take home salary will be $27 276 as he is under 21 years old, and he estimates that he spends on average $410 per week on other living costs. The apprentice needs to ensure that the plan is within his financial capacity.

Your task

Of the two mobile plans recommended which plan should he choose that is both cost effective and would meet his likely needs? Explain all reasoning.

Useful information

- Table A: Average usage pattern for an under 21 year old.

Type of usage	Totals per month
Phone calls	525 minutes per month
Text messages	1683 texts per month
Face book	420 minutes per month
Web browsing	420 minutes per month

- Table B: Typical data usage for different activities

Type of usage	Data used
Facebook	1MB per minute
Web Browsing	400 kB per minute

- Data usage conversions
  - 1024 kilobytes (kB) = 1 megabyte (MB)
  - 1024 megabytes (MB) = 1 gigabyte (GB)

- Table C: Two different phone plans.

Plan	Cost per month ($)	Length of contract	Included value per month	Text	Phone calls	Data per month
1	19.90	no contract	$500 (for phone and text)	25.3c/ text	$1.54 per minute	100 MB (excess 10c/MB)
2	95	24 months	$950	Unlimited	$2.38 per 2 minutes	2.5 GB

Steps to help you answer the question

- Determine the mobile phone usage for the apprentice each month.
- Using the monthly usage plan determine the cost associated with each plan.
- Given the apprentice's other living costs, determine which phone plan is most appropriate and within his means.

Figure 2: Example of a typical problem solving question presented to students.

the problem to show their understanding, interpret and give meaning to any numerical data and select and describe a method of solution. In the 'calculations' section, students perform the necessary calculations and communicate their solution(s) in a logically sequenced way. Finally, in the 'conclusion' section, students evaluate and analyse their results and methods, comment on any assumptions or limitations that affected these results and justify the practicality of their method(s) of solution. This section is then

completed with a discussion of alternative methods that could have been used to solve the problem and the relevance of the overall problem to the real world.

These non-routine problem solving questions occur within the two high stakes assessments, but students are given the opportunity to self-assess their skills and knowledge prior to attempting these problems through at least two graded quizzes before week 6 and another two graded quizzes prior to week 13. Students have an option to resubmit the first written assessment for a passing grade and extensions are provided on a case-by-case basis.

In summary, this case exemplifies how a subject can be delivered with problem solving at its heart without compromising the development of routine mathematical skills. A summary of how this fits into the model is displayed in Table 1.

Table 1. Curriculum elements and how they fit into the proposed framework.

Curriculum organiser	Curriculum elements
Attitudes, beliefs, resilience	'Welcome' relationship-building video from the subject leader and topic on learning mathematics Welcome topic on how to learn mathematics Video and introductory games Discussion board/on campus classes Teachers with belief in student's success
Skills	Using tools e.g. calculator, spreadsheets Arithmetical, algebraic and data manipulation Visual representations
Concepts	Integers, fractions, decimals and percentages Units and conversions Rates and ratios Geometry Powers Introductory algebra and linear relationships Introductory statistics
Processes	Question analysis Reading and writing in mathematical contexts Checking practices Estimation practices
Metacognition	Self- assessment – practice quizzes Practice problem solving Mathematics checking practices Estimation as a way of knowing

Evaluation

The effectiveness of the curriculum design was evaluated in three ways:

- Post-PSP maths students' performance in two undergraduate subjects were compared with the performance of students who came through traditional pathways.
- Student feedback on the subject was collected through an anonymous feedback process administered by the university. This was supplemented by unsolicited comments received by tutors.
- Tutors' reflections on the subject were gathered from three tutors who had taught the subject for one, two, and six years. Three or four tutors are employed each session to teach the subject.

Student performance

The most significant evidence supporting that the subject has achieved its aims lies in students' undergraduate results. Anecdotal reports from undergraduate teachers indicated that students were performing well in mathematics-based undergraduate subjects. To confirm this result two cohorts of students were tracked and monitored in their undergraduate studies in 2015. One cohort of 61 students studied a traditional first year mathematics subject (subject A) while the second cohort of 32 students studied a first year subject that had mathematics deeply embedded within its context (subject B). The pass rates of students in the undergraduate subjects are displayed in Figure 3.

Within the traditional first year mathematics subject, the pass rate of 62% was slightly higher than that of students who entered the program through traditional pathways (usually school leavers). In the subject in which the mathematics was highly contextualised, former PSP students' performance exceeded that of students who entered through traditional pathways (100% compared with 87%). Although there may be other factors involved, these results indicate the possible impact of completing the PSP unit.

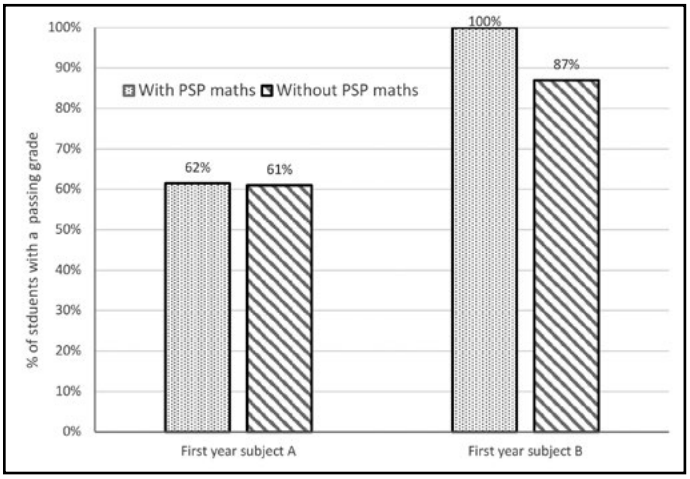

Figure 3. Performance in two first year undergraduate subjects for students with and without PSP mathematics.

Student feedback

Student feedback on the subject is collected routinely using the university institutional instrument at the end of each session. This instrument includes

open ended questions and questions based on a five-point Likert scale where 1 represents strong disagreement and 5 strong agreement with a statement. Responses to selected questions relevant to the curriculum from the last three sessions are displayed in Table 2. Overall students' mean scores of above 4 were reported indicating that by the end of the session students were satisfied with the subject.

Table 2. Mean student feedback collected by SCU institutional instrument using a Likert scale where 1 is strongly disagree and 5 strongly agree with the given statement. The response rate is reported as a percentage.

	2015 S2 (29%)	2015 S3 (30%)	2016 S1 (23%)
The objectives and performance standards in this unit were made clear	4.13	4.30	4.21
This unit helped me to develop some valuable skills/ attributes	4.08	4.34	4.10
Overall, I am satisfied with this unit	4.03	4.37	4.30

In response to the open ended question, students made a variety of comments. Some students restated experiences about their previous mathematical learning and background and how it impacted on their learning:

> I have struggled for all of my life with Maths. <Tutor> has help me to appreciate and understand maths… the way <the tutor> teaches has helped me immensely and has demystified most of the confusing concepts of maths.

Other comments provided by successful post-PSP undergraduate students included the following comments:

> I did maths in 1992 and I don't think I even passed it… but to be able to know how to do <maths> is pretty important in the midwifery degree. You know giving someone medication, administering drugs and if you've got a calculator or an iPhone where it's just gone flat or turned off and you can't work it out, that can be the matter between someone's life and someone's death. It's pretty important stuff. So that was pretty crucial for me and very beneficial.

It is clear from student comments that a number of students struggled with this subject, but with this struggle came success and the satisfaction that they had achieved beyond their expectations.

Tutors' reflections

Three tutors were asked to reflect on their experiences in teaching the subject. One tutor had six years' experience while the other two tutors had one and

two years' experience. All were experienced teachers in their own right prior to teaching in this subject.

The students' experience of school mathematics, whether recent or in the past appears not to have prepared them for this type of mathematical thinking and writing. The most difficult thing for the students was not necessarily the mathematical skills but the positioning of these skills within the problem context. Students had significant difficulty writing about what they did to obtain a solution and many had difficulty unpacking the assumptions they made that impacted on the solution. Two tutors reflected:

> Initially, many students may not see the value of proposing solution strategies, explaining *why* these strategies are followed and then taking a step back and considering the *assumptions* and *limitations* behind the problem being solved. By developing and practicing the skill of writing about mathematics (as opposed to just performing calculations), students come to see the value and significance of the mathematics itself and are in a far stronger position to critically reflect upon and appreciate its relevance to everyday life. (Tutor with two years' experience in the subject)

It was the general observation of the tutors that the expansion of critical analysis enables students to transfer the skills and practices acquired in this subject to the deeply contextualised mathematics they will encounter in their undergraduate studies. A tutor says:

> Many students who enrol in this subject come into it with the preconceived idea that 'obtaining the correct answer is the be all and end all of mathematics'. However, being able to talk about and write about the mathematics itself is for many a missing link in their notion of what mathematics is all about… This in turn strengthens their skill set going into undergraduate level study. (Tutor with two years' experience in the subject)

One tutor also works with students once they have progressed to undergraduate studies. This tutor reflects:

> <The students> response is without fail that the inclusion of the problem solving responses in the Preparing for Success Program has helped them when they are asked to complete assessments involving any sort of calculations. Students quickly realise that their answers depend on the understanding of the premises and that answers are not simply focused on a successful calculation but rather the justification and explanation of the assumptions and limits of their answer. (Tutor with six years' experience in the subject)

Discussion

Students enrolled in enabling programs face academic and adjustment issues that are not necessarily encountered by undergraduate students entering directly into university (Hodges et al., 2012). Further, the additional challenges of engaging students in mathematics-based courses is widely documented (Galligan & Taylor, 2008; Bonham & Boylan, 2011). Taken together these challenges may present significant obstacles for students. However, the results presented in this paper indicate that the development of an enabling curriculum for mathematics with problem solving at its heart can build students' capabilities, skills and knowledge to a level that overcomes their previous experiences, changes attitudes towards mathematics and promotes individual student success in their chosen undergraduate program.

The student's journey is not an easy one and involves struggle for both the students and staff. However, it is clear that students engaged, albeit hesitantly, with the problem solving approach later acknowledged its value upon progression into their degree studies. This approach allowed them to extend their mathematical competence beyond a purely procedural level towards a deeper level of understanding of and having greater confidence in their own mathematical ability. The inclusion of a formal written component within the high stakes assessment tasks explicitly connected concepts with procedures and solutions and enhanced students' academic numeracy capabilities in line with previous work of Taylor and McDonald (2007).

The work presented here also supports the investigations and reflections of Richland, Stigler and Holyoak (2012, p. 1970), who in their analysis of developmental mathematics in the USA hypothesise that 'students will be best served by learning to represent mathematics as a system of conceptual relationships in which problems and concepts must be connected'.

This paper, through the presentation and implementation of a curriculum framework for enabling mathematics, shows that if problem solving is seriously considered at the heart of the curriculum, as the main organiser for student actions, then mathematics courses can enable students who have previously not been successful to succeed in undergraduate studies.

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