The tail of a whale: A real-world problem for the maths classroom

Geoff Woolcott
Southern Cross University, NSW
<geoff.woolcott@scu.edu.au>

Southern Cross University (SCU) educators and local teachers have developed a five-lesson instructional sequence built around fluke identification as a way of resolving the question: How fast do humpback whales travel up the east coast of Australia?

Introduction

Every year, humpback whales move up and down the east coast of Australia from their feeding grounds in the Antarctic to their breeding grounds in tropical waters of eastern Australia, near Hervey Bay in Queensland. Peta Beeman, a research student at Southern Cross University (SCU), is recording the patterns of whale flukes, the powerful swimming fin or tail of a whale. (Figure 1 shows a breaching humpback whale.) The pattern of each fluke is distinctive for each whale and, when people send images to Peta, she is able to process them using pattern recognition software called *Fluke Matcher*. This allows Peta and her team to recognise where each whale is at a particular time.

Figure 1. A breaching humpback whale on its migration north, seen off the coast of New South Wales.

In 2014 SCU initiated a team project that developed resources for teachers and school students designed to involve them in real-world investigations being undertaken by some of our scientists. Peta worked with a team of university educators and local teachers to develop a five-lesson instructional sequence built around fluke identification as a way of resolving the question: “How fast do humpback whales travel up the east coast of Australia?”[1] The idea was for students to go through similar processes to a scientist who was trying to answer this question, to see how they would respond to being involved in a real-world scientific inquiry.

Why tail flukes

There were several reasons for presenting students with this problem. One was that there was a lot of information already available about humpback whales. A team led by Daniel Burns at the Marine Ecology Research Centre, a SCU centre with researchers based at Coffs Harbour and Lismore NSW, had already documented the migration of humpback whales and their behaviours during their journey up and down the migratory corridor off the east coast of Australia (Burns et al., 2014). It was this team that developed the *Fluke Matcher* software, using it to identify whales from their fluke patterns—scars and pimentation patterns which act like fingerprints due to their uniqueness for each individual (Figure 2)[2] .

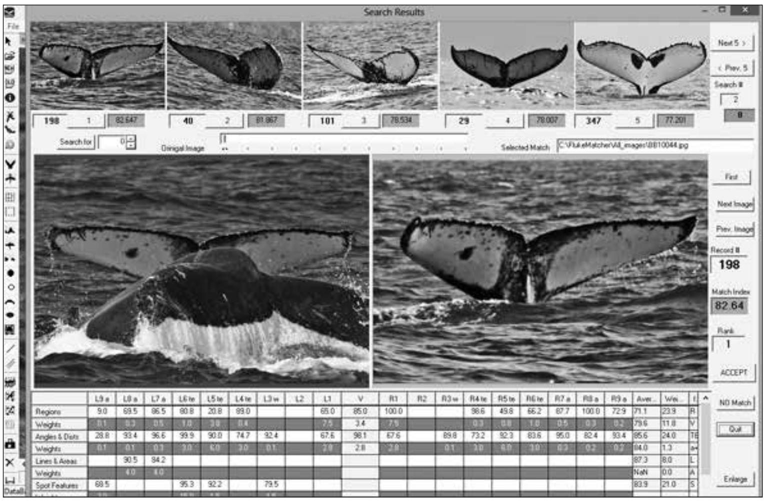


Figure 2. Peta used *Fluke Matcher* to identify a humpback whale from the pattern on its tail fluke.

Humpback whales had been part of a once-flourishing industry based at whaling stations along the migration route, including near the modern-day observation point at Byron Bay, which is as far east as the Australian coastline extends. Humpback whale numbers were estimated to be as low as 100 when whaling ended in the 1970s, but numbers had increased to 10,000 in 2015 (Bejder et al., 2016) and Peta has reported that numbers are expected to be more than 30,000 in 2018. Whale watching is now a major tourist attraction and people flock to various places along the east coast to observe them in the wild, particularly at Hervey Bay (Queensland), where the whales stay for up to two months, mating and giving birth to young calves before their return to the Antarctic.

1. Although travel speed was an important part of Peta’s study, its overall focus was looking at patterns of migration timing, such as year-to-year consistency, and residence time in the northern and southern termini of the migration.
2. Similar software can also be seen at https://happywhale.com/.

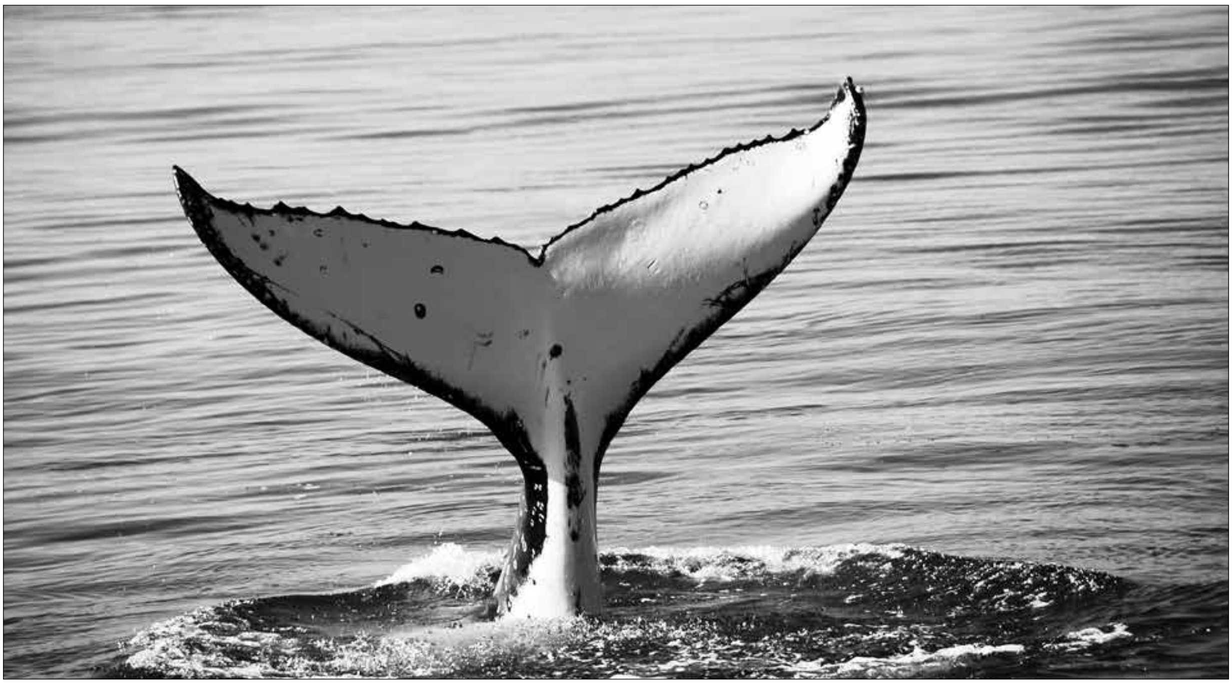
What makes a good fluke photo?

- Photo must show the underside surface of the fluke.
- Focus and contrast should be sharp enough that the markings can be clearly seen.
- The angle of the fluke should not be so sharp that the markings are obscured.
- At least 50% of the fluke should be showing above the waterline.
- Photos should be high resolution, saved as digital files (e.g., .jpeg or .tiff files).

To contribute photos to the East Coast Whale Watch Catalogue, please use the online form at www.eastcoastwhales.com.au

✗ Over 50% of fluke submerged.

✗ Oblique angle, low contrast.



✓ Markings clearly visible.

The photos to the right, are of the same whale travelling south near Ballina in 2004 (left) and then again in Byron Bay in 2009 (right).

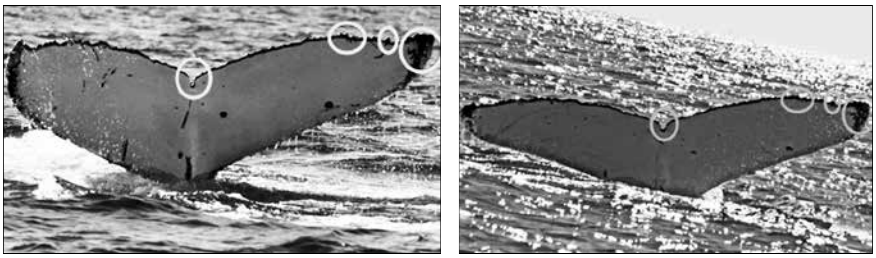


Figure 3. What makes a good fluke photo?

Peta's question was designed to find out how fast the whales travelled on their migration. This included considering the questions: Do they travel at a constant speed?, Do they vary their speed at night?, Do they all leave Antarctic waters at the same time?. Knowing that fluke identification could be used to log the whales' journey, Peta invited contributions of fluke images from whale watchers all along the east coast of Australia for the East Coast Whale Watch Catalogue, a citizen science project. Since images uploaded also note a date and location, identification in *Fluke Matcher* allows her to create a catalogue that shows an individual whale at particular times during its journey up and down the coast.

Analysis of photographs of flukes, such as those contributed by people on whale-watching boats, enable researchers to identify individual whales in repeated sightings from year to year along their migration path (Figure 3). The catalogue has been used to analyse data on over 1000 humpback whales (between 2005 and 2014) enabling researchers to obtain valuable information about life histories, population size, migration timing, travel speeds, movement and association patterns (who swims with whom, and when).

Whale tail lessons

The topical issue of whale migration was included as part of the Regional Universities Network's (RUN)[3] Digital Classroom project conducted from 2013 to 2015. The aim of this project was to bring scientific and mathematical research into the secondary classroom, using collaborations within each of the six universities of RUN, and with teachers and secondary school students within each university's educational footprint. Peta's research question was considered a good example of how science and mathematics[4] could be integrated across a real-world problem, using appropriate levels of knowledge, skills and experiences from each subject. The five-lesson whale migration sequence is shown in Table 1, although it can easily be reconfigured to take up more or less time in a school program.

Table 1. The lesson sequence for the whale migration research question.

Lesson one	How are whales identified? Whale fluke patterns as identification criteria.
Lesson two	How can we identify how fast individual whales are travelling? Using rate/ratio, latitude and longitude.
Lesson three	Whale-song graph analysis.
Lessons four and five	Tourist Operator Analysis—investigating the tourism industry surrounding whale sightings.

Each lesson plan provided an opportunity for an instructional practice based on student-centred inquiry around a central goal statement, rather than an expositional

3. The Regional Universities Network (www.run.edu.au/) is based in eastern Australia and comprises Southern Cross University (SCU) and the University of New England (UNE) in New South Wales, Central Queensland University (CQU), the University of Southern Queensland (USQ) and the University of the Sunshine Coast (USC) in Queensland, and Federation University Australia (FedU) in Victoria.
4. This paper focuses on mathematics and science but recognises that this focus may lie within the overall framework of Science, Technology, Engineering and Mathematics (STEM).

or didactic framework (teacher explaining and students listening). In this way student participation can be optimised verbally and through interaction with peers as well as with the teacher. The lesson materials provided opportunities for students to develop their own conclusions individually and in groups, and compare them with those of their peers and their teacher.

A focus on spatial reasoning

Although real-world problem solving was the central focus of the five-lesson whale sequence, several of the study components (lesson plans and resources) in the first two lessons incorporated spatial reasoning. A number of studies have focused on the importance of spatial reasoning (or spatial thinking) in secondary schools, as well as its importance in STEM (Wai, Lubinski, & Benbow, 2009). Importantly, Uttal et al. (2013) have shown that spatial skills are malleable, that is, they can be taught. Although the use of spatial skills is integral with human-lived experience and spatial negotiation in a 3D world, school curricula have only recently begun to recognise the value of teaching that encompasses these skills (Bruce et al., 2017).

The first two lessons in the sequence have a strong spatial skill component: lesson one is dedicated to students finding a way to identify whales from patterns on their tail flukes; and lesson two is dedicated to using spatial information to document the whales' journey and their speed of travel. The two lessons involve categories of spatial skills in teaching tasks described by Uttal et al. (2013) as either intrinsic or extrinsic, and as involving either static or dynamic tasks (see Figure 4).

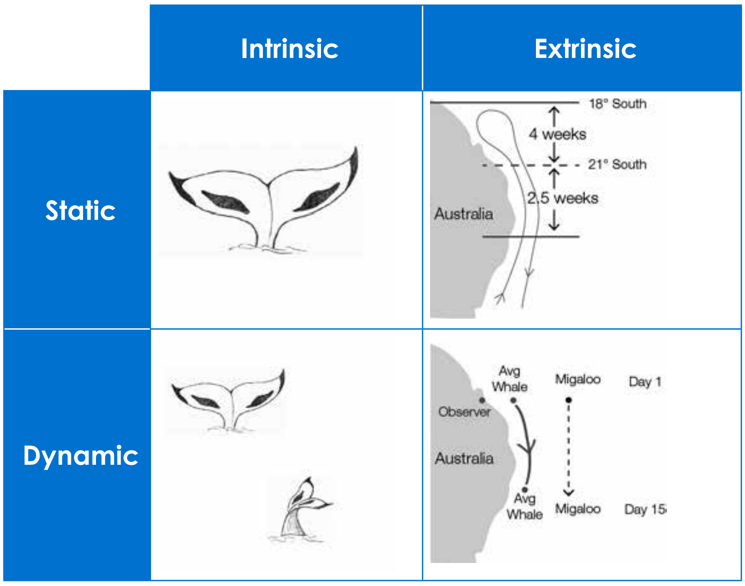


Figure 4. Illustrations of the four categories that combine the intrinsic/extrinsic and static/dynamic categories of Uttal et al (2013).

'Intrinsic' specifies the parts of an object and their relations whereas 'extrinsic' refers to the relations among objects in a group. 'Static' refers to fixed information and 'dynamic' refers to objects that are moving or moved, or viewed from a moving reference point. The classroom identification process required that students synthesise and differentiate the patterns represented in a set of given whale fluke images, as well as visualise the identification patterns. In other words, students combine elements from several sources to identify a commonality in pattern, but also separate elements according to difference to determine which fluke patterns are different. The second lesson, to do with the whales' speed of travel, required that students use identification data as well as annotated maps of travel pathways.

How did this work in the classroom?

The lesson sequences, along with the associated resources, were trialled successfully in several schools in the SCU region. The researcher acted as a passive observer in each lesson of the sequence, but took extensive notes during the lessons. At the end of each lesson the researcher interviewed the teacher, engaging them in discussion of how the lesson was received. A summary of the first two lesson outlines in the sequence is provided in Table 2.

Lesson observations

Lesson one: Fluke matching

Classes were not accustomed to this type of real-world inquiry (even though an expectation of the curriculum) but teachers were interested in seeing how students handled an issue clearly relevant for them through their familiarity with, and interest in, whales and whale migration. In the first two parts of the lesson, teachers employed the lesson format as recommended, with the introductory video followed by Whale Fluke Bingo done in competing groups: some of the flukes shown by the teacher have images matching those in the image set provided to each group. Teachers asked students how they identified similar whale flukes in the game and provided information from university scientists to confirm student responses.

The third part of the lesson, also conducted in small groups, engaged students in using the images and an acetate grid (unit squares) to find percentages of dark or light pigmentation (Figure 5). Teachers used the fourth part of the lesson, as recommended, for a guided discussion around the similarities and differences in fluke patterns that could be used for identification.

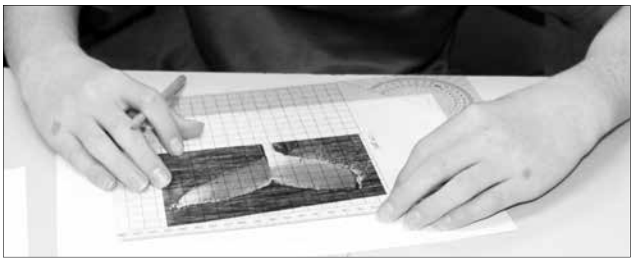


Figure 5. A whale fluke image being examined for patterns using an acetate grid of unit squares.

Lesson two: Whale travel

Each teacher used the given lesson materials to develop this part of the investigation in different ways. Despite having a narrative context for the whale journey, students experienced particular difficulty in interpretation of the visual material on the maps provided. Students also had difficulty in trying to provide reasons for changes in whale speed and how this related to latitude markings (see Figure 4 right-hand images). The teachers used a diagram from Burns et al. (2014) showing a map of the migration as well as locations and times (summarised in Figure 4). The materials provided were designed to assist in interpretation of a diagram representing the entire journey of Migaloo (an albino whale with a dedicated fan base and website, migaloo.com.au) and an 'average' whale, through use of partially completed interpretative tables.

Table 2. A summary of the four parts of each of the first two lessons in the Whale Fluke sequence.

Lesson 1: How are whales identified? Whale tail Bingo.	Lesson 2: How can we identify how fast individual whales are travelling?
Part 1. Presentation and introduction of the question. Research background in the form of a **five-minute video**, presented by the project scientist, on humpback migration and identification.	**Part 1. This lesson involves looking at how fast whales travel up and down the east coast.** The teacher introduces using **diagrams outlining the movements of whales** from their Antarctic feeding grounds to their breeding grounds near the Great Barrier Reef off Queensland.
Part 2. Identification of matching and non-matching tail flukes. **Whale fluke bingo using a set of 50 printed whale fluke images.** The teacher displays ten images on a projector/screen and students work to match their group's images with those displayed. Played as a game with three correct matches of flukes being a win for a team. Teacher then encourages students to note how the identified image was the same: What markings, shapes, etc did they use?	**Part 2. Calculating distance and time of travel.** Students, in pairs, **interpret the resource sheet information** to obtain distance and time data and use it to complete a table provided. Table to be completed for both Migaloo and for the 'average whale'. The sheet shows the east coast of Australia and extends to the Antarctic waters. The latitude lines, labeled with degrees, are included for significant locations on the map. The teacher presents the **key question:** *What assumptions/approximations about time must we make about residence times to get travel times for both the feeding range and the breeding range?*
Part 3. Calculating the estimated area of a fluke. Teacher/students overlay fluke images with an **acetate grid.** Teacher demonstrates how to calculate the area with a grid. Teacher presents **key questions:** *How can we calculate the percentage of dark pigmentation using the grids acetate?* *What area of the whale fluke was black? White?* *What percentage was white or black?*	**Part 3. Constructing travel graphs.** Students, in pairs, interpret **Figure 3 in the resource sheet** to obtain distance and time data and use it to complete the travel graphs for both Migaloo and for the 'average whale'.
Part 4. Teacher-guided group discussion around two main questions. *How can we use the information collected to build a model for identifying whale flukes?* *Can we generate a rule, pattern, or set of guidelines for identifying whale flukes?*	**Part 4. A teacher-led discussion** Using the graphs, interpret the slopes and identify the fastest segment and slowest segment of the whale's journey. Students examine and interpret their graphs to determine the fastest and slowest segments of the trip. The teacher presents the **key question**: *What factors could influence the speeds at which whales travel?*

What does this say about spatial reasoning?

In this section, we are looking to examine what spatial reasoning skills were integral to each lesson. Clearly the scientists working on this problem had an excellent command of the skills required to answer their real-world research questions. In observing student activities in lessons one and two, we were able to see what spatial reasoning skills students already had and what skills they were yet to obtain, at least in this problem-solving context. The spatial reasoning skills indicated in Figure 4 can be organised into four categories as outlined in Table 3. In observing student behaviour we focused on the four combinations of: intrinsic-static; intrinsic-dynamic; extrinsic-static; and, extrinsic-dynamic.

Table 3. The four categories of spatial reasoning skills adapted from Newcombe and Shipley (2015).

Category of spatial skill	
Intrinsic-static	Coding the spatial features of objects, including their size and the arrangement of their parts i.e., their configuration (e.g., to identify objects as members of categories).
Intrinsic-dynamic	Transforming the spatial codings of objects, including rotation, cross-sectioning, folding, plastic deformations (e.g., to imagine some future state of affairs).
Extrinsic-static	Coding the spatial location of objects relative to other objects or to a reference frame (e.g., to represent configurations of objects that constitute the environment and to combine continuous and categorical information).
Extrinsic-dynamic	Transforming the inter-relations of objects as one or more of them moves, including the viewer (e.g., to maintain a stable representation of the world during navigation and to enable perspective taking).

Lesson one: Fluke matching

In the whale problem context of lesson one, intrinsic refers to spatial activities that involved describing objects, such as pigment markings or scars, tail or fin shape and edge markings on the whale flukes. The patterns on the fluke did not change with respect to each other and hence were considered static. There was almost no requirement in lesson one for students to consider objects that were dynamic, for example, in predicting what the fluke might look like if it was seen from another angle. Extrinsic here refers to spatial activities where the students determined relations; in lesson one these were the relations between the flukes, fluke shapes and markings.

In the trials, students were able to refer to intrinsic-static features, such as overall shape, the colour and shape of distinctive markings from fluke images provided, as well as edge patterns, including scarring. In the exercise with acetate grids (Table 2 and Figure 5), many students were also able to calculate relative percentages of coverage of black or white patches, and some students were also able to recognise symmetry in the fluke markings. Extrinsic-static features recognised included patterns of configurations of shapes and colours within the fluke. Some students were able to additionally recognise these in the same fluke in a different position. Although there was little opportunity for observing dynamic spatial reasoning skills in lesson one, a few students (three or less) were able to model an identity pattern for a whale fluke and use this to identify whales from twisted or rotated fluke images. These students were also able to predict the distance of the same fluke viewed from a greater distance, by comparing the sizes of the grid of black or white areas on current images.

Lesson two: Whale travel

In lesson two the exercise was essentially dynamic, developed around graphing spatial positions and predicting time of travel and spatial position against a pattern of differing sets of information that served as a distraction. The static object was the map of eastern Australia showing degrees of latitude in only some parts of the journey, with lines showing the migratory path of the humpback whales, and duration markers (directional arrows) showing travel times, north and south, between the given latitude lines. For example, for the breeding range of the whales (latitudes 21°S to 15°S), the duration markers indicated travel times up and back as 2.5 weeks for the average whale, but 1.1 weeks for Migaloo.

Most students were able to interpret that the intrinsic-static features of latitude lines (degrees) and duration markers (weeks) on the resource sheet (annotated map) were members of a spatial category, such as static-extrinsic (although they did not need to know the names of such categories). The difficulty arose from the irregular spacing of the latitude lines and duration markings (Figure 1, top right image). The intrinsic-dynamic skills were less often observed where the skill needed was to transform the spatial codings of latitude and duration to predict whale positions for either the average humpback or for Migaloo. Most students had difficulty in interpreting the latitude lines in relation to time travelled and, hence, did not generally demonstrate this skill.

In lesson two, extrinsic refers to spatial activities where the students determined relations between a whale's location relative to other whales, or to the latitude and time markings, or places labelled on the map. Demonstration of extrinsic-static features required that students relate latitude of whale sighting to travel time, which was not in equal increments (e.g., a relatively long time in the breeding area). Few students were able to estimate distances between locations of the map by spatial partitioning, or to convert degrees to kilometres as required in this activity, even when the teacher provided exemplars for this context (for Migaloo). When provided with a completion table (Table 4) over the route taken, students fared better and were able to convert the information to a distance/time graph. The extrinsic-dynamic category was even more difficult. Students needed to predict the position of a whale based on the slope of a distance time graph derived from the sighting data and only three students were observed to do this.

How did teachers overcome the difficulties in lesson two?

Teachers reported that, although they had conceived this lesson as having a group activity component, the difficulty of the interpretation of these kinds of largely extrinsic spatial skills was best dealt with as a teacher-guided activity. One teacher, for example, prepared additional graphs and projected these from her laptop for the whole class to view. These graphs related to translating the distance travelled in given time periods and using this to compare parts of Migaloo's journey with that of an average whale.

One of the issues apparent from the lesson observations was that most students did not appear to have sufficient prior knowledge related to the map context and were consequently unable to engage completely with potential solutions to the problem. As the teachers indicated, prior knowledge of both spatial and non-spatial skills in such contexts may have assisted students in identifying and using latitude measurements to calculate distances. The lessons also required an overlap of cognitive functions, for example, integration of spatial skills with calculation or reading skills. Students, however, seemed to understand the nature and context of the problem, even where they were not always able to use the data provided for its resolution.

Table 4. Migaloo's journey compared with an average humpback provided as part of lesson two.

'Average' humpback whale migration patterns				
Location	Distance from origin	Degree latitude	Time between points	Time since start
Feeding range		70	25 weeks	0 weeks
Top of feeding range		60		
Feeding range to Ballina	3500 km	29	9 weeks	
Ballina to Hervey Bay	1000 km	21	2.5 weeks	
Breeding grounds		15	4 weeks	
Migaloo's migration patterns				
Location	Distance from origin	Degree latitude	Time between points	Time since start
Feeding range		70	28.6 weeks	0 weeks
Top of feeding range		60		
Feeding range to Ballina	3500 km	29	4 weeks	
Ballina to Hervey Bay	1000 km	21	1.1 weeks	
Breeding grounds		15	11.7 weeks	

Note: 1 degree latitude represents approximately 112 km

Conclusion

Overall, the project was successful in engaging world-class researchers, specialist educators, teachers and students in co-creation of digital teaching resources based on real-world problems and up-to-date solutions. The 'tail of a whale' lesson sequence is of particular interest to mathematics teachers since it is one of the few resources that aligns elements of the *Australian Curriculum* in both science and mathematics for Years 9 and 10. Some of our teachers have shown, however, that the whale lesson sequence can be separated as lessons for Years 7 to 10. The financial mathematics required for Lessons four and five are best suited to Years 9 and 10, as is the analysis of sound graphs in Lesson three.

Our trials of Lessons one and two indicated that our students may need a stronger background in spatial reasoning skills, albeit in a context of particular problems. A broader view of spatial reasoning skills across the curriculum may benefit from the overarching view, at least from the viewpoint of school learning, that all students may have skills that are based in their lived experiences in the 3D world, but which need to be adapted to the contexts required in school subjects. The approach taken in developing this lesson sequence suggests that spatial reasoning skills can be built into classroom problem solving, with motivational aspects provided by use of real-world contexts that are familiar to the students.

Acknowledgement

The Australian Government funded project, Regional Universities Network (RUN) maths and science digital classroom: A connected model for all of Australia (the RUN Digital Classroom), involved participants from the Regional Universities Network (RUN) and three associated organisations working with community-based educators and secondary school students within the educational footprint of each RUN institution. RUN is based in eastern Australia and comprises Southern Cross University and the University of New England in New South Wales, Central Queensland University, the University of Southern Queensland and the University of the Sunshine Coast in Queensland, and Federation University Australia in Victoria. The organisations involved in the collaboration were the Commonwealth Scientific and Industrial Research Organisation (CSIRO), the Australian Mathematical Sciences Institute (AMSI) and the Primary Industry Centre for Science Education (PICSE). These lesson sequences are available, along with the other educational resources developed for the Virtual Centre, on http://www.usq.edu.au/research/research-at-usq/institutes-centres/adfi/digital-classroom.

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