The maths is right!

Micah Stohlman

University of Nevada, Las Vegas, USA
<stohlman@unlv.nevada.edu>

The mathematics in some of the games on The Price is Right is discussed.

The Price is Right is a long-running television show that has been on the air for forty-six years in the United States of America. There was also an Australian version of the show for sixteen years. It is an energetic show and can be used as an exciting context for some great mathematics. The general format is that four contestants are called forward at the start of the show to Bidder's Row and are given the opportunity to bid on a prize, with the goal to be the closest to the actual retail price without going over. The contestant that is the closest without going over then plays a game for cash or prize. After this contestant has played a game, a new contestant is called down to Bidder's Row so that there are always four contestants trying to get on stage to play a game. Six contestants are given a chance to play a game. After three contestants have played a game, they then spin the big wheel in the Showcase Showdown. The two contestants that have the highest total in their group of three at the wheel compete in the Showcase round where the biggest prizes of the day are offered.

The Price is Right Bidder's Row and games provide many opportunities for mathematical connections. The Price is Right website has videos or pictures for students to see how each game is played (http://www.priceisright.com/games/). This article will describe the mathematics of probability and statistics that is integrated in the famous television show.

Bidder's Row

Guessing prices of products is an engaging activity that connects well to a Year 10 *Australian Curriculum: Mathematics* standard:

- Use scatterplots to investigate and comment on relationships between two numerical variables. (ACMSP251)

First, students are shown an item and guess the price. Like the television show, the goal is to be the closest without going over. Students record their guesses for the ten items that are shown to them. The students are then told the actual price and record this. Students use the guessed prices as the independent variable (x-axis) and the actual price as the dependent variable (y-axis) to produce a scatterplot. In doing this activity with middle years students, the following items were used: television, tablet, trampoline, man's watch, woman's watch, moped, two video game consoles, basketball hoop, and a ping-pong table. Any items that could be relevant to a teacher's student population could be used and prices can be readily found on the internet.

Next, students are given a piece of spaghetti and asked to show the "line of a person that was a perfect guesser; where every guessed price matched the actual price". Spaghetti is used because it is thin, helps with visualisation, and allows students to easily make adjustments to the line of best fit. Students then answer the following questions:

1. How many of your points fall on the spaghetti line? What does this mean?
2. Where do you see the points where your guess was too high? Too low?
3. Describe any patterns or trends in your scatterplot.
4. Draw in an "estimated" line of best fit that fits your data well. How does it compare to the line $y = x$?
5. Produce an equation for the line of best fit. Use your line of best fit equation to predict the actual price of an item if you guessed $350.
6. What does the slope of your line of best fit mean in this context?
7. What does your y-intercept mean in this context?

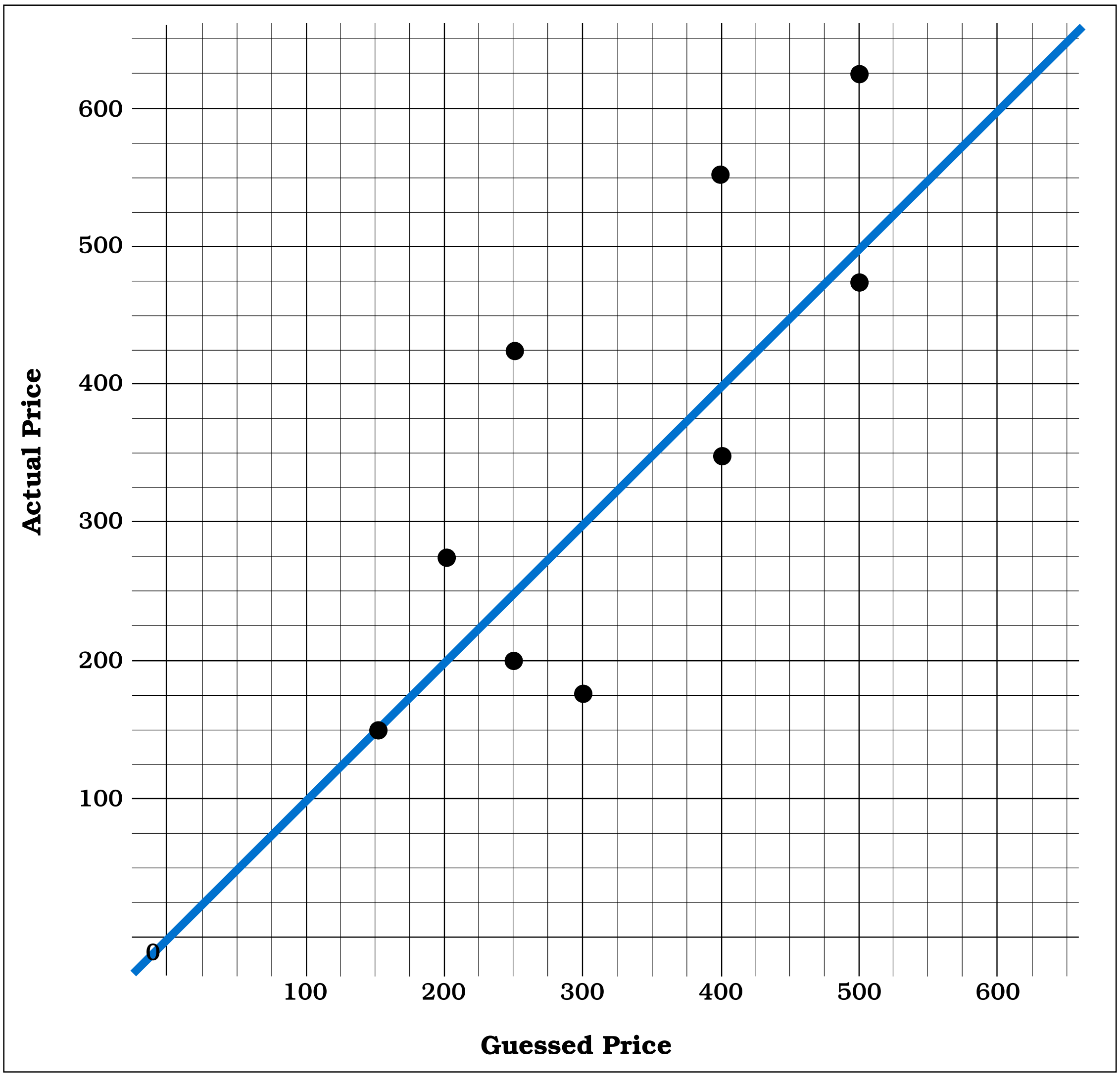

Figure 1. Scatterplot of data with line y = x showing perfect guesses.

For question 1, the line $y = x$ is the line that students' spaghetti should be showing. For question 2, it may be counter intuitive to students that points that fall below the spaghetti line are over-bids and points above the spaghetti line are under-bids. Students can use a few of their points that they know were under-bids or over-bids to check this as they determine where the points appear. The y-intercept for the line of best fit for the students' data would not have a sensible interpretation in this context as no contestant would bid zero dollars.

Temptation game

In this game, contestants have a chance to win a car and four other prizes or have the option to just take four prizes without playing the game. If contestants go for the car and do not get it, they end up winning nothing. This game connects to a Year 7 and Year 10 statistics and probability standard.

- Assign probabilities to the outcomes of events and determine probabilities for events. (ACMSP168)
- Describe the results of two- and three-step chance experiments, both with and without replacements, assign probabilities to outcomes and determine probabilities of events. Investigate the concept of independence. (ACMSP246)

In the game the first digit in the price of a car is revealed. Four prizes are then shown to the contestant and their prices are revealed. Each prize has two distinct numbers in the price, for example, \$525 or \$1010. Contestants must then pick one of the two digits as the number to be used in the price of the car. The contestant can then take the prizes, which usually total between \$3000 and \$5000, or go for the car and all the prizes. Students are asked what decision a contestant should make. Since the contestant has a 50% probability of getting each of the four numbers right, the probability (P) of winning the car and all the prizes is:

$$P = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} = 0.0625$$

The expected value can be used for further support to help students make their decision. Expected value is an average value. It is calculated as the sum of all possible values each multiplied by the probability of its occurrence. Let's say a contestant had a chance to win a \$20000 car plus \$3000 in prizes. He or she has a $\frac{1}{16}$ probability of winning \$23000 total in prizes, but also a $\frac{15}{16}$ probability of winning nothing. To figure out the expected value (EV), we take the probability multiplied by the values of the winnings: $EV = \frac{1}{16}(\$23\,000) + \frac{15}{16}(0) = \$1437.50$. Thus the expected winning, if going for the car, is \$1437.50 but the guaranteed prizes (without the car) are \$3000, which should lead to an easy decision. However, too often people go for the car and all the prizes. The temptation turns out to be too much. However, in doing this game with middle year students, most of the students wanted to take the guaranteed prizes before discussing the mathematics of the game. All of the students then said they would take the guaranteed prizes after discussing the mathematics.

Pathfinder game

The pathfinder game similarly deals with probability of compound events and the chance for contestants to win a car. Contestants stand in the middle of a five by five grid of numbers and are given the first number in the price of a car, which they stand on. They then have four choices for the second number in the car, three choices for the third number, two or three choices for the forth number depending on the show, and two choices for the fifth number. Contestants also have the opportunity to win three more guesses by knowing the price of three smaller prizes. Students are asked if they think it is easier to win a car in this game or the Temptation game. If students need more guidance the following questions could be used to help students determine which game is the better chance to win a car.

1. What is the probability of correctly guessing the second number of the price of the car?
2. What is the probability of correctly guessing the third number of the price of the car?
3. What is the probability of correctly guessing the fourth number of the price of the car?
4. What is the probability of correctly guessing the fifth number of the price of the car?
5. What is the probability of correctly guessing in a row the second, third, fourth, and fifth number of the price of the car?
6. If a contestant were able to have three additional guesses, what would be the probability of correctly guessing all of the numbers of the price of the car?
7. Is it likely that a contestant would guess the correct price of all three smaller prizes?
8. Is it more likely to win a car in the Temptation game or the Pathfinder game?

For question 5, depending on the show the probability (P) would be $P = \frac{1}{4} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{72}$ or $P = \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{48}$. This probability does not, however, factor in the three additional guesses that can be won. For each prize, the contestant is shown two prices and must determine which is the correct price. If the correct price is chosen, the contestant gets an additional guess. If the contestant were to get all three smaller prizes correct, he or she could incorrectly guess the price of the car three times and still win the car by being correct with the fourth opportunity to guess. The probability (P) would then become $P = \frac{4}{(72)} = \frac{1}{18}$ or $P = \frac{4}{48} = \frac{1}{12}$ depending on the show. The contestant may have some idea on the price of the three items, but if it was just a guess, the probability would be $P = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ to get the price of all three smaller prizes. Overall, most of the time the Temptation game would provide the better opportunity to win the car.

Punch a Bunch Game

In this game, contestants can earn up to four chances to punch on a fifty-hole punch-board arranged in five rows of ten. Contestants earn chances by being shown a price for a prize and correctly stating if the actual price is higher or lower than the price shown. Each hole on the punchboard has a value from $25 000 down to $100 and there is a certain number of each value (Table 1). After seeing the amount in one punched hole, contestants can keep the money or go on to see what value is in another punched hole.

Table 1. Punch a Bunch game values.

Value	$25000	$10000	$5000	$2500	$1000	$500	$250	$100
Number of holes with the value	1	2	4	8	10	10	10	5

Students answer the following questions on probability and expected value to determine the best strategy for playing this game.

1. What is the probability of winning each money amount with one punch?
2. Which amount has the greatest probability? The least?
3. What is the expected value for this game with one punch?
4. Suppose a contestant has passed on $1000 and $500. They get $2500 on their third punch. Should they keep this or see what value is in their fourth punch?

The expected value of the game with one punch is $2060 and contestants should keep any value $2500 or over.

$$\text{Expected value} = \tfrac{1}{50} \times \$25\,000 + \tfrac{2}{50} \times \$10\,000 + \tfrac{4}{50} \times \$5\,000 + \tfrac{8}{50} \times \$2\,500 + \tfrac{10}{50} \times \$1000 + \tfrac{10}{50} \times \$500 + \tfrac{10}{50} \times \$250 + \tfrac{5}{50} \times \$100 = \$2060$$

For question 4, the expected value changes to $2106.38 because there are only 47 money values left.

$$\text{Expected value} = \tfrac{1}{47} \times \$25\,000 + \tfrac{2}{47} \times \$10\,000 + \tfrac{4}{47} \times \$5\,000 + \tfrac{7}{47} \times \$2\,500 + \tfrac{9}{47} \times \$1000 + \tfrac{9}{47} \times \$500 + \tfrac{10}{47} \times \$250 + \tfrac{5}{47} \times \$100 = \$2106.38$$

In this case the contestant should still keep the $2500. If students get a different expected value, they can be asked if they removed one $500, one $1000 and one $2500 value from the probability calculations. For example, the probability of getting a value of $2500 on the fourth punch would be $\frac{7}{47}$.

Showcase Showdown

Students can explore the probability of spinning a total of $1 on the wheel and then getting $1 on a bonus spin. If a player spins a $1 on the wheel with one spin, or a combination of two spins (e.g. 40 cents and 60 cents), they earn $1000. For getting a total of $1, contestants also get a bonus spin where they can get $10 000 if it lands in one of two green spaces (the 5 cent or 15 cent space), or $25 000 if it lands on $1 again. Students are asked how likely it would be to get $26 000. A probability tree can be used for this (Figure 2). There are twenty numbers on the wheel with values starting at 5 cents and increasing in 5 cent increments. Thus, the probability of spinning the $1 in one spin is $\frac{1}{20}$ or 0.05. If a person does not get a $1 on the first spin, then there is only one number that they can spin that will total $1. For example, if a contestant spun 40 cents, then they would only have one option out of twenty again to get the dollar by spinning 60 cents.

1. Create a probability tree diagram to represent the different paths for the Showcase Showdown depending on if a contestant gets $1 on the wheel or not.
2. How many paths result in a contestant getting a total of $1 on the wheel and then getting $1 on a bonus spin?
3. What is the probability of a contestant getting a total of $1 on the wheel and then getting $1 on a bonus spin?
4. What is the final outcome with the greatest probability from the probability tree diagram?

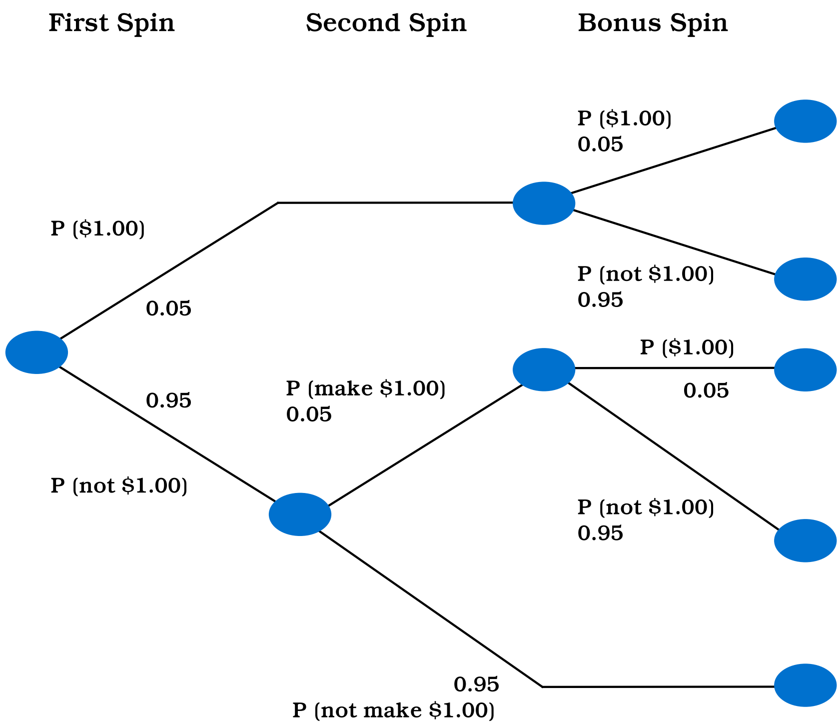

Figure 2. Probability tree for Showcase Showdown.

To answer question 3 and calculate the probability (P) we can combine the probabilities from path one and path three which would lead to spinning $1 and getting $1 on the bonus spin: $P = 0.5 \times 0.5 + 0.95 \times 0.05 \times 0.05 = 0.0025 + 0.002375$ which is approximately 0.0049 or 0.49%. The path with the greatest probability is path 5 in which a contestant has two spins that do not total $1. This probability ($P$) is: $P = 0.95 \times 0.95 = 0.9025$.

Other mathematical connections

There are different strategies for winning games that have been investigated by others that go beyond lower secondary mathematics. These could be explored further by accessing the listed references (Master Key game–Biesterfeld, 2001; Plinko game–Burks & Jaye, 2012; Spelling Bee game–Butterworth & Coe, 2002; Showcase Showdown–Coe and Butterworth, 1995).

Conclusion

The Price is Right has many wonderful mathematical connections with probability and statistics. The show itself is interesting, and allowing students to see the mathematics that is embedded in the show can help them to look for mathematics in their daily lives. The power of mathematics is that it is used in almost any field and the mathematics that students learn can help them in their current and future lives.

References

Biesterfeld, A. (2001). The price (or probability) is right. *Journal of Statistics Education, 9*(3). http://www.amstat.org/publications/jse/v9n3/biesterfeld.html

Burks, R. & Jaye, M. (2012). The price is right again. *Journal of Statistics Education, 20*(2). www.amstat.org/publications/jse/v20n2/burks.pdf

Butterworth, W. & Coe, P. (2002). The prizes rite. *Math Horizons, 9*(3), 25–30.

Coe, P. & Butterworth, W. (1995). Optimal stopping in "The showcase showdown". *The American Statistician 49*(3), 271–275.