The ELPSA framework

A learning design for developing mathematics understanding: The ELPSA framework

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Experience-Language-Pictorial-Symbolic-Application [ELPSA] is a learning framework that presents mathematical ideas through a cyclic design. The ELPSA framework which can be used to develop mathematics lessons and units of work that build students' concept understanding is described here.

Introduction

This paper describes a learning framework that has been developed and modified over a 25-year period, by members of our team. A description of the history of the framework is provided elsewhere (see Lowrie & Patahuddin, 2015). More recently, we have been using the framework in lesson design with secondary teachers in large nationally-funded projects in Indonesia and Australia. In working with the classroom teachers, the framework has been influential in developing (1) units of work and (2) sequences of lessons within a mathematics topic. The framework also affords opportunities for teachers to better understand the way in which pedagogical practices and learning experiences align to students' concept development.

Theoretical and conceptual underpinnings

The Experience-Language-Pictorial-Symbolic-Application [ELPSA] framework is underpinned by theories about learning that are considered constructivist and social in nature —especially concerned with the notion that learning is an active process where students construct their own ways of knowing (developing understanding) through social interactions with peers and individual learning differences.

Our framework assumes that experiences, both personal and collaborative, are the foundations for the introduction of new learning opportunities. The central idea to the work of social theorists is based on the premise that learning occurs through participation (Kolb, 1984; Lerman, 2003) and that participation should encourage high levels of engagement and interaction. That is, classroom practices that allow students to develop mathematical ideas from personal experiences and understandings increase the likelihood that content can be introduced in meaningful ways. Such a viewpoint is frequently taken by those who adhere to the realistic mathematics model (Gravemeijer, 2010; Heuvel-Panhuizen, 2003). This social foundation is embedded in the way that language is utilised in promoting learning. Social theories associated with how experiences are scaffolded (Vygotsky, 1978) and the influence of daily language on mathematics language (Adler, 1998; Setati & Moschkovich, 2010) highlight the importance of connecting personal experiences to mathematical terminology in order to ensure that sense-making can be promoted.

Psychology-based theories are also influential in our learning framework. The manner in which mathematical ideas are represented is critical to sense making. Dienes (1959) argued that concrete representation and manipulatives supported students' learning as they move towards more abstract concepts and ideas. Concrete and visual representations often provide the learner with the mental model of how pictures and symbols can be represented, which is essential for progression to sophisticated levels of analytic reasoning (Pirie & Kieren, 1994). In our learning framework, the connections between visual and analytical reasoning take place in the pictorial component—where teachers are encouraged to represent mathematics ideas in different ways. Typically, mathematics representations can be classified within two systems, namely internal and external (Goldin & Shteingold, 2001). Internal representations are commonly classified as pictures 'in the mind's eye' (Kosslyn, 1983) and include various forms of concrete and dynamic imagery (Lowrie & Logan, 2007) associated with personalised, and often idiosyncratic, ideas, constructs and images. External representations include graphical representations (e.g., graphs and maps), schematic representations (e.g., networks) and conventional symbolic systems of mathematics (e.g., algebraic notation or number lines). These two systems do not exist as separate entities and are seen as "a two-sided process, an interaction of internalisation of external representations and externalisation of mental images" (Pape & Tchoshanov, 2001, p. 119).

In our framework, the symbolic component involves the students' capacity to represent, construct, and manipulate analytic information in a symbolic manner. Mathematical symbols include number sentences, algebraic expressions, and other external representation that use symbolic notations. According to De Cruz and De Smedt (2013), mathematical symbols enable us to perform operations and actions that would not be possible without such supports. In this component of our learning framework, it is necessary for students to construct, and practise using, symbolic operations and notations in order to develop fluency, often with explicit instruction (Uttal, Scudder, and DeLoache, 1997). If mathematical symbols are introduced too early, or without support from pictorial representations, students become overly reliant on rote understandings of symbols (Lowrie & Clement, 2001).

Most learning frameworks do not explicitly recognise the role of application in the learning process. However, the manner in which students are able to apply mathematical ideas to new situations is considered critical to the enhancement of students' mathematical literacy. There is an evidence base that suggests students do not utilise mathematical understandings developed in the classroom in out-of-school situations. In our model, the application component also serves as a transition to a new (related) topic of mathematics.

Components of the framework

The ELPSA framework follows a learning design approach which is cyclic in nature. This design presents mathematical ideas through lived experiences, mathematical conversations, visual stimuli, symbolic notations, and the application of the applied knowledge (Lowrie & Patahuddin, 2015). In this learning design, teachers are encouraged to introduce concepts from what the students know.

Experience

This aspect of the model is about finding out what the students know—how students have used mathematics, what particular concepts they know, how they can acquire that information, and how mathematics has been experienced by individuals, both in and outside of classrooms. As a consequence, this component of the design involves assessment, since the teacher should be aware of individual differences and what new information needs to be introduced to scaffold students' understanding. Feedback and revision loops are critical to this component of the learning design. Learning experiences can include brainstorming, general discussions, the use of visual stimulus, and rich stories from the teacher or students.

Language

This component of design follows Experience and focuses on both the generic and specific language required to represent mathematical ideas. Mathematical language is used to convey meaning and sense making—both for everyday literacy demands and specific mathematics terminology. This component of the design is also associated with particular pedagogy practices, since it is important for teachers to model appropriate terminology and for students to use this language to describe their understanding, and converse with peers and teachers, to both explain and reinforce understanding.

Pictures

The third component of the learning design is associated with the use of visual representations to represent mathematical ideas. In general, there are two types of pictorial representations used in the classroom: (1) those constructed by the teachers or from learning resources; and (2) those constructed by the students. The former would could include representations of different parallelograms, including rectangles, squares, and parallelograms from the textbook. Such representations become a mental model to support student's understanding of two-dimensional shapes within the quadrilateral family. The latter, student constructed representations, might be encoded on paper, computer, or in 'the mind's eye'. Students might imagine transforming a square into a rectangle in their 'mind's eye', or they might draw a diagram to solve a geometric problem. Pictures are often used to help to scaffold their understanding and to provide stimulus to complete mathematical tasks before the introduction of the symbolic notation.

Symbolic

The next component of the learning design is the most recognisable aspect of teaching mathematics, that is, the use of symbols to represent mathematical ideas. To some degree, this component makes mathematics different from other discipline areas, and is sometimes referred to as a universal language. If moved too quickly toward symbolic thinking, misconceptions in students' sense-making can emerge, especially if students lack the conceptual depth to represent particular concepts in different ways. An over-reliance on this component of a learning design can also lead to an emphasis on rote learning or drill-and-practice. The component of the learning design should include scaffolded worked examples, refined practice examples, modelling fluent notation, open-ended investigations that encourage flexibility, and opportunities for students to discuss different ways of representing the same mathematical expressions.

Application

The component of the learning design highlights how symbolic understanding can be applied to new situations. New situations could be within the mathematics domain (e.g., relationships between area and volume or volume and surface area) or across other discipline areas (e.g., transformations in visual arts). The application component also provides opportunities for students to see how mathematics can be used in out-of-school contexts (e.g., how triangles are used as the foundation in engineering and architecture to design roofs). This component of the model allows teachers to add breadth and depth to conceptual understandings and in particular used in improving students' affect and stimulating creativity.

Tenets and principles of the ELPSA framework

In the following section, we describe the operational principles we use to frame the ELPSA learning design. The cyclic nature of the design is used to develop sequences of lessons within units of work (see Figure 1). It is uncommon for the five components of the design to be included in a single lesson—rather the process from Experience to Application might typically take two or three 50-minute lessons. The framework allows for multiple entry and exit points within the design. Capable students are afforded opportunities to progress through the respective components of the cycle quickly, allowing for the development of conceptual breadth at the Application phase. By contrast, some students may require more time at the Pictorial representation phase and may not engage in learning opportunities at the Application phase in a first cycle.

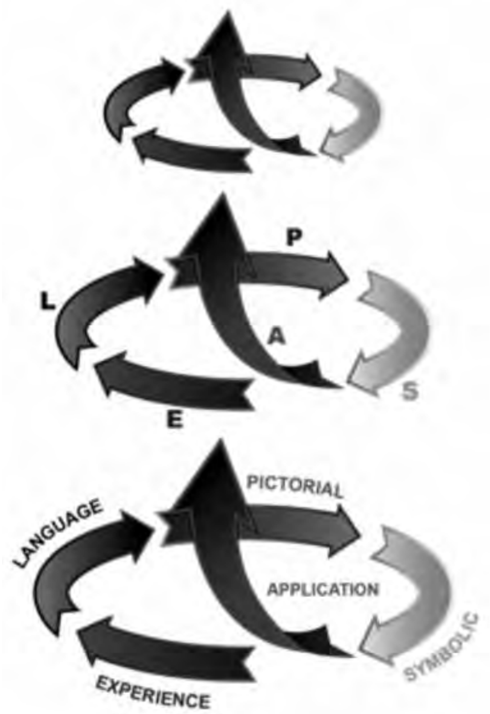

Figure 1. Multiple cycles of the ELPSA framework illustrate a series of lessons within a topic or unit.

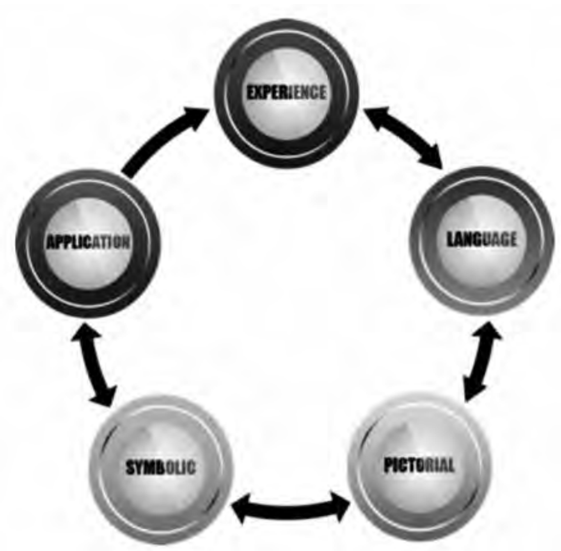

Figure 2. The five components of the ELPSA cycle, illustrating the recursive-progressive nature of the framework.

One of the fundamental tenets of the learning cycle is its progressive-recursive nature. Throughout the learning design, we encourage teachers to develop activities without 'skipping' components of the cycle, that is, only moving one step forward or one step back throughout the design process (see Figure 2). This has both pedagogical and theoretical implications for the learning design. If students are not understanding concepts or the mathematical ideas presented in a lesson, we encourage the movement back to more visual or concrete representations. In a similar vein, we do not advocate for leaps in mathematical representation—sometimes common practice in secondary classrooms where students are encouraged to reason symbolically before strong conceptual foundations are developed. In our framework, we encourage students to build on existing knowledge in ways that allow for the reconstruction and elaboration of understandings (moving back one component) in what Pirie and Kieren (1994) maintained to be a new level of understanding and depth—when students are ready to engage in new activities one component forward.

Each component of the learning design has a different function or purpose. The "L" and "P" components can be considered bridges or springboards in the cycle. The Language phase has purposeful links between and across the Experience and Language components. Mathematical language and terminology are drawn from students' existing understandings, with progressive concepts established through concrete and pictorial representations of these mathematical ideas. The Pictorial phase provides teachers with the opportunity to fold back to more concrete understandings when students are unable to access symbolic representations with sufficient accuracy or fluency. A representation of the distinctive features of each component of the framework is presented in Figure 3.

E	• Evoke out-of-school experience to build on understanding. • Reinforcing existing understandings to new concepts. • For new concepts, provide physical experiences if possible.
L	• Reinforce mathematics terminology throughout the lesson. • Foster conversations that link experiences with language. Build bridges between E & L. • Encourage students' own language while modelling precise terminology.
P	• Includes concrete manipulatives, external representations and students' encoded understandings. • Ensure multiple representations are provided including non-prototypical representations. • Progressively model effective pictorial heuristics.
S	• Introduce symbolic expressions alongside pictorial representations. • Encourage multiple appropriate symbolic representations. • Model fluency and flexibility with efficient symbolic representations.
A	• Apply symbolic reasoning to real-life situations. • Apply symbolic reasoning to related mathematics concepts. • Consider the application of the mathematics concepts outside of the classroom.

Figure 3. The central practices in each of the five components of the ELPSA framework.

Concluding comments

The ELPSA framework is presently being utilised by classroom practitioners, policy makers and university lecturers across Indonesia and Australia. As we have suggested elsewhere (Lowrie & Patahuddin, 2015), the respective components of the framework enhance students' sense-making by following a structure that is akin to natural concept development. Moreover, each component sequence provides a logical sequence to scaffold, reinforce and apply mathematics knowledge through a learning design that helps secondary teachers organise learning activities with mathematics topics or units of work. We describe how a series of algebraic lessons can be organised within the ELPSA framework in another manuscript within this edition of the journal (see Patahuddin, Lowrie & Lowrie, 2018, pp 32–40).

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