

Misconceptions of primary school students about the subject of fractions

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ABSTRACT

This study was conducted with the aim of investigating the current knowledge of Primary Teachers and Primary Pre-service Teachers on the misconceptions of primary school students about the subject of fractions. The qualitative research method of case study was used to conduct the research. The data were collected with semi-structured forms that were developed by the researcher to collect the views of Primary Teachers and Primary Pre-service Teachers on the topic. The participants stated that, regarding the subject of fractions, primary school students had difficulties the most in representing fractions by models, the concepts of denominator and numerator, when they need to rank fractions, when they need to solve problems, reading and writing concepts that express fractions, distinguishing types of fractions – converting these, showing a given fraction on number lines and operations on fractions, and they had misconceptions about these.

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1. INTRODUCTION

Instruction that is suitable for the structure of mathematics should have the purpose of helping students understand mathematics-related concepts and operations and establish links among operations [1]. Conceptual comprehension is having knowledge related to basic ideas and relationships regarding a subject. Operational comprehension is the symbolism that is used while performing mathematical operations. Conceptual comprehension is an important component of operational capability in learning mathematics [2]. The general objectives of the Turkish Mathematics Curriculum include students' capabilities to understand mathematical concepts, use these concepts in their daily lives and express concepts with their different ways of representation [3]. However, students cannot always form correct concepts, and unfortunately, they keep incorrect concepts throughout their lives. Some of these misconceptions are unique to students, while some are caused by the instruction methods used by teachers. Teachers should first be aware that they lead to misconceptions in their students and also know the reasons of these misconceptions [4].

When students learn a mathematics-related subject incorrectly or incompletely, they experience problems and these problems are reflected in their future education lives, and if these issues are not overcome, incomplete or incorrect learnings turn into misconceptions [5]. A misconception is 'a product of incomplete comprehension, and in most cases, it is an incorrect application of a rule or a mathematical generalization.' The best clues to understand whether something is a misconception or only a mistake are the frequency and consistency of the mistake (Spooner, 2002, cited in [6]). It is understood that the underlying reasons for misconceptions are not having learned concept knowledge and mathematical operation knowledge to an extent that they would complement each other, students' inability to acquire knowledge and skills related to problem-solving to the required degree, and shortcomings of students such as using the

incorrect rules and performing operations carelessly [7]. Misconceptions lead students to develop negative attitudes towards mathematics [8].

Fractions and operations with fractions are among the top subjects in mathematics because of both their structure and their relationship with other subjects [9]. The subject of fractions is highly important due to its relationships to subjects like rational numbers, ratio and proportion, decimals, percentages and probability [10], [11]. Fractions, which constitute one of the hardest topics for students and teachers, are among difficult mathematical concepts children encounter in primary school [11]-[21]. One main reason that students find fraction operations difficult is that they memorize formulae and algorithms instead of understanding fractions, while another is that they perceive the denominator and the numerator in fraction as two separate integers [22]. Each piece of information in mathematics generally has three different ways of expression as verbal, numerical and visual. The ability of students to make transitions among these expressions depends on their level of comprehension. Fractions have four different ways of expression as verbal, symbolic, objective and model. It is important in the subject of fractions and operations with fractions that students are able to make strong connections among these expressions. In this sense, it may be stated that there are serious problems in the transition between visual expression and the other ways of expression [9].

In Turkey, the subject of fractions is introduced beginning with the first years of primary school. The Primary School Mathematics Curriculum consists of four learning fields as 'Numbers and Operations', 'Geometry', 'Measurement' and 'Data Processing'. While all fields of learning are included in all class levels, some sub-fields are introduced after a certain grade. In the primary school mathematics curriculum (1st-4th grades), the subject of fractions is discussed under the fractions sub-field within the field of 'Numbers and Operations'. The subject of fractions is distributed along the grades as the following: raising awareness on whole and half fractions in the 1st grade; introducing the relationships of the whole and the half with the quarter grade and introduction to division (grouping, dividing into pieces) in the 2nd grade; introducing fraction-related terms by emphasizing the whole-piece relationship and reinforcing the understanding on the relationship between the numerator and the denominator by discussing the concept of unit fractions in the 3rd grade; defining/using proper, compound and mixed fractions, performing addition and subtraction operations on fractions and solving suitable problems [3]. Although these are not in the primary school curriculum, students are expected to understand compound and mixed fractions and convert these to each other, rank fractions, perform addition and subtraction operations with these fractions, make sense of these operations, and at the same time, solve problems.

Several studies have reported that students have difficulty in learning the concept of fractions and fraction-related operations [14], [17], [18], [23]-[32]. As students think the characteristics of fractions are the same as those of other numbers, they try to apply the same rule, and this leads to the development of their misconceptions [33], [34]. In the primary school mathematics curriculum, fractions are after the introduction of integers. However, fractions have different characteristics. While operations with fractions conceptually resemble those with integers, they are different to integer operations due to their numbers of operational steps. Teaching fractions in mathematics classes requires care and attention because of their complexity and conceptual richness. Understanding fractions and related concepts well in primary school and gaining skills of performing operations with fractions fast by understanding them will not only make this enjoyable subject of mathematics meaningful for students and contribute to their success in using fractions in other classes, but it will only set a solid preliminary learning basis for advanced mathematics subjects [35].

Another reason for misconceptions in students is the instructional approach of their teachers [36], [37]. Moreover, knowing about the misconceptions of students is very important also for teachers to diversify and use their instruction techniques [38]. As a result of the interviews they held with teachers and students, [39] reported two reasons for the very low accessibility levels of the targeted outcomes in the fractions unit as the misconceptions of the students and the learning-teaching process that is carried out by the teachers without taking the students' preliminary knowledge into account. Hence, it is important for teachers to know about misconceptions and take the precautions that are necessary to avoid them in students. This is because teachers may prevent the likely mistakes or misconceptions of students if they know about misconceptions and their reasons [40]. Likewise, if teachers determine the learning difficulties and misconceptions of students in the subject of fractions and prefer an instruction strategy based on these, learning of fractions may take place on a conceptual level [29]. Thus, it is important that teachers and prospective teachers know about the misconceptions of students. When the literature on misconceptions was reviewed, no study was found to investigate the views of both teachers and prospective teachers on the issue.

2. RESEARCH METHOD

2.1. Design

This study, which aimed to reveal an existing situation, was in the form of a descriptive case study. In this study, which was conducted with the aim of examining the current knowledge of primary teachers and primary pre-service teachers on the misconceptions of primary school students in the subject of fractions, the qualitative research method of multiple case study was used. The problem with the generalizability of case studies may be overcome by carefully designed multiple case studies [41]. This study utilized open-ended questions to collect the views of primary teachers and the primary prospective teachers on determining the misconceptions in the subject of fractions.

2.1. Sample

The sample consisted of 26 primary teachers employed at three different primary schools in Istanbul, Turkey and 73 3rd and 4th year primary pre-service teachers enrolled at a faculty of education in the same province and were doing internship within the scope of the 'Teaching Practice' course. The demographic information of the form teachers is provided below.

Among the primary teachers who participated in the study, 5 were 1st grade, 2 were 2nd grade, 10 were 3rd grade and 9 were 4th grade teachers. 15 of the teachers had 1-10 years, 5 had 11-20 years, 3 had 21-30 years and another 3 had 31-40 years of professional experience shown in Table 1.

Table 1. Descriptive characteristics of the primary teachers

Grades	f	Years of professional experience	f
1 st grade	5	1-10 Years	15
2 nd grade	2	11-20 Years	5
3 rd grade	10	21-30 Years	3
4 th grade	9	31-40 Years	3
Total	26		26

2.1. Data collection

The data were collected by two semi-structured forms that were prepared by the researcher as data collection tools.

Teacher Form: This form is a measurement instrument that was prepared to learn about the misconceptions of students determined by the teachers as a result of their experience. In this form, the primary teachers were asked about their demographic information and to provide three situations under the question 'what are the misconceptions your students have about the subject of fractions based on your experience?' to reveal the misconceptions that they thought their students had.

Prospective Teacher Form: The prospective teacher form was adapted for the prospective teachers, and the participants were asked to provide three situations under the question 'what are the misconceptions primary school students have about the subject of fractions based on your observations?'

In order to achieve content validity for these forms that were developed by the researcher, as reported by [42] opinions of three academics who are experts in the subject of mathematics and primary education were received. The forms were organized by examination based on the expert opinions and finalized.

2.1. Data analysis

The purpose of analysis in qualitative studies is to discover patterns, views, explanations and meanings [43], [44]. The data about the misconceptions of primary school students that were obtained in the study by the open-ended forms that were applied with both teachers and prospective teachers were analyzed with the method of 'descriptive analysis' [41]. The written explanations provided to respond to the open-ended questions were content-analyzed. The responses of the teachers and the prospective teachers were interpreted with descriptions by the researchers, and similarities and difference in the comments were revealed. The similarities and differences that were determined was named by various codes and tabulated based on their frequency of occurrence. In order to increase the reliability of the study, the determined categories and codes were separately examined by two experts experienced in qualitative research and a mathematics educator other than the researcher, and the categories were given their final shape. Including direct quotes from the views of participants and explaining the results based on these is also an important issue for the reliability of a study [41]. This study used this strategy and included direct quotes from the responses of the participants. The participants' views were presented by assigning codes for the two types of

participants as PT for the primary teachers and PPT for the primary pre-service teachers. The analysis of the demographic information in the study was achieved by SPSS 16.0.

3. RESULT AND DISCUSSION

This section of the article presents the respective views of the primary teachers and the primary prospective teachers on the misconceptions that they thought were present in primary school students in the subject of fractions. The primary teachers were asked the question ‘what are the misconceptions your students have about the subject of fractions based on your experience?’ and the responses are shown in Table 2.

Table 2. Primary teacher’s responses to the question ‘what are the misconceptions your students have about the subject of fractions based on your experience?’

Categories	f
Showing with models	14
Concepts of the numerator and the denominator	13
Ranking	13
Problem Solving	10
Concept of fractions and related reading and writing	8
Distinguishing types of fractions and conversion	8
Representation on the number line	7
Operations	5

Accordingly, the teachers stated that the students had difficulty mostly in showing with models, in concepts of the numerator and the denominator, when they needed to rank, when they needed to solve problems, in reading and writing at the same time in concepts that express fractions and in distinguishing types of fractions and converting them to each other, and they had misconceptions in these issues (Table 2). Some teacher views on these are listed below in the order from the most frequently stated misconception to the least frequently stated one:

‘They make mistakes in showing with models’ (PT 3).

‘Students find it difficult to divide a whole into unit fraction pieces’ (PT 17).

‘They see the numerator and the denominator in the fraction as separate, and they cannot notice that these numbers are related’ (PT 9).

‘They confuse the concepts of greatness-smallness. They cannot decide upon which fraction is greater as they still cannot comprehend the natural number and piece-whole sub-titles’ (PT 13).

‘They fall into a misconception by considering the natural number values while ranking fractions’ (PT 3).

‘As their conceptual knowledge about fractions are not internalized, students may experience confusion in algorithmic operations or forget easily’ (PT 9).

‘While comparing two fractions, they say the one with larger numbers is greater’ (PT 7).

‘In the case that the greatness of the fraction is dependent on a whole, students are not aware that the wholes will not have the same greatness, so they make mistakes in ranking’ (PT 12)

‘They do not understand specific problems’ (PT 12).

‘They perform operations from memory while solving problems’ (PT 14).

‘Not being able to notice fractions in different wholes may express different greatness’ (PT 25).

‘They cannot conceive that the numerator and the denominator represent a whole together’ (PT 6).

‘They might not comprehend the piece-whole relationship between the numerator and the denominator’ (PT 24).

‘They cannot conceive that pieces constitute the whole and the numerator is a piece of the whole, and they cannot realize that both are related’ (PT 3).

‘They have difficulty in understanding the concepts of piece-whole’ (PT 17).

‘Mistakes in showing the whole by dividing it into pieces and failure to draw each piece identically’ (PT 6).

‘They cannot read fractions correctly as they do not fully understand the concepts of numerator and denominator’ (PT 4).

‘They make mistakes while converting fractions into each other’ (PT 4).

‘Proper fractions are easier to understand than compound and mixed ones’ (PT 18).

- 'Failure to determine the intervals while showing on the number line' (PT 18).
 'Failure to show compound or proper fractions on the number line' (PT 21).
 'When they are asked to show a compound fraction on the number line, they do so by firstly converting it to a mixed fraction' (PT 20).
 'They might miss the integer part while showing mixed fractions on the number line' (PT 12).
 'There are problems in addition and subtraction as they do not see fractions as an entirety or a number and perform separate operations' (PT 5).
 'Trying to perform operations as in natural numbers' (PT 13).
 'Using the rules in natural numbers while performing operations' (PT 11).
 'Thinking that addition is adding numerators and denominators separately' (PT 1).

The primary prospective teachers were asked the question 'what are the misconceptions primary school students have about the subject of fractions based on your observations?' and the responses are shown in Table 3.

Table 3. Primary prospective teacher's responses to the question 'what are the misconceptions primary school students have about the subject of fractions based on your observations?'

Categories	f
Concept of fractions and related reading and writing	76
Ranking	40
Operations	35
Representation on the number line	25
Represent with models	16
Concepts of numerator and denominator	14
Distinguishing types of fractions and conversion	10
Problem solving	3

As a result of their teaching observations for three semesters, the primary prospective teachers stated that primary school students had difficulty in the concept of fractions and related reading and writing, ranking fractions, fraction operations, representation on the number line, when they need to represent fractions with models, in the concepts of numerator and denominator, in distinguishing types of fractions and conversion, and while solving problems, and they may have misconceptions on these issues (Table 3). Some prospective teacher views on these are listed below in the order from the most frequently stated misconception to the least frequently stated one:

- 'Students treat fractions as integers, they consider the numerator and the denominator as separate numbers' (PPT 3).
 'While expressing fractions with shapes, they cannot draw identical lines or squares. This shows that there is a problem in dividing the fraction into identical pieces in the subject of piece-whole' (PPT 16).
 'They cannot understand the concepts of whole and piece' (PPT 3).
 'Students may reflect their generalizations on natural number onto fractions and consider the numerator and the denominator as separate numbers' (PPT 29).
 'Considering the numerator and the denominator as separate natural numbers' (PPT 33).
 'They make mistakes while verbally expressing a fraction' (PPT 46).
 'Students cannot conceive that identical pieces represent a whole, and thus, they cannot divide whole into identical pieces' (PPT 48).
 'While comparing fractions, students generalize their preliminary information on natural numbers in fractions' (PPT 7).
 'Assuming that the fraction with higher numbers is greater as in natural numbers' (PPT 59).
 'While ranking fractions with equal numerators, they think the one with larger denominator is greater' (PPT 18).
 'They might not think of a fraction as pieces of a whole in comparisons' (PPT 63).
 'They separately add numerators and denominators in addition operations' (PPT 3).
 'In operations on fractions, they act like they are natural numbers' (PPT 71).
 'In operations on fractions, they generalize operations in natural number to this subject' (PPT 43).
 'They make mistakes in representation on the number line, especially representing operations like addition and subtraction' (PPT 42).
 'They confuse the number of pieces to divide intervals while showing fractions on the number line' (PPT 17)

- ‘They have difficulty in showing especially mixed fractions on the number line’ (PPT 72).
- ‘They have difficulty in showing compound and mixed fractions on the number line’ (PPT 58).
- ‘They make mistakes in representation by models, especially showing mixed fractions’ (PPT 69).
- ‘Failing to divide into identical pieces while representing fractions with a model’ (PPT 5).
- ‘They can form inaccurate displays as they cannot convert compound fractions into mixed ones while representing them with models’ (PPT 13).
- ‘They have difficulty in distinguishing the denominator and the numerator’ (PPT 72).
- ‘They make mistakes in converting a compound fraction into a mixed fraction’ (PPT 39).
- ‘They make mistakes in converting a mixed fraction into a compound fraction’ (PPT 3).
- ‘They have difficulty in reflecting problems on shapes for solution’ (PPT 62).
- ‘They cannot solve problems’ (PPT 1).

One result of the study was the statements that primary school students ‘confused fractions with their knowledge on integers.’ It was found that a significant part of misconceptions in students were caused by students’ generalization of observations that are valid in integers for fractions [35]. An attentive education process may lead students to understand and learn fractions to the required depth and prevent misconceptions caused by such generalizations. In fact, it was observed that most misconceptions were caused by inattentive approaches to fraction instruction. There are findings that suggested that an early and hasty transition to representation of fractions in the classroom with abstract symbols without dependence on student experience and a basic conceptual framework leads to misconceptions (Bezuk & Bieck, 1993; cited in [35]).

According to the views of the primary teachers and the primary prospective teachers, another situation among primary school students was that they had mistakes ‘while showing fractions with models’. It is a known fact that visual representations or shapes affect comprehension of mathematical concepts positively. Fractions and operations with fractions are a top subject among those where using shapes is the most important [9]. Shapes that are drawn and models that are used in relation to a problem provides ease of reaching the correct solution by making it easier to understand [45]. As using a set of models and manipulative tools in introduction to fractions makes fractions tangible for primary and secondary school students who are still in their tangible operation stage, it leads to easier learning of the concept of fractions and students to perform fraction-related operations more easily [22]. Davis, Hunting and Pearn Vergnaud and Kieren recommended the use of schemas that show the characteristics of rational numbers and fractions (geometrical models) and emphasize that using especially schemas of dividing a whole into pieces is highly important for establishment of knowledge on rational numbers [22]. In many studies, as in the case of this study, students experienced difficulty in expressing the fractions on given models and reaching the correct answer in problems of finding the model that is suitable for a given fraction [10], [46-48].

Based on their experiences and observations, the participants stated that students confused the denominator and the numerator, the concept of fractions was not fully established and especially the whole-piece relationship was not understood, and students had difficulty in writing and reading fractions. While providing students with knowledge on expressing a fraction in writing, the questions of how many identical pieces the whole is divided into and how many of these pieces are colored/selected should be answered, and the answers these answers should be represented with symbols [49]. Students think the numerator and the denominator are separate values. The main reason for this is that student use their knowledge on natural numbers when they encounter fractions [2]. The reason for this misconception is that, while fractions are taught, their representation with symbols is introduced without setting the conceptual basis for the symbols [35]. This agrees with the result found by [26] that students find it difficult to solve some fraction problems as they cannot fully understand piece-whole relationships. Although ‘piece-whole’ is the most basic concept in fractions [37] and it is in the nature of fractions [11], some students experience difficulty in learning this concept [51], [52] investigated the mistakes of fourth-and fifth-grade students in fractions and found that the students had problems in the principle of identical pieces in the piece-whole relationship. In the qualitative study by [53] with primary school students, the students thought of fractions like they are integers, expressed the concepts of the numerator and the denominator as separate numbers, and confused these with each other. Several studies reported findings that students confuse numerators and denominators [7], [26], [45]-[57]. In order to overcome these difficulties, firstly the piece-whole relationship should be taught to students while teaching fractions and a basis should be formed in students regarding the concept of fractions [50]. Baykul argued that models should also be used in teaching fraction-related concepts [1]. Doğan-Temur emphasized that for good development of a concept of fractions, education processes should be carried out by using real-life situations, tangible tools and equipment and models, rather than teaching rules to students [58].

According to the experiences and observations of the primary teachers and the primary prospective teachers, it is believed that the basis of the misconceptions on ranking fractions is that students think of fractions as if they are integers. As the quantity represented by a number is proportional to its greatness in

integers, this is mistakenly generalized for fractions, and some think that fractions with larger numbers are greater [35], Doğan-Temur reported that students made mistakes in several subjects such as ranking fractions, equivalency in fractions and mixing denominators and numerators [58]. Biber stated that, while ranking fractions, students rank denominators amongst each other, rank numerators amongst each other, and then, they apply these operations to the fractions [45]. Several other studies also reported that students had misconceptions about ranking fractions [7], [31], [48], [55]-[61]. In order to prevent this misconception, it should be shown that the quantities represented by equivalent fractions are the same by using tangible and pictorial materials in the classroom [35].

The primary teachers and the primary prospective teachers stated that primary school students had difficulty in fraction problems and made mistakes. They wrote that there were issues especially in the parts while understanding the problems, students made mistake in the solutions of the problems. Soylu observed that, in fraction problems, students experienced difficulty in understanding the problem, and therefore, determining the operations and order of operations in the problems [29]. Başgün found that students made mistakes in applying concepts about fractions in problems [62].

The statements of the participants suggested that students could not distinguish types of fractions and had difficulty in converting them into each other. As in this study, the teachers who participated in the study by Demiri reported that students had misconceptions about converting fractions into different types. 'Representing fractions on the number line' is one of the problems experienced by students [10]. In activities of drawing a number line model that is used to express fractions, the necessity to divide the whole into identical pieces should be emphasized with care. As in the case of the area model, showing fractions on the number line also requires careful consideration of the number of pieces to be shown in a whole and the number of pieces among these to be selected. There should be no hurry to transition to symbols so that the shape-related meaning of fractions on the number line is not neglected [63], Doğan-Temur conducted interviews with primary teachers, and the teachers stated that students made mistakes most frequently in ranking fractions, showing fractions on the number line, problems, reading the fraction, fraction rules, equivalency in fractions and confusing numerators and denominators [58]. Kutluca stated that it is important to use transparent fraction cards, which are tangible materials, to provide students with skills of comparing fractions, ranking, identification, addition and subtraction, multiplication and division operations [64]. Many other studies also reported that students experienced problems while showing fractions on the number line [6], [7], [57], [63], [65]-[67].

Finally, primary teachers and the primary prospective teachers stated that students made mistakes in addition and subtraction operations. Schumacher and Malone prepared problems to determine the mistakes of fourth-grade students in the subject of fractions, and it was found that the students performed addition operations by adding the numerators and denominators separately [68]. The same mistake was also found in the study by Okur and Çakmak-Güler that was carried out with fifth-grade students [69]. The study by Idrisand Narayanan revealed that, while students could perform addition and subtraction operations with fractions that had equal denominators, they had difficulty in operations with fractions that had different denominators [47]. Misconceptions of students in addition and subtraction operations have been reported by several other studies [6], [10], [45], [55], [70], [71]. As addition and subtraction in fractions require a common denominator, it is necessary to have comprehended basic fraction-related concepts in order to perform these operations by understanding them. That is, it is highly difficult to learn and teach addition in fractions without understanding what fractions mean and what equivalent fractions are [35]. Therefore, there is a need for a common denominator to add fractions. If the denominators are not equal, the fractions should be converted into equivalent fractions to make the denominators equal. This is how misconceptions in addition and subtraction will not be encountered, or it will be to overcome misconceptions if they exist [2].

The study by Moss and Case that was conducted with the aim of determining whether or not prospective primary and secondary school mathematics teachers are aware of the mistakes of students in the subject of fractions revealed that most prospective teachers were aware of the mistakes of students [72]. While the prospective teachers were aware of the mistakes of the students, they could provide superficial reasons for the mistakes. For solving the mistakes of students, they recommended strategies of verbal explanations, area models, daily-life examples, repetition of preliminary knowledge, teaching standard solutions, asking leading questions, using easy examples, using opposite examples, exercises and practices, leading students to notice their mistakes and increasing students' motivation.

4. CONCLUSION

Consequently, this study showed that primary teachers and the primary prospective teachers did not have much difficulty in identifying the misconceptions of students regarding operations in fractions. It is believed that, in order to minimize misconceptions in students, the concept of fractions should be introduced starting with identical piece problems and developing the concept with student-centered activities by using various methods will be appropriate. It will also contribute greatly to the field to conduct qualitative studies that involve the analysis of the mistakes primary school students make in the subject of fractions.

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