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### **Difficulties Faced by the In-service Mathematics Teachers Planning Lessons based on Questioning during a Training Course**

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## Difficulties Faced by the In-service Mathematics Teachers Planning Lessons based on Questioning during a Training Course

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### Abstract

In this paper, we analyse how  $N = 31$  in-service teachers study the question that could generate a Study and Research Path (SRP): How does a parabolic antenna work? We also consider how they design lessons based on this question, by adopting the pedagogy of questioning the world according to the Anthropological Theory of the Didactic (ATD). The teachers investigated the question individually and in groups during an on-line course on Mathematics Didactics. Then, we asked to the teachers to organize a possible instruction oriented to an institution known to them, based on the question they analysed before. Written texts produced by the teachers in both roles, studying the question and planning lessons are analysed using qualitative techniques and the concept of RSP. The results describe the main difficulties of in-service teachers to planning instruction according to the paradigm of research and questioning of the world proposed by the ATD.

### Introduction

The importance given to the fact that training of the mathematics teacher includes knowledge that exceeds the mathematical contents a teacher should teach, has been emphasized by numerous authors (Chevallard & Cirade, 2009; Chevallard, 2012; Llinares, Valls & Roig, 2008; Ribeiro, Monteiro & Carrillo, 2010). The report of the Education Committee of the EMS, 2012 remarks as crucial notions to be developed in teacher education the “pedagogical content knowledge” (PCK) (Shulman, 1987) and the different dimensions of the “mathematical knowledge for teaching” (MKT) (Ball, Thames, & Phelps, 2008). Both approaches reject the dominant paradigm that conceive teaching as transmission of knowledge. The Anthropological Theory of the Didactic (ATD) approach advocates an epistemological and didactic revolution (Chevallard, 2012) of the teaching of mathematics and school disciplines that calls for the dropout of the traditional teaching paradigm. Traditional teaching has replaced the study of questions by the study of answers, enforcing a non-motivating encounter with pieces of knowledge, which have an unknown rational. According to Chevallard (2012), the study gestures as characteristic of the research pedagogy are fully experienced if the questions studied are strongly co-disciplinary, suggesting design more complex, management and implementation of teaching. According to the epistemological foundations of the ATD the most relevant didactics and mathematics activities are referred to questioning and reorganizing knowledge to be taught. Most of in-service mathematics teachers are not aware of this aspect because they tend to assume knowledge as transparent, and given, or as ATD has pointed out, as a monument. The research in the framework of the ATD (Barquero, Bosch & Romo, 2016; Ruiz, Sierra, Bosch & Gascón, 2014; Otero, Llanos, Gazzola, Arlego, 2016) have highlighted the relevance of disposing of relatively tested SRP for the teacher training. Here, we have selected a SRP designed by the IREM of Poitiers (Bellenoué et al. 2014) driven to secondary school teachers.

We designed a course for in-service mathematics teacher training at university, based on the Anthropological Theory of the Didactic, where we taught the fundamental ideas SRP’s and the attitudes and gestures involved in the Paradigm of Questioning the World (PQW). One of the aims of the course is train in-service mathematics teachers to conceive and design mathematics lessons involving at least some gestures of the PQW (Chevallard, 2012). The generating question  $Q_0$  of the SRP proposed in this case is “How does a parabolic antenna work? The question will be partially responded at the end of the process allowing teachers to live a “study and research path” (SRP) as a student. The main goal is to make teachers encounter an unfamiliar inquiry-based activity related to  $Q_0$  that could exist in a normal classroom of the considered educational level. Teachers studied the question individually and in groups producing a written answer in both instances. Then, we asked them to

analyze and plan a possible instruction proposal adapting the lived SRP to be hypothetically experienced in a real school.

This work aims to describe the potential and difficulties of the teachers while they plan lessons according to certain gestures of the PQW. In the long term, the research aims to propose possible devices to the in-service teachers training. Here we are mainly interested in how teachers reorganize mathematical contents to be taught and which organization of the instruction they propose. Below we briefly present the main ATD aspects, the generating question  $Q_0$  and the approach adopted in this work.

## ATD, Monumentalism and In-service Teachers Training

The Anthropological theory of the didactic (Chevallard, 2008) considers that any teaching situation leads to the emergence of a didactic system  $S(X, Y, \heartsuit)$  where  $X$  represents a community of study,  $Y$  represents one or more teachers helping to study, and  $\heartsuit$  designates the object to study. This concept of system allows introducing the notion of teaching paradigm. Regarding the so-called monumental paradigm, which we will also name "traditional", the system adopts the form  $S(X, Y, O)$  where  $O$  is a theory or a work that a set of pupils  $X$  must learn helped by a unique teacher.

Monumentalism is a metaphor proposed by the ATD, which describes a didactic phenomenon that consists in treating mathematical knowledge as a monument. In general, someone is summoned to admire, visit, preserve, immortalize and even love those monuments, as if they had always been there. Consequently, the monumental paradigm conceives and treats knowledge in that way. Teachers naturally invite students to visit knowledge, without altering it, transforming it or deconstructing it. When someone encounter a monument, is supposed to discover it, at most to live an aesthetic experience with it. Monuments are rigid and non-adaptable, remaining always at the same place. In a monumental epistemology, something similar happens with mathematical knowledge, which is considered immutable through time, to know him it is enough to show it, hence the ostensive treatment of the mathematical objects. Teachers living in the traditional paradigm promote monumental encounter with knowledge, that is, the students meet knowledge in advance, without need to use it. This kind of meeting is named unmotivated by the ATD. Another characteristic of monumentalism, predominant in educational systems, is to split knowledge into small units. The in-service mathematics teachers participating in this research developed their professional life in the framework of the traditional paradigm and were trained there. They are not aware of the variety of monumental gestures they perform.

The ATD advocates for substituting the traditional paradigm by another yet emerging so-called the paradigm of questioning the world. The theory define SRP's as devices allowing the study of mathematics focusing on questioning. The ATD establish that the starting points of mathematical knowledge are questions, named generating questions, because its study should generate new questions. In this case, the didactic system adopts the form  $S(X, Y, Q_0)$ , where pupils  $X$  investigate and study a question  $Q_0$  under the direction of a teacher ( $y$ ) or a set of teachers ( $Y$ ). The purpose of this kind of didactic system is to develop and provide a possible answer to  $Q_0$ , which is produced under certain constraints, but there is no universal or universally valid answer (Chevallard, 2009).

During a SRC, the entire didactic system and not just the teacher produces an answer, to do this, the system use tools, resources and works. It performs actions such as searching, analysing, describing, developing and evaluating objects, works, resources, information, etc. that is the system generates a didactic environment  $M$ . The medium  $M$  is composed of some answers called "pre-constructed" or available answers, because they are within reach of the community of study - for instance, a book, Internet, course notes, etc. It also includes questions derived from  $Q_0$ , formulated from searching for answers to  $Q_0$ . This process describes the type of epistemological activity developed in the didactic system into the paradigm of questioning the world. In other words, SRC's are didactics devices created by the ATD to face with monumentalism, because they possess, among others, the following characteristics:

- They are developed from a so-called generating question  $Q_0$ , because it does not admit an immediate response. That is, it will be necessary to formulate deriving questions, and untag the available answers.
- The didactic medium  $M$  is not built *a priori*, but from the elaboration of answers. Resources are incorporated when they are needed, at any time, under the condition that they have to be validated by the study community.

- The teacher directs the study process, without having a preponderant role constructing M, and their contributions may or not be incorporated into M. In a SRP, the principle of authority does not apply; there are no privileged information systems or with more authority than others, unlike what happens in the monumental paradigm.
- The study group formulates questions, except the generating question  $Q_0$ , which is proposed by the teacher. The diffusion of the possible response to  $Q_0$  includes proofs and it has a strongly epistemological component, unlike the narrative character of diffusion within the monumental paradigm, where the teacher's role is more similar to that of a guide in the visit to a museum, than to the director of a study whose path is unknown in advance.
- Students formulate questions, propose resources, develop responses, evaluate, disseminate, defend and critically answer other students' responses.

In a SRC, modelling is an essential activity; it indeed produces a model that makes it possible to answer the question  $Q_0$ . Modelling can be intra-mathematical or extra-mathematical. In the first case, the SRC mobilizes only mathematical knowledge. In the second case, one or more different disciplines of mathematics intervene in the SRC, for example physics, biology, geography, etc. Thus SRC's can be mono-disciplinary (see for example Parra, Otero & Fanaro, 2015, Gaud & Minet, 2009; Fonseca, 2011) or co-disciplinary (Otero, Llanos, Parra & Sureda, 2014; Barquero, 2011; Ruiz-Higueras & García García, 2011).

### The SRP and an Epistemological Model of Reference

A possible epistemological model of reference to study  $Q_0$ : *how does a parabolic antenna work?* refers to the problem of the construction of the tangents to a curve from the analytical geometry. In addition to re-discovering some properties of synthetic and analytical geometry, the reflection of light on different surfaces from geometric and wave Optics, could be studied (see the Figure 1).

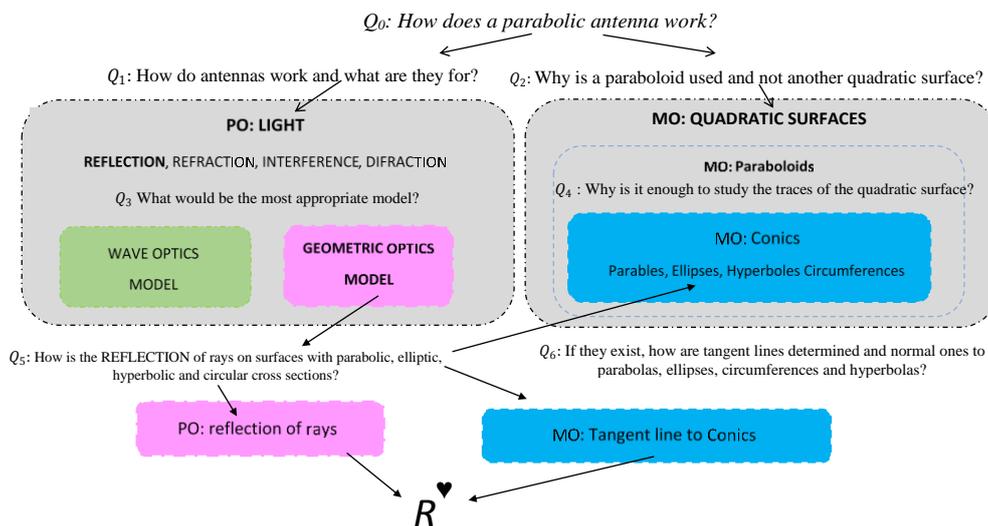


Figure 1: Scheme of a possible epistemological model of reference

In secondary school, the main mathematical know-how to deal with, would be: determine the equation of a circle from its characteristic elements and determine the equation of a line, the relative position of two lines; find the canonical form of a second-degree trinomial; solve a second degree equation; determine algebraically the coordinates of the points of intersection of two curves; show that a line is tangent to a circle, a parabola, a hyperbola and find its analytical expression. In addition,  $Q_0$  allows to study about the historical analysis of the problem of the tangents to a curve, and the development of mathematical knowledge linked to this problem. The questions concerning the reflection of light in different surfaces, lead to the study of the conics, and of the tangents to those curves. Experiments of the reflection on different surfaces could be carried out, considering several kinds of mirrors: cylindrical, parabolic or hyperbolic and questions like: Why a surface could be considered as a mirror? What types of antennas exist? What are antennas for? Which mathematical and extra-mathematical knowledge could be necessary to study the problem? Possible answers could include tools of the

synthetic or analytical geometric framework in  $R^2$  or  $R^3$ , among others. On the other hand, if the curves were unknown, the insufficiency of the geometric-analytical framework to determine tangent lines would require studying differential calculus. In this case,  $Q_0$  was selected because it allows studying an important part of mathematical contents involved in the teachers training that are also relevant contents of the secondary school syllabus in Argentine.

## Method

The research has an exploratory, descriptive and ethnographic character. This work involves ( $N = 31$ ) in-service mathematics teachers who attended the second year of the Bachelors in Mathematics Education (BME) at a National University in Argentina. This course, allows teachers of mathematics graduates of Institutes of Teacher Education, which are non-university institutions of teacher training, to complement their mathematical and didactic training. The BME curriculum consists of eight four-month courses over two years: three are Mathematics courses and the others corresponding to Didactics of Mathematics, Information and communications technologies (ICTs), Epistemology, Methodology and Cognitive Psychology. The instruction was provided completely online by means of the Moodle platform. The course was in charge of three teachers (one teacher per ten students). The fundamental notions of ATD were thought and some examples of various SRPs available and widely disseminated in the literature were analyzed.

The last month of the course was devoted to exploring the question  $Q_0$ : “How does a parabolic antenna work? Teachers grouped in six teams carried out the following tasks:

- T1: Study  $Q_0$  as a student and prepare a possible individual written answer,
- T2: Analyze and discuss the individual answer with the group, proposing a possible group written answer to  $Q_0$ ,
- T3: Propose in each group a possible instruction adapting the lived SRP (T1 and T2) to be hypothetically experienced in a real school. By means of the written answers given to T1, T2 and T3, we seek to identify and describe the abilities, difficulties and the most relevant drawbacks found by the in-service teachers while studying  $Q_0$  and planning lessons based on questioning, according to the paradigm of questioning the world.

We not asked teachers to test their proposal in classroom, due they could not introduce a SRC in a period of one week, nor the course team could help them properly. The teachers responsible for the course interacted with the students and made returns of each task. Then, we analyzed the responses given by the six groups of teachers to the tasks T2 and T3 through the components of an SRP. The aim was to identify, describe and understand the most important difficulties and obstacles faced by teachers when they study  $Q_0$  with the hypothetical intention of organizing a teaching in accordance with the paradigm of questioning the world. We built six tables comparing T2 and T3 in columns and the derivating questions, available responses, mathematical and physical knowledge linked and the possible answer elaborated by each group, in rows. We also use lexicometric statistical methods (Lebart Morineau and Fenelon, 1985) to triangulate the data. The lexical statistic analyze which terms are the most used, the associations between them and how the words chosen depend on the type of document analyzed or on who is expressed. Both analysis support the results.

## Questions

1. How in-service teachers transform knowledge when they respond the tasks T2 and T3 in the role of student or teacher, respectively?
2. What are the main difficulties teachers faced planning lessons according to the questioning paradigm?

## Results

Due to the lack of space, we cannot present the tables performed according to the SRP definition to compare how each group solved tasks T2 and T3, which are available on request.

### Group A

Regarding to T2, first, they asked physics questions and then mathematics ones. The questioning referred to the functioning of the antennas, the parabolic antennas, the electromagnetic waves and their propagation and

reflection. Finally, they adopted the geometry optics model, they considered rays reflection and questioned about the equality between the incident and reflected angle. This was justified by means of Fermat principle. Then, they asked about the property of paraboloids, which would direct the reflected rays on the focus, due their shape. Once arrived here, they defined parabolas in the synthetic framework and they gave a central role to the geometric proof of the existence of the tangent line to any point of a parabola. Then, they used this knowledge to justify the concentration of the reflected rays on the focus. The written answer of this group was based on mathematics and physics knowledge. On the other hand, responding to T3 this group eliminated eight questions, but preserved the synthetic framework of analysis. The teachers proposed ask to the students about the reflection on the parabolic antennas. The group A teachers designed specific tasks to construct parabolas based on their geometric definition, or their elements, and to find the tangent line by means of GeoGebra. The teachers validated characteristics of ray reflexion on a parabolic surface by means of an already made Geogebra video (<https://www.geogebra.org/m/xxeRSH7H>), showing the geometric construction. Thus, adopting the teacher role they removed questions, and knowledge to be taught mainly focusing on parabolas.

### Group B

Responding to T2, first this group asked: Mathematically, what is a paraboloid? and about its definition, characteristics, classification and canonical equations of quadratic surfaces and paraboloids. Then, group B teachers questioned the connection between antennas and electromagnetic waves (EW) as well as how signals received by satellites would be concentrated on the focus. They also asked about the definition of parabolas, their Cartesian equations and about how to demonstrate the so-called "*reflective property of the parabola*". The last question was about the reason to use a paraboloid as an antenna. Considering task T3, the group kept almost all the questions, but left aside the parabolas, including ellipses and hyperbolas. The teachers designed traditional lessons based on exercises to teach the contents, and avoided questions. There were almost no didactic differences between T2 and T3, which reveals that these teachers did not regard the difficulties of the students. This group did not propose any transformation of knowledge to be taught, beginning with definitions followed by application exercises.

### Group C

First, responding to T2, they proposed physics questions and later mathematics ones, as what characteristics EW have and what types of antennas do exist? They also questioned why parabolas should be used making satellite antennas. Group C teachers highlighted the study of parabolas calculating their canonical equations. Regarding parabolic antennas, teachers analyzed ray reflection and refraction on these surfaces. They used the so called "*property of focus convergence of the parabolas*" in order to justify that any beam parallel to the axis will be reflected passing through the focus. Regarding T3, teachers eliminated many questions, and particularly physics questions were almost abandoned. Selecting knowledge to be taught, they only considered parabolas in the functional framework and designed lessons based on traditional exercises about polynomial functions of second degree, that are not strictly related to the generating question, which finally was not answered.

### Group D

Regarding T2, the group asked about the paraboloids, parabolas and their characteristics related to the emission and detection of signals. Then, they formulated the so called "*reflective property of the parabola*" by means of a GeoGebra's applet obtained in Internet. Responding to T3 this group changed the generating question into: "*How to do a solar kitchen?*" Teachers stated this question was best suited to the students that they had in mind. The group proposed to construct a solar kitchen using cardboard guided by a video ([https://youtu.be/F\\_fZEBw8r-c](https://youtu.be/F_fZEBw8r-c)). After that, teachers proposed to build parabolas by means of four manual techniques. Finally, the group formulated the answer as follows "*By means of a paraboloid and placing in the focus the object to be cooked, the solar cooker is obtained*".

Group E. Responding to T2, GE first asked about the satellite antennas and the wave fronts. Then, they asked how is a parabola defined? Which are their properties? and about the characteristics of paraboloids. Both, parabolas and the paraboloids, were introduced by means of definitions. The so-called "*focal property of the parabola*" was analytically justified by means of GeoGebra tools. The group formulated the response as follows: "*Any beam going out of the focus is reflected on the parabola with direction parallel to the axis; any beam parallel to the axis is reflected passing through the focus*". Considering T3, teachers replaced questions by

specific exercises to study parabolas in the analytical framework. The group concluded: *"To study the parabola is sufficient to understand the functioning of antennas"*.

### Group F

Regarding task T2, the group asked more than 17 questions about physics and mathematics, in this order. For instance: How is an EW defined? What is a wave front? What happens when an EW encounters an obstacle? How does reflection occur? What geometrical characteristics have a parabola? How do the antennas transmit and receive EW? What is the connection between "parabolas" and "parabolic" antennas? What happens when an EW hits a parabolic surface? Physically and mathematically, what is reflection? Which equation represents a right line? How to determine its slope? What is the equation of a parabola? What are an axis of symmetry and the focus of a parabola? The group studied parabolas and paraboloids in the analytical framework, and finally adopted the ray optics model. The teachers based the entire study on a single book of Physics and also Mathematical justifications were proposed. Regarding T3, the group eliminated many questions which were transformed in traditional exercises. The parabolas were studied to explain that the reflected rays will be concentrated on the focus, if the antenna has a paraboloid shape.

### Discussion

Regarding questions asked by the teachers, many are "essential" questions that begin by: *What is?* These questions not only promote closed answers as definitions, but also reveal that teachers conceive mathematics as immutable. On the other hand, their responses are mostly coming from the internet; they copied and pasted the answers without questioning. The majority of the groups seem to know enough mathematics to deal with the problem, but they do not question what they get, in correspondence with a crystallized viewpoint of mathematical knowledge. In these cases, we could not say that is the mathematical ignorance, which hinders reorganization of knowledge to be taught, but how teachers conceive mathematics. There are differences between the groups that first asked physics or mathematics questions. When physics questions were asked first, the groups explored different areas and subjects on physics, and then asked mathematics questions related to physics subjects. But when the groups started by mathematics, it would assumed that first, it is necessary to know and define mathematics they label as linked to the problem and then, ask questions to find the determined knowledge in advance, as possible answers. These groups generally conserved the same organization of knowledge in both tasks, proposing an instruction framed in the traditional paradigm. This happens with the groups B and E. However, monumentalism would adopt various forms, for instance the group D changed  $Q_0$  by: *"How to build a solar kitchen?"* While it could be considered auspicious, finally, they proposed to build a cardboard kitchen. Thus, they promoted a merely manipulating teaching activity, disregarding science and mathematics knowledge, and, in their own understanding, simpler and friendlier to the students. In addition, they proposed to construct parabolas by means of four handmade activities, but without planning any questioning during the lesson. In most of groups, the monumental viewpoint turns out evident: only teachers should control and manage the didactic environment M. Also, they eliminated real questions when passing from T2 to T3, which are replaced by traditional and directed activities. For instance, Group E would propose to the students *"Observing all these paraboloids: Could you say which ones have parabolas as cross sections?"*

Regarding how is the response  $R^v$  formulated in both tasks; it could be an epistemic (group A, B, F) or a narrative response (groups C, D, E). The so-called diffusion of answers is a type of mathematical activity alien of the mainstream traditional paradigm. This activity involves communicating and discussing a possible answer based on validated knowledge proposed by the entire class, not only the teacher. The groups considered the communication and discussion of the answer in T2 relatively, but not in T3. This indicates that when teachers have to reorganize knowledge, they cannot avoid monumentalism. That is, knowledge to be taught is transparent and questioning is not required. On the other hand, regarding to the groups starting with Physics questions, group A is remarkable. This group adopted the ray optics model and decided on studying the parabola in the synthetic geometry framework. When they tried to justify the reflection of rays, this is the only group that noticed the relevance of demonstrate the existence of the tangent to the curve. They developed a geometric technique to find the tangent. However, when they resolved T3, eliminated questions and adopted a much more conducted teaching. The group F differs from the A because although it attached great relevance to physics, it studied only parabolas in the analytical geometry framework. They made an epistemic diffusion of the answer in T2, which would be in charge of the teacher in T3. The Group C, also started by asking physics questions, but when responding T3 they limited the knowledge to be taught to the *"quadratic function"*. Excepting the Group B, all confined the knowledge to be taught to parabolas. Only Group A noticed and solved the problem of the

tangent to the parable. This shows the relevance of building the epistemological model of reference as an essential didactic-mathematical activity. The in-service teachers have difficulties developing the EMR, because it is alien of the traditional pedagogy. Thus, students not considered necessary doing the didactic analysis a priori, nor noticed that it is one of the most relevant professional practices, although during the course it was carried out several times.

## Conclusion

The main outcome of this work is that most of teachers faced strong difficulties designing hypothetical lessons based on the questioning paradigm. The reason is that the teachers avoid losing control of the didactic medium and conceive themselves as solely responsible for it. Therefore, teachers planned traditional lessons and limited the knowledge to be taught to parables topic. The questioning paradigm requires an open didactic medium and non-monumental encounter with knowledge. The epistemological viewpoint of teachers would have great influence in this fact, given that half of the teachers groups decided to find mathematics first and then, afterwards, propose related questions, exactly in contrast to the research and questioning paradigm.

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