

Analysis of Education Systems Performance in European Countries by Means of PCA-DEA

Audrone JAKAITIENE, Antanas ZILINSKAS, Dovile STUMBRIENE

Institute of Data Science and Digital Technologies, Vilnius university

Akademijos str. 4, LT-08663 Vilnius

e-mail: {audrone.jakaitiene, antanas.zilinskas, dovile.stumbriene}@mii.vu.lt

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Abstract. Many countries have focused on the improvement of education system performance. Small number of studies consider system of a country as unit of assessment where indicators represent all levels of education system. In the paper, we propose the methodology for the performance analysis of education systems as a whole hybridizing Data Envelopment Analysis and Principal Component Analysis. Its applicability is illustrated by the analysis of the data collected for 29 European countries. In the analysis we used publicly available data from EUROSTAT and OECD which European Commission uses for the performance monitoring of education in European Union. No prior assumptions were made or expert judgements included. We demonstrated good performance of the method on limited data set. The proposed methodology of hybrid Data Envelopment Analysis and Principal Component Analysis allows researchers analyse education systems quantitatively. The recommendations for improvements and assessment of real world education systems should be based on the analysis of a sufficiently large data set comprehensively representing the considered education systems.

Keywords: data envelopment analysis, principal component analysis, education systems, performance.

1. Introduction

Performance assessment of education systems plays a vital role worldwide. The European Commission monitors the performance of education systems in Member States according the strategic framework “Education and Training 2020¹” (ET2020). However, the importance of performance of education systems are still underestimated. The evaluation of schools is a widely researched topic, but limited number of papers analyse the education sector as a whole (not only school level analysis). Also there are not so many

¹ For more information:

[https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:52009XG0528\(01\)](https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:52009XG0528(01))

papers whose focus on multi-country comparisons (Witte and López-Torres, 2017). This topic is especially urgent because of structural reforms of education systems in the new EU countries. Enlargement of the EU and accession of new member states in 2004–2013 was the beginning of the development of education systems in the new EU countries along the similar lines like in the Western countries. Post-communist countries in Europe after the structural reforms do not composed one type of education system and moved in different directions (Beblavy *et al.*, 2011, Želvys *et al.*, 2017). Taking into account that, there is no one way to improve the education systems in all EU countries, it is important to estimate the performance of each country and to provide guidelines that particular country should follow to improve her performance of education system. As pointed out (Silva and Camanho, 2017) measuring performance in absolute terms is often less valuable than making comparisons with other countries, and provide examples of good education practices that under-performance countries should follow to improve their performance of education systems. Data Envelopment Analysis (DEA) have been proved as appropriate for the analysis of different sectors of the education systems. It is natural to investigate the applicability of DEA methodology to the analysis of an education system of a country as a whole.

DEA proposed by Charnes, Rhodes and Cooper (Charnes *et al.*, 1979) is a widely used technique to analyse relative performance of systems in a large variety of fields. The original DEA model allows total exibility of the weights, i.e. each decision making unit (DMU) maximizes its efficiency score, given the inputs consumed and the outputs attained. The exibility of the weights' selection is strength and weakness of a DEA, as it allows some indicators to be assigned a zero weight. Due to full exibility, many DMUs will be able to achieve the maximum DEA efficiency score (Liu *et al.*, 2006). DEA loses discriminatory power when the number of indicators increase compared with the number of the DMUs. Also the large dimension of the data set and correlations among indicators reduce the discrimination power of DEA and introduces bias (Nunamaker, 1985, Dyson *et al.*, 2001). The first attempt to restrict the exibility of the weights and improve the discriminatory power of the DEA model was made by Thompson *et al.* (Thompson *et al.*, 1990). They improved the discrimination between the DMUs' efficiency scores by defining ranges of acceptable weights. Assurance regions and Cone-ratio constraints are often used to restrict the weights and improve the discrimination power of the DEA. These techniques requires prior information, which is often difficult to attain (Adler and Golany, 2001). For the increase of discriminatory power of DEA, Torgersen *et al.* proposed slack-adjusted efficiency measure for the ranking of efficient units (Torgersen *et al.*, 1996). The two-stage method did not use any prior information.

Other way dealing with discrimination issue, the dimension reduction of indicators before running DEA. Principal components analysis (PCA) could be employed where original number of indicators would be replaced by smaller number of principal components with a minimal loss of information (Adler and Golany, 2001). The idea to combine PCA and DEA was developed independently by Ueda and Hoshiai (Ueda and Hoshiai, 1997) and Adler and Golany (Adler and Golany, 2001, Adler and Golany, 2002). They showed that PCA can improve discriminatory power in DEA, which often fails when

there are too many of indicators in relation to the number of DMUs, and give more reliable efficiency measurement in small samples.

The use of PCA-DEA approach for performance assessment can be found in different fields (aviation (Adler and Golany, 2001); manufacturing (Azadeh *et al.*, 2007, Pöldaru *et al.*, 2014); logistics (Andrejić *et al.*, 2013, Chen *et al.*, 2016, Jiang and Fu, 2009); ecology (Liang *et al.*, 2009, Nieswand *et al.*, 2009); agriculture (Dong *et al.*, 2016); finance (Jothimani *et al.*, 2017); health (Annapoorni and Prakash, 2017)). To our knowledge there is one paper (Adler and Golany, 2002) where hybrid PCA and DEA approach was applied for education data (assessed performance of seven university departments).

For the selection of an appropriate DEA method, we perform a preprocessing of the available data. Since the number of indicators selected is relatively large with respect to the number of countries analysed, involvement of PCA seems rational. The analysis of the Pareto frontier of the data set encouraged the application of convexity assumptions based methods. Since the analysis is oriented to aid structural reforms, the proportional improvement of indicators is not a concern. Thus PCA and the Additive Model based DEA were hybridised and investigated.

The objective of the study is to analyse performance of education systems by means of hybrid PCA and DEA approach. We quantitatively analyse performance of education systems in 29 European countries. For the analysis we use publicly available data for year 2013, 2014 and 2015. No prior information or expert judgement were used in the analysis.

The rest of the paper is organised as follows: we present the data analysed first, next we describe methodology of PCA-DEA, then we provide numerical example and discussion with conclusions finalise the paper.

2. Available Data

As described above, European Commission has The Education and Training Monitor initiative for monitoring and fostering performance of education systems in EU. Seven key indicators are selected for the monitoring and benchmarks reached by 2020 are set (ET2020²). In the analysis we use 6 out of 7 key indicators, as learning mobility indicator still waits for appropriate compilation of cross-national data (Flisi *et al.*, 2014). We added two additional indicators for the reflection of a wider range of learning activities (the higher-achievement in reading, maths, science and the minimum necessary qualifications to actively participate in social and economic life). In the further text y_{ij} denotes the j th indicator for i th country. The titles of variables and detailed description are the following:

- $y_{.1}$: Early leavers from education and training (the percentage of the population aged 18–24 with at most lower secondary education and who were not in further education or training during the last four weeks preceding the survey);

² [https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:52009XG0528\(01\)](https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex:52009XG0528(01))

- $y_{.2}$: Tertiary education attainment (the share of the population aged 30–34 years who have successfully completed tertiary-level education);
- $y_{.3}$: Early childhood education and care (the share of the population aged 4 to the age when the compulsory education starts who are participating in early education);
- $y_{.4}$: Employment rate of recent graduates (the share of employed graduates (20–34 years) having left education and training 1–3 years before the reference year);
- $y_{.5}$: Adult participation in lifelong learning (participation rate of adults (25–64 years) in education and training in last 4 weeks);
- $y_{.6}$: Low achievement in reading, maths and science (the percentage of the PISA (The Programme for International Student Assessment) average score in reading, mathematics and science below Level 2);
- $y_{.7}$: Top achievement in reading, maths and science (the percentage of the PISA average score in reading, mathematics and science at Level 5 or 6);
- $y_{.8}$: Upper secondary or tertiary education attainment (the percentage of people at aged 25–64 who have successfully completed at least upper secondary education).

All variables were extracted from EUROSTAT and OECD (The Organisation for Economic Co-operation and Development) databases for 29 European countries over 2013–2015 year. European Commission monitors performance of 28 European countries (EU-28), we used 26 out of 28 countries, because Cyprus and Malta have insufficiently data set, also we added three European countries (Iceland, Norway and Switzerland), that are not in European Union. Missing data were replaced with the most recent year available. The indicators were adjusted as the profit type *the larger the better*, so that higher values of all indicators correspond to better performance. The values of indicators $y_{.1}$ and $y_{.6}$ were converted using the complement to 100 percent. The data used in the study is provided in Appendix Tables 7, 8 and 9.

Education performance can be analysed as the degree to which an education system achieve desired goals and effects. In the context of education systems in European countries, goals and effects might be represented in terms of education systems achievement according to the framework ET2020, an education system that contributes to greater levels of these achievement is considered more effective than another education system.

From Table 1 we see that on average all variables except $y_{.6}$ and $y_{.7}$ indicate better performance of European countries in 2015 compare to 2013. An average the target of $y_{.1}$ is already reached in 2014 however at country level there are some countries those have not reached the target. The performance of education systems should be improved with respect to $y_{.2}$ – $y_{.6}$ variables in almost all countries. The correlation analysis for selected variables revealed that some of variables are medium to high correlated (see Appendix Table 10) which indicates that PCA application is plausible. Next we describe hybrid PCA and DEA approach which was employed to quantify the performance of each country in the analysis. The selected indicators will serve as output indicators only in DEA.

Table 1
Descriptive statistics of analysed indicators for 29 European countries with ET2020 targets

		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
ET2020	targets	90.0	40.0	95.0	82.0	15.0	85.0	-	-
2013	Mean	89.6	38.8	91.3	74.1	12.5	80.2	8.8	79.1
	Std.	5.0	9.3	7.9	12.2	9.0	7.1	3.4	12.0
	Dev.								
	Median	90.3	42.3	94.1	75.8	10.0	80.8	8.9	81.8
	Max	96.1	52.6	100.0	89.8	31.4	91.8	15.3	93.4
	Min	76.4	22.5	71.4	39.6	2.0	60.0	1.9	39.8
2014	Mean	90.2	40.5	91.7	75.1	12.4	78.5	8.3	79.8
	Std.	4.7	8.8	7.0	11.6	9.3	7.1	3.1	11.4
	Dev.								
	Median	91.0	42.3	94.4	77.4	9.6	80.2	8.8	82.7
	Max	97.3	53.3	100.0	89.0	31.9	89.8	13.2	93.3
	Min	78.1	23.9	72.4	44.0	1.5	59.5	2.0	43.3
2015	Mean	90.2	41.4	92.0	76.4	12.7	78.5	8.3	80.2
	Std.	4.3	8.9	6.9	11.2	9.4	7.1	3.1	11.1
	Dev.								
	Median	90.8	43.4	95.0	78.7	9.7	80.2	8.8	82.7
	Max	97.2	57.6	100.0	92.1	32.1	89.8	13.2	93.5
	Min	80.0	25.3	73.8	45.0	1.3	59.5	2.0	45.1

3. Analysis of Data by Means of PCA-DEA

Because of different historical obstacles, economical and political conditions, the achievements of different countries with respect of the considered criteria are quite different. The comparison of these achievements is a problem of multicriteria evaluation. In the present section we will analyse data about education systems by means of DEA. For the convenience of references, here the terminology of DEA will be used. Let Y_i , $i = 1, \dots, 29$, denote the vectors the components of which $y_{ij} > 0$, $j = 1, \dots, 8$, are criteria representing the efficiency of i -th DMU. These criteria represent the performance of the education system of the corresponding country. The values of y_{ij} for the considered years are given in Appendix Tables 7, 8 and 9. As explained in the previous section performance is an increasing function of all criteria. The crudest evaluation is the partition of the set Y_i , $i = 1, \dots, 29$, into subsets of dominating (Pareto optimal) and dominated vectors. We have applied a standard algorithm to find Pareto optimal vectors for the data analysed. The Pareto optimal vectors are defined by the shorthand of names of the corresponding countries in Table 2 for each year.

The obtained results show that only a third of DMUs are not Pareto-optimal. In the case when Pareto optimal points belong to the convex hull of the data set, the application of a standard DEA technique, e.g. CCR (Cooper *et al.*, 2006), defines the unit efficiency

Table 2
The countries' performance vectors of which are non-dominated

2013	CZE	DNK	DEU	EST	IRL	FRA	LVA	LTU	LUX	NLD	AUT
	POL	SVN	SVK	FIN	GBR	SWE	ISL	NOR	CHE		
2014	BEL	CZE	DNK	DEU	EST	IRL	FRA	HRV	LVA	LTU	LUX
	NLD	AUT	POL	SVN	FIN	SWE	GBR	ISL	NOR	CHE	
2015	BEL	CZE	DNK	DEU	EST	IRL	FRA	HRV	LVA	LTU	LUX
	HUN	NLD	AUT	POL	SVN	FIN	SWE	GBR	ISL	NOR	CHE

of all Pareto optimal DMUs. In our case, this happens as shown by the results of a numerical experiment. The results of the experiment, although itself not very interesting, encourages the application of methods based on convexity assumptions. Therefore, the convex hull based frontier can be preferred against the free disposal hull frontier. For the discussion on potential disadvantages of the weighted sum aggregation of criteria in the case of non-convex feasible objective region we refer to (Pardalos *et al.*, 2017).

For the discrimination of the Pareto optimal DMUs additional assumptions should be made with respect to the involved criteria. Several DEA methods have been proposed where the different treatment of criteria is implied implicitly by formulating restrictions for weights in the DEA models. This idea is implemented in the assurance region (Thompson *et al.*, 1986) and cone-ratio methods (Charnes *et al.*, 1990). However, in the considered problem we do not have rational argument to substantiate a magnitude of restrictions.

An alternative option is to aggregate criteria potentially reducing the scatter of data in the criteria space. Principal component analysis is widely used for reducing dimensionality of data in various applications. The hybridization of PCA with DEA has been proposed quite recently (Ueda and Hoshiai, 1997), and yet thoroughly investigated and applied (Adler and Golany, 2002, Adler and Berechman, 2001, Adler and Golany, 2001, Adler and Yazhemy, 2010, Azadeh *et al.*, 2007, Pöldaru *et al.*, 2014, Chen *et al.*, 2016, Annapoorni and Prakash, 2017, Jothimani *et al.*, 2017). This hybrid method seems attractive since a priori does not require any assumptions about relations between the considered criteria.

We have applied an algorithm of PCA to the data in the three considered years. Since the first four components explain about 90% of variance (see Table 3) they are considered as representing the data sufficiently well. As we see in Fig. 1 according to the first and the second principal coordinates all Pareto optimal countries (marked as diamond) except Croatia come into one group.

Table 3
Variance explained by principal components

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8
2013	46.31	26.00	10.42	7.12	6.15	2.14	1.37	0.48
2014	48.30	25.25	8.73	7.21	5.81	2.85	1.43	0.42
2015	46.16	24.34	11.66	7.26	6.06	2.79	1.30	0.43

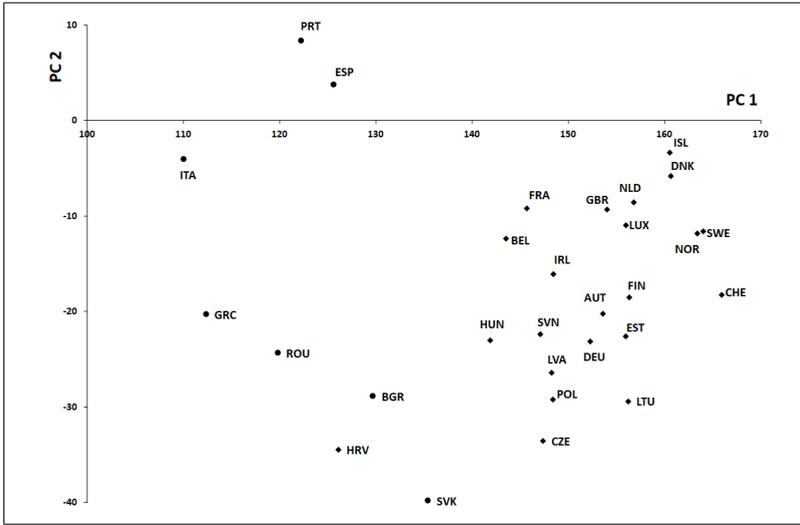


Fig. 1. The first and the second principal coordinates (2015 year).

Further we apply DEA for the projection of the original data to the subspace defined by four eigenvectors of the covariance matrix (computed by means of a PCA algorithm). For the details we refer to (Adler and Golany, 2001) where an excellent description of the algorithm is presented. The four dimensional projections of data are denoted by $\tilde{Y}_i, i = 1, \dots, 29$. The attention should be paid to the unfavorable results of PCA where some elements of the transformed data sets are negative. Recall that the data with negative elements is not appropriate in the context of the original formulation of the DEA problems. However, alternative DEA models have been developed which are translation invariant (Banker *et al.*, 1984, Ali and Seiford, 1990). We will use the so called Additive Model which maintains translation invariance in the analysis of entirely output data (Ali and Seiford, 1990). Let us note, that the Additive Model is also used in (Adler and Golany, 2001).

Since we consider entirely the output data, the Additive Model is defined as the following problem of linear programming

$$\begin{aligned}
 \max_{\Lambda, s^+} z_i &= e s^+, \quad i = 1, \dots, 29, \\
 \tilde{Y} \Lambda - s^+ &= \tilde{Y}_i, \\
 e \Lambda &= 1, \\
 \Lambda &\geq 0, \quad s^+ \geq 0,
 \end{aligned}
 \tag{1}$$

where $\tilde{Y} = (\tilde{Y}_1, \tilde{Y}_2, \dots, \tilde{Y}_{29})$, $e = (1, \dots, 1)^T$.

The solution of (1) z_i is equal to the L_1 (city block) distance from the vector \tilde{Y}_i to the efficiency frontier, i.e. to the Pareto optimal segment of the convex envelop of the data vectors. Thus, the equality $z_i = 0$ is valid for the efficient DMUs. Correspondingly, the non-zero distance is a measure of inefficiency. The values of z_i for all DMUs are

summarized in Table 4 and together with slacks for each of four constraints of the linear programming problem (1) are provided in Appendix Tables 11, 12, 13.

The distance from \tilde{Y}_i to the efficiency frontier can be used as a criterion for ranking the inefficient DMUs. The non-zero slacks show the potential improvement quantities along directions of principal components. However, these potential efficiency improvements does not have proper interpretation. To obtain estimates of potential efficiency improvements with respect of original criteria the slacks can be expressed in terms of original data using loadings of principal components. However, this problem has no unambiguous solution. For example, the PCA coordinates of vectors in the original space are obtained by linear projecting where a “shadow effect” can emerge. The mutual distances of images can be better preserved using non-linear projecting methods, e.g. Multidimensional Scaling (MDS) (Borg and Groenen, 1997, Žilinskas and Žilinskas, 2006). The attractive idea of hybridization of DEA and MDS, however is very new and still not mature.

Therefore we have applied an alternative option to express desired improvements for non-efficient DMUs. Let Λ_i denote the optimal vector of weights corresponding to the solution of linear programming problem (1) with \tilde{Y}_i as left hand side of the constraints where $z_i > 0$. Since the i -th DMU is inefficient its improvement seems desirable. A reasonable target to achieve can be formulated as

$$Y_i = \sum_{j=1}^{29} \lambda_{ij} Y_j \quad (2)$$

where $\lambda_{ij}, j = 1, \dots, 29$, are components of Λ_i .

Table 4
The distances to the efficiency frontier

Country	2013	2014	2015	Country	2013	2014	2015
PRT	0.0	0.0	0.0	FRA	10.7	14.5	13.7
ESP	0.0	0.0	0.0	LUX	3.8	9.4	14.0
GRC	0.0	0.0	0.0	BEL	17.0	17.7	15.6
ISL	0.0	0.0	0.0	POL	29.8	24.2	19.4
NOR	0.0	0.0	0.0	LVA	26.0	28.4	25.7
IRL	0.0	0.0	0.0	AUT	42.3	26.8	28.7
DNK	0.0	0.0	0.0	DEU	26.7	35.4	29.9
CHE	0.0	0.0	0.0	ITA	49.7	40.7	34.3
SWE	4.6	0.0	0.0	HUN	45.0	46.9	45.1
FIN	2.8	0.0	0.0	CZE	54.5	50.0	46.5
EST	11.8	10.6	0.0	HRV	77.5	50.0	53.9
NLD	0.0	9.0	2.4	SVK	73.5	69.0	69.2
GBR	7.4	9.3	4.6	BGR	72.4	68.8	72.4
SVN	31.6	15.4	11.6	ROU	82.8	79.1	79.8
LTU	0.0	0.0	13.1				

4. Numerical Example

The methodology described in the previous section is illustrated by its application in formulating targets to the inefficient DMUs. For a numerical example, let consider Bulgarian (BLG) and Latvia (LVA). We propose to define a peer by (2) where λ_i are obtained as the solution of (1). The non zero $\lambda_{i,j}$ defines j -th DMU as a peer for i -th DMU. For Table 4, Bulgaria is one of the countries with the largest distance to the efficiency frontier ($L = 72.4$ of the year 2015). The peers (defined by the non-zero weights) are Denmark (weight is equal to 0.8896) and Ireland (weight is equal to 0.1104). The targets for Bulgaria are presented in Table 5.

Only one indicator ($y_{.8}$) is already reached in 2015, so the performance of education system in Bulgaria should be improved with respect to $y_{.1}$ – $y_{.7}$ indicators. The largest potential for the improvement of the education system in Bulgaria can be reached by improving indicators $y_{.5}$ and $y_{.7}$. These indicators are much worse than in peers Denmark and Ireland.

Similarly for Latvia, peers are Denmark (weight is equal to 0.2864) and Ireland (weight is equal to 0.7137). The targets for Bulgaria are presented in Table 6.

Three indicators ($y_{.3}$, $y_{.4}$ and $y_{.8}$) out of eight are already reached in 2015 and the largest potential for improvement education system in Latvia can be reached by improving the same indicators ($y_{.5}$ and $y_{.7}$) like in Bulgaria.

5. Discussion and Conclusions

In the analysis of sectors of education systems the most frequently radial DEA models and CCR modifications are applied. Radial efficiency measure and proportional improvement of efficiency are natural for the units with fixed structure operating in identical conditions. Education systems of different countries do not correspond to

Table 5
Observed data and formulated targets for Bulgaria (BGR)

	$y_{.1}$	$y_{.2}$	$y_{.3}$	$y_{.4}$	$y_{.5}$	$y_{.6}$	$y_{.7}$	$y_{.8}$
Observed (2015)	86.6	32.1	89.2	73.6	2.0	59.5	3.6	81.9
Target	92.3	48.1	97.9	79.9	28.6	85.3	8.5	80.3

Table 6
Observed data and formulated targets for Latvia (LVA)

	$y_{.1}$	$y_{.2}$	$y_{.3}$	$y_{.4}$	$y_{.5}$	$y_{.6}$	$y_{.7}$	$y_{.8}$
Observed (2015)	90.1	41.3	95.0	78.7	5.7	81.2	4.4	90.1
Target	92.9	51.0	94.4	76.6	13.6	86.1	9.0	80.0

such a characterisation. Improvement of education systems means mainly improvement in its structure: a reform means not only increasing values of indicators but also changing ratios between their values. Therefore BCC type model seems the most appropriate for education systems analysis. Our experiments show the applicability of PCA with Additivity Model base DEA. The proposed methodology of PCA-DEA allows researchers analyse education systems quantitatively. We demonstrated good performance of the proposed methodology on limited data set of ET2020 indicators. The assessment of the performance and recommendations for the improvements of education systems could be based on the analysis conducted using sufficiently large data set which would comprehensively represent the considered education system of the country as a whole.

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A. Jakaitiene is professor and chief researcher at Vilnius university. She defended her PhD thesis in physical sciences (Informatics 09P) in 2001. A. Jakaitiene has experience of statistical analysis in social, physical and biomedical areas from working in Lithuanian and EU institutions. As of 2005, her research paper topics could be split in three fields: 1. modelling and forecasting of Lithuanian, European Union and global economic indicators; 2. analysing biomedical data, with special interest to modelling of large-scale genetic data; 3. conducting secondary analysis of international large-scale assessment databases in education. She is a board member of Lithuanian Statistical Society, a member of Lithuanian Mathematical Society, International Epidemiology Association and International Biometric Society.

A. Zilinskas is a member of Lithuanian Academy of Sciences, holds the positions of a principal researcher, and is professor-emeritus of Vilnius University. His scientific interests are statistical theory of global optimization, optimization based modelling and design, and analysis of multidimensional data by means of visualization. His research is oriented to develop statistical models of global optimization, implement and investigate the corresponding algorithms, and apply them to practical problems. He is a member of editorial boards of Journal of Global Optimization, Informatica, Baltic Journal of Modern Computing, International Journal of Grid and High Performance Computing, Control and Cybernetics, Statistics, Optimization and Information Computing, and The Open Cybernetics and Systemics Journal. He is a member of several working groups of international scientific societies, and of American Mathematical Society. In 2017 he was awarded Constantine Caratheodory Price of International Society of Global Optimization for his contribution to the field of global optimization.

D. Stumbriene, PhD student in Informatics and junior researcher from the Institute of Data Science and Digital Technologies of Vilnius university Faculty of Mathematics and Informatics. Her main research interest is the largescale analysis of education data. She is focusing on creating the methodology and instruments for education monitoring.

Appendix

Table 7
Indicators for year 2013

Country	$y_{.1}$	$y_{.2}$	$y_{.3}$	$y_{.4}$	$y_{.5}$	$y_{.6}$	$y_{.7}$	$y_{.8}$
BEL	89.0	42.7	98.1	78.1	6.9	82.4	13.5	72.8
BGR	87.5	29.4	87.8	67.6	2.0	60.0	3.8	81.8
CZE	94.6	26.7	85.7	80.3	10.0	82.8	8.9	92.8
DNK	92.0	43.4	98.3	81.0	31.4	83.9	7.4	78.3
DEU	90.2	32.9	97.0	88.2	7.9	85.2	12.9	86.7
EST	90.3	42.5	90.4	75.8	12.6	91.8	11.9	90.6
IRL	91.6	52.6	98.1	72.4	7.6	87.5	10.9	76.7
GRC	89.9	34.9	76.9	39.6	3.2	72.1	3.8	67.2
ESP	76.4	42.3	97.1	55.9	11.4	80.8	6.1	55.5
FRA	90.3	44.0	100.0	74.7	17.8	80.0	11.2	75.0
HRV	95.5	25.6	71.4	53.8	3.1	78.0	5.3	81.3
ITA	83.2	22.5	98.7	48.3	6.2	79.0	7.6	58.2
LVA	90.2	40.7	94.1	77.6	6.8	83.6	5.5	89.4
LTU	93.7	51.3	86.5	74.7	5.9	78.9	5.5	93.4
LUX	93.9	52.5	99.4	77.1	14.6	77.1	9.4	80.5
HUN	88.1	32.3	94.5	73.4	3.2	78.1	6.9	82.5
NLD	90.7	43.2	99.5	83.9	17.9	86.0	13.6	75.8
AUT	92.5	27.1	93.9	89.5	14.1	82.0	9.2	83.0
POL	94.4	40.5	84.8	72.7	4.3	88.7	12.5	90.1
PRT	81.1	30.0	93.9	65.6	9.7	79.1	7.0	39.8
ROU	82.7	22.9	86.4	67.2	2.0	61.5	1.9	75.7
SVN	96.1	40.1	89.8	73.5	12.5	82.0	9.4	85.5
SVK	93.6	26.9	77.5	70.3	3.1	72.5	6.7	91.9
FIN	90.7	45.1	84.0	79.8	24.9	89.6	15.3	85.9
SWE	92.9	48.3	95.7	84.2	28.4	76.0	7.4	83.2
GBR	87.6	47.4	95.9	82.8	16.6	82.2	10.6	78.3
ISL	79.5	43.9	96.2	86.7	26.3	77.8	7.4	72.2
NOR	86.3	48.8	97.4	89.8	20.8	80.6	9.1	82.4
CHE	94.6	46.1	79.1	84.3	30.4	87.0	13.3	87.2

Table 8
Indicators for year 2014

Country	$y_{.1}$	$y_{.2}$	$y_{.3}$	$y_{.4}$	$y_{.5}$	$y_{.6}$	$y_{.7}$	$y_{.8}$
BEL	90.2	43.8	98.1	77.4	7.4	80.2	11.4	73.6
BGR	87.1	30.9	89.3	65.4	2.1	59.5	3.6	81.1
CZE	94.5	28.2	86.4	81.3	9.6	78.5	8.5	93.2
DNK	92.2	44.9	98.1	82.3	31.9	85.2	8.4	79.6
DEU	90.5	31.4	97.4	88.4	8.0	83.2	11.7	86.9
EST	88.6	43.2	91.7	79.3	11.6	89.8	12.9	91.2
IRL	93.1	52.2	96.0	73.5	6.9	86.5	9.2	78.8
GRC	91.0	37.2	84.0	44.0	3.2	68.1	3.4	68.4
ESP	78.1	42.3	97.1	61.1	10.1	81.1	5.9	56.6
FRA	91.0	43.7	100.0	73.0	18.4	77.7	10.6	76.7
HRV	97.3	32.2	72.4	62.0	2.8	74.5	5.1	82.9
ITA	85.0	23.9	96.5	44.8	8.1	77.5	6.8	59.3
LVA	91.5	39.9	94.4	76.6	5.6	81.2	4.4	89.5
LTU	94.1	53.3	88.8	80.1	5.1	74.9	5.2	93.3
LUX	93.9	52.7	98.4	82.7	14.5	74.2	8.3	82.0
HUN	88.6	34.1	94.7	78.0	3.3	72.9	5.7	83.1
NLD	91.3	44.8	97.6	83.3	18.3	82.2	12.5	75.9
AUT	93.0	40.0	94.0	86.9	14.3	78.3	9.1	83.9
POL	94.6	42.1	87.1	75.2	4.0	84.0	9.2	90.5
PRT	82.6	31.3	93.5	66.7	9.6	80.5	8.8	43.3
ROU	81.9	25.0	86.4	65.9	1.5	60.9	2.0	72.8
SVN	95.6	41.0	89.4	70.0	12.1	84.6	11.0	85.7
SVK	93.3	26.9	77.4	72.7	3.1	69.8	5.0	91.0
FIN	90.5	45.3	83.6	77.0	25.1	88.0	13.2	86.5
SWE	93.3	49.9	95.9	84.5	29.2	79.7	9.6	83.7
GBR	88.2	47.7	98.2	82.3	16.3	80.9	10.2	79.2
ISL	80.9	46.4	96.2	89.0	26.3	76.3	6.9	73.6
NOR	88.3	52.1	97.2	86.1	20.1	83.1	10.3	82.7
CHE	94.6	49.2	80.6	87.3	31.7	81.9	12.3	88.0

Table 9
Indicators for year 2015

Country	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
BEL	89.9	42.7	98.0	77.9	6.9	80.2	11.4	74.7
BGR	86.6	32.1	89.2	73.6	2.0	59.5	3.6	81.9
CZE	93.8	30.1	88.0	82.0	8.5	78.5	8.5	93.2
DNK	92.2	47.6	98.5	80.5	31.3	85.2	8.4	80.4
DEU	89.9	32.3	97.4	88.9	8.1	83.2	11.7	86.8
EST	88.8	45.3	91.6	79.2	12.4	89.8	12.9	91.1
IRL	93.1	52.3	92.7	75.0	6.5	86.5	9.2	79.8
GRC	92.1	40.4	79.6	45.0	3.3	68.1	3.4	70.4
ESP	80.0	40.9	97.7	62.2	9.9	81.1	5.9	57.4
FRA	90.8	45.0	100.0	70.9	18.6	77.7	10.6	77.5
HRV	97.2	30.9	73.8	62.4	3.1	74.5	5.1	83.3
ITA	85.3	25.3	96.2	48.3	7.3	77.5	6.8	59.9
LVA	90.1	41.3	95.0	78.7	5.7	81.2	4.4	90.1
LTU	94.5	57.6	90.8	81.1	5.8	74.9	5.2	93.5
LUX	90.7	52.3	96.6	83.5	18.0	74.2	8.3	76.0
HUN	88.4	34.3	95.3	80.0	7.1	72.9	5.7	83.2
NLD	91.8	46.3	97.6	86.6	18.9	82.2	12.5	76.4
AUT	92.7	38.7	95.0	86.7	14.4	78.3	9.1	84.6
POL	94.7	43.4	90.1	76.8	3.5	84.0	9.2	90.8
PRT	86.3	31.9	93.6	70.5	9.7	80.5	8.8	45.1
ROU	80.9	25.6	87.6	68.0	1.3	60.9	2.0	75.0
SVN	95.0	43.4	90.5	71.1	11.9	84.6	11.0	86.8
SVK	93.1	28.4	78.4	75.2	3.1	69.8	5.0	91.4
FIN	90.8	45.5	83.6	75.5	25.4	88.0	13.2	87.7
SWE	93.0	50.2	95.0	85.5	29.4	79.7	9.6	84.3
GBR	89.2	47.9	100.0	85.0	15.7	80.9	10.2	79.7
ISL	81.2	47.1	97.6	92.1	28.1	76.3	6.9	75.0
NOR	89.8	50.9	97.3	89.7	20.1	83.1	10.3	82.7
CHE	94.9	51.4	81.3	84.8	32.1	81.9	12.3	88.2

Table 10
Spearman's correlation of indicators

		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
2013	Y1	1							
	Y2	0.116	1						
	Y3	-0.348	0.391*	1					
	Y4	0.146	0.462*	0.279	1				
	Y5	0.098	0.626*	0.417*	0.726*	1			
	Y6	0.252	0.287	0.129	0.391*	0.412*	1		
	Y7	0.254	0.417*	0.327	0.531*	0.552*	0.759*	1	
	Y8	0.648*	0.046	-	0.325	-	0.305	0.145	1
				0.481*		0.038			
2014	Y1	1							
	Y2	0.202	1						
	Y3	-0.35	0.378*	1					
	Y4	0.146	0.546*	0.334	1				
	Y5	0.008	0.611*	0.447*	0.632*	1			
	Y6	0.131	0.383*	0.194	0.299	0.508*	1		
	Y7	0.12	0.437*	0.296	0.502*	0.647*	0.753*	1	
	Y8	0.635*	0.09	-	0.359	-	0.231	0.185	1
				0.425*		0.032			
2015	Y1	1							
	Y2	0.252	1						
	Y3	-	0.311	1					
		0.455*							
	Y4	0.081	0.508*	0.365	1				
	Y5	0.009	0.600*	0.476*	0.574*	1			
	Y6	0.186	0.426*	0.217	0.239	0.475*	1		
	Y7	0.172	0.441*	0.291	0.420*	0.649*	0.753*	1	
Y8	0.609*	0.115	-	0.328	-	0.251	0.171	1	
				0.454*		0.051			

*Correlation is significant at the 0.05 level (2-tailed).

Table 11
DEA results for 2013

Country	Distance to frontier	Slack 1	Slack 2	Slack 3	Slack 4
BEL	17.04	1.88	0.00	11.68	3.48
BGR	72.42	20.40	19.03	20.70	12.29
CZE	54.45	0.00	24.31	15.83	14.32
DNK	0.00	0.00	0.00	0.00	0.00
DEU	26.67	0.00	11.96	14.58	0.13
EST	11.83	0.00	11.68	0.00	0.15
IRL	0.00	0.00	0.00	0.00	0.00
GRC	0.00	0.00	0.00	0.00	0.00
ESP	0.00	0.00	0.00	0.00	0.00
FRA	10.68	0.00	0.00	5.01	5.67
HRV	77.48	27.57	27.95	1.93	20.03
ITA	49.67	31.11	0.00	10.44	8.12
LVA	25.95	0.00	14.52	8.76	2.67
LTU	0.00	0.00	0.00	0.00	0.00
LUX	3.83	0.00	1.48	0.75	1.60
HUN	44.95	9.19	12.98	16.96	5.82
NLD	0.00	0.00	0.00	0.00	0.00
AUT	42.29	0.00	11.05	20.09	11.16
POL	29.82	0.26	20.63	2.48	6.45
PRT	0.00	0.00	0.00	0.00	0.00
ROU	82.76	26.41	15.76	24.87	15.72
SVN	31.58	0.09	13.57	6.07	11.85
SVK	73.45	11.92	31.26	14.08	16.19
FIN	2.84	0.00	1.13	0.00	1.71
SWE	4.61	0.00	0.00	0.00	4.61
GBR	7.39	0.00	0.00	3.16	4.23
ISL	0.00	0.00	0.00	0.00	0.00
NOR	0.00	0.00	0.00	0.00	0.00
CHE	0.00	0.00	0.00	0.00	0.00

Table 12
DEA results for year 2014

Country	Distance to frontier	Slack 1	Slack 2	Slack 3	Slack 4
BEL	17.72	10.84	0.00	6.88	0.00
BGR	68.84	22.83	16.45	13.27	16.29
CZE	50.03	0.00	21.26	9.57	19.19
DNK	0.00	0.00	0.00	0.00	0.00
DEU	35.38	0.00	11.14	15.53	8.72
EST	10.59	0.00	9.86	0.00	0.73
IRL	0.00	0.00	0.00	0.00	0.00
GRC	0.00	0.00	0.00	0.00	0.00
ESP	0.00	0.00	0.00	0.00	0.00
FRA	14.46	14.39	0.00	0.06	0.00
HRV	49.96	26.68	20.26	0.00	3.02
ITA	40.66	40.66	0.00	0.00	0.00
LVA	28.44	0.65	13.05	7.91	6.83
LTU	0.00	0.00	0.00	0.00	0.00
LUX	9.36	0.00	2.90	4.66	1.80
HUN	46.94	7.94	11.74	16.00	11.26
NLD	9.00	3.17	0.00	5.83	0.00
AUT	26.81	0.00	7.95	8.87	9.98
POL	24.17	0.00	16.53	1.16	6.48
PRT	0.00	0.00	0.00	0.00	0.00
ROU	79.06	29.51	12.07	16.43	21.04
SVN	15.38	6.19	7.50	0.00	1.68
SVK	68.99	12.56	28.02	7.23	21.18
FIN	0.00	0.00	0.00	0.00	0.00
SWE	0.00	0.00	0.00	0.00	0.00
GBR	9.28	6.88	0.00	2.36	0.03
ISL	0.00	0.00	0.00	0.00	0.00
NOR	0.00	0.00	0.00	0.00	0.00
CHE	0.00	0.00	0.00	0.00	0.00

Table 13
DEA results for year 2015

Country	Distance to frontier	Slack 1	Slack 2	Slack 3	Slack 4
BEL	15.56	5.23	0.00	10.34	0.00
BGR	72.41	30.91	23.08	16.68	1.74
CZE	46.49	9.98	24.95	11.56	0.00
DNK	0.00	0.00	0.00	0.00	0.00
DEU	29.86	0.14	10.39	19.33	0.00
EST	0.00	0.00	0.00	0.00	0.00
IRL	0.00	0.00	0.00	0.00	0.00
GRC	0.00	0.00	0.00	0.00	0.00
ESP	0.00	0.00	0.00	0.00	0.00
FRA	13.72	11.05	0.09	2.58	0.00
HRV	53.91	31.91	22.00	0.00	0.00
ITA	34.31	33.98	0.00	0.32	0.00
LVA	25.72	3.19	12.85	9.68	0.00
LTU	13.13	0.00	13.13	0.00	0.00
LUX	13.96	4.75	1.92	7.29	0.00
HUN	45.13	14.93	13.96	16.24	0.00
NLD	2.41	0.00	0.00	2.41	0.00
AUT	28.65	3.88	11.74	13.04	0.00
POL	19.36	1.28	14.16	3.92	0.00
PRT	0.00	0.00	0.00	0.00	0.00
ROU	79.84	40.82	18.56	17.44	3.02
SVN	11.63	4.90	6.72	0.00	0.00
SVK	69.22	25.22	34.04	8.71	1.25
FIN	0.00	0.00	0.00	0.00	0.00
SWE	0.00	0.00	0.00	0.00	0.00
GBR	4.56	0.00	0.00	4.56	0.00
ISL	0.00	0.00	0.00	0.00	0.00
NOR	0.00	0.00	0.00	0.00	0.00
CHE	0.00	0.00	0.00	0.00	0.00

