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Melinda S. Eichhorn¹, Lindsey E. Perry², Aarnout Brombacher³

¹Gordon College

²Southern Methodist University

³Brombacher and Associates

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Students' Early Grade Understanding of the Equal Sign and Non-standard Equations in Jordan and India

Melinda S. Eichhorn, Lindsey E. Perry, Aarnout Brombacher

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Abstract

Many students around the world are exposed to a rote teaching style in mathematics that emphasizes memorization of procedures. Students are frequently presented with standard types of equations in their textbooks, in which the equal sign is immediately preceding the answer ($a + b = c$). This exposure can lead to many misconceptions, such as thinking that the equal sign means “do something” or “the answer is.” This paper describes students’ understanding of the equal sign when solving nonstandard equations in Jordan on the Early Grade Mathematics Assessment (EGMA) and in India on a number sense screener. Common misconceptions are shared, as well as strategies for improving instruction with the equal sign and non-standard equations to prevent future errors.

Introduction

What number goes in the box to make this number sentence true?

$$3 + 8 = \square + 6$$

- A. 17
- B. 11
- C. 7
- D. 5

(Trends in International Mathematics and Science Study (TIMSS), 2011)

When fourth grade students were asked to solve this problem on the TIMSS mathematics assessment, an average of 39% of students chose the correct answer. This ranged from 85% correct among students from Singapore, 47% correct among American students, and 16% correct among Moroccan students (TIMSS, 2011). Often times, textbooks present equations in a standard format, $a + b = c$, and students fail to develop a true understanding of the equal sign. Students’ incorrect responses can reveal their current understanding and uncover misconceptions about the equal sign. How can teachers learn from students’ errors to change their instruction to develop deeper conceptual understanding of such a paramount symbol in mathematics in the early grades before it is deeply rooted in the late elementary grades?

The Importance of the Equal Sign

Students need a relational understanding of the equal sign; interpreting it as a relational symbol that shows equivalence is correlated to their equation-solving performance. However, students incorrectly view the equal sign as a “do something” symbol or “put the answer next” (Harbour, Karp, & Lingo, 2016). Equality does not only mean “the same as,” and teachers can use more accurate language with students. It means that two things that are not the same exactly (an exception is the identity principle), but they are equal in value (Faulkner, 2009). The equal sign is more than the “=” command on the calculator (Li, Ding, Capraro, & Capraro, 2008).

Carpenter, Franke, and Levi (2003) outlined a progression describing how students develop correct conceptions about the equal sign, identifying four important stages of development. First, students must learn to verbalize their conceptions of the equal sign. Next, students recognize that not all number sentences will be of the form $a + b = c$. Third, students recognize that the equal sign represents a relation between two quantities. Finally, students can utilize their numeric relational reasoning skills to find an answer to a number sentence without calculating, using number properties or composition.

Consider the way math facts and early addition problems are typically presented by teachers and textbooks. How can students be expected to learn what the teacher intends if they are not correctly viewing, let alone interpreting, the instructional materials (Booth, 2011)? Students need further experience with nonstandard types of equations, as well as concrete and representational experiences, to develop a relational and conceptual understanding of the equal sign to support their mathematical thinking and algebraic reasoning in the years to come. If students do not develop conceptual understanding of the equal sign, they can continue to struggle with more advanced math concepts, such as interpreting complex algebraic equations, and be at-risk for learning difficulties in mathematics. Inadequate knowledge of the equal sign is “one of the major stumbling blocks in learning algebra” (Carpenter et al., 2003, p. 22).

Without a conceptual understanding of the equal sign, students will likely solve equations based on rote knowledge and procedures (e.g., if a value is added to one side of an equation, it must be added to the other side) (Byers & Herscovics, 1977), which may lead to frequent errors since students do not know why the procedures they are using work. Being able to reason relationally requires many skills, including understanding that the equal sign expresses a relation and seeing relationships between expressions. Unfortunately, these misconceptions are widespread in young children. In fact, only 5% of grades 1 and 2 students responded that the answer to $8 + 4 = \square + 5$ was seven (Falkner, Levi, & Carpenter, 1999). Most students thought the missing value was 12, and others thought it was 17, 12 and 17, or another number.

The purpose of this article is to continue to broaden the understanding of the equal sign in international contexts. Cross-cultural comparison can lead to better understanding of the way children learn and comprehend mathematical ideas (Capraro et al., 2007). Previous studies have been conducted in China, South Korea, Turkey, and the U.S. (see Capraro et al., 2011). This article extends the current knowledge to the countries of Jordan and India.

The study focuses on the following research questions:

- What are the common misconceptions about the equal sign in international contexts?
- How do students in Jordan and India perform on composing and/or decomposing tasks?
- How do misconceptions about the equal sign exhibited by students in Jordan and India match those exhibited by students in prior literature?
- How are equations presented in the textbooks in Jordan and India and how might this reinforce students' misconceptions?

Background

Common Misconceptions with the Equal Sign in International Contexts

The equal sign means “is the same value as,” or both sides of the equation are balanced. However, elementary students may believe the equal sign tells you “and the answer is” to the right of the equal sign. This misconception arises and becomes over-generalized by only seeing examples of number sentences with an operation to the left of the equal sign and the answer on the right, otherwise known as the “standard context” for presenting the equal sign (Capraro et al., 2011, p. 190). Overexposure to standard equations of the form $a + b = c$ may lead students to conclude that they should always read equations from left to right and should perform calculations from left to right as well (Molina & Ambrose, 2006; 2008).

Instead of recognizing that the equal sign is a relational symbol, most children see it as a “do something symbol” (Behr, Erlwanger, & Nichols, 1980, p. 15), believing that they must calculate or compute something when they see an equal sign (Baroody & Ginsburg, 1983; Denmark, Barco, & Voran, 1976; Falkner, Levi, & Carpenter, 1999). For example, when given a problem such as $12 + 4 = \square + 3$, many students determine that the answer is 16 because they believe the equal sign signals them to complete the operation on the left side of the equation. These students believe that the “answer” always follows the equal sign. With this level of understanding, the equal sign is nothing more than an operator symbol that connects a problem with its answer (Kieran, 1981); students do not recognize it as a symbol that expresses a relation between two expressions. For example, consider the equation $4 + 5 = \square + 4$. Students who see the equal sign as a “do something” symbol are unable to reason relationally and recognize that they can apply the commutative property to solve the equation; instead, only the left side of the equation is considered. The notion of the equal sign as an operator is common

not only in early elementary years (Behr et al., 1980; Denmark et al., 1976), but also in later elementary (Saenz-Ludlow & Walgamuth, 1998), middle school (Knuth, Stephens, McNeil, & Alibali, 2006), and high school grades (Kieran, 1981).

While some students see the equal sign as a “do something” symbol, other students disregard the placement of the equal sign in an equation and perform an operation on all of the numbers (Carpenter et al., 2003; Denmark et al., 1976). In this conception, the equal sign means “the total” (McNeil & Alibali, 2005). For example, to solve $4 + 3 = \square + 5$, students add all of the given numbers, $4 + 3 + 5 = 12$, to determine the value of the unknown. This misconception also arises even when students are not solving for an unknown. For instance, in an interview with a 7-year-old, Behr et al. (1980) observed that the student changed the equation “ $3 + 2 = 2 + 3$ ” to “ $3 + 2 + 2 + 3 = 10$ ” because the student believed that all numbers should be used to find the total.

Similar to “the total” conception of the equal sign, students also “extend the problem” (Carpenter et al., 2003, p. 11) by performing calculations from left to right. For example, to solve $4 + 3 = \square + 5$, students first add $4 + 3 = 7$, believing that this sum is the missing value, indicating that they view the equal sign as a “do something” symbol. Next, they add $7 + 5$ to get the “answer” to the equation. Students who exhibit this misconception read the equation from left to right and do not understand that both sides of an equation must equal the same amount (Carpenter et al., 2005). These examples indicate that some students do not understand that an equal sign describes a relationship between two expressions. Instead, students see equations or number sentences as a prompt to use all the numbers to find an answer. This is particularly problematic for numeric relational reasoning since the relationships between values within equations are disregarded.

These misconceptions, common from grade one through six, prevent students from seeing relationships between the expressions within an equation and inhibit students’ development of number sense. Believing that the equal sign should be placed in a specific location impedes students’ ability to reason relationally because they do not understand the equal sign as a relational symbol. To prevent these types of misconceptions, first and second graders need to see equations written multiple ways, such as $4 + 3 = 7$ and $7 = 4 + 3$, in which the equal sign is in other orientations, also known as “non-standard context” (Capraro et al., 2011, p. 190; Kansas Association of Teachers of Mathematics (KATM), 2014). In other words, students may think “the only correct format for a problem is $a + b = c$ or $a - b = c$, not recognizing it can also be $c = a + b$ or $c = a - b$ ” (SciMathMN, 2015, para.1). Teachers can begin to predict and anticipate misconceptions and ways students typically respond to a problem, while also researching and anticipating the way they can clear up and resolve the misunderstanding(s) in their instruction. Identifying misconceptions about the equal sign enables teachers and leaders to modify instruction and intervene in order to improve student learning. Some students will need explicit and direct instruction regarding the meaning of the equal sign to develop strong conceptual understanding (Capraro et al., 2010).

Composing and Decomposing

Related to the concept of equality is the task of composing and decomposing. Students with strong number sense are able to decompose, or take apart, and compose, combine, numbers in a variety of ways (Van de Walle, Karp, & Bay-Williams, 2016). This is especially important with place value, as students begin multi-digit computation. In the early grades, students need number sense to develop their understanding of magnitude comparison - greater than, less than, and equal to. Students can recognize that 7 is “three more than 4, two less than 9, composed of 4 and 3, three away from 10...” (Van de Walle et al., 2016, p. 142). Students can then extend their knowledge of composing and decomposing single digit numerals to multi-digit numerals. In kindergarten and first grade, students should practice composing and decomposing numerals to include multiple combinations of the whole, such as 5 and 10 (Van de Walle et al., 2016).

This is also described as a “take apart situation,” in which a total quantity is taken apart to form two or more addends, or $c = a + b$ (National Research Council, 2009, p. 32; Massachusetts Department of Elementary and Secondary Education, 2017). This problem situation is mentioned in the 1st grade Common Core standard, CCSS.MATH.CONTENT.1.OA.A.1,

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

(Common Core State Standards Initiative, 2014, para.1)

Previous International Studies

Previous research examined second graders' understanding of the equal sign in international contexts (Capraro et al., 2011). Equal sign tests were administered to students in China, South Korea, Turkey, and in the U.S. (Texas). The questions on the equal sign test were as follows:

$$\begin{array}{cccc}
 3+5 = 5+ \square & 4+2+3 = 4+ \square & 3+ \square = \square +1 & 6 = \square \\
 8+3 = \square + 5 & 4+ \square = 5 & 8+3 = 5 +\square & \square+3 = 5+7 = \square \\
 7 = \square + \square & 3+5 = 4+ \square & &
 \end{array}$$

Based on prior international comparison tests, such as the TIMSS and PISA, it was hypothesized that the U.S. students would outperform the Turkish students. However, the Turkish students were more accurate than American students on the equal sign tests (Capraro et al., 2011). In general, there were statistically significant differences between samples for the equal sign test scores for China and all others (South Korea, Turkey, and the U.S. (Texas)). The overall math achievement of the Chinese and Korean students was substantially higher than the Turkish and American students (Capraro et al., 2011).

Li et al. (2008) argue that one of the contributing factors for the difference in understanding the equal sign is due to textbook presentation. Chinese teaching materials have traditionally used multiple problem contexts, types, and arrangements as compared to U.S. materials (Li et al., 2008). In several international studies (see Yaman, Toluk, & Olkun, 2003; Choi, 2004; Choi & Pang, 2008; Do & Choi, 2003), manipulatives and concrete materials helped students to develop conceptual understanding of the equal sign (Capraro et al., 2011).

The Influence of Textbooks

Textbooks play a critical and prominent role in classrooms around the world (Reys & Reys, 2006; Doabler, Fien, Nelson-Walker, & Baker, 2012). Teachers often heavily rely on the textbook as an instructional tool and have students complete problems from the textbook in class and for homework. The textbook often influences how teachers will teach the content (Capraro et al., 2011).

Powell (2012) examined eight different K–5 textbook programs in the U.S. to study the location and placement of the equal sign and found that most of the equations in the textbooks were in standard equation form (i.e., equal sign is located in the second place from the right in an equation). While nonstandard equations (i.e., anything different from a standard equation, such as $5 = 3 + \square$, $1 + 4 = 3 + 2$) were present in some of the textbooks, Powell determined that none of the textbooks provided a “complete package of equal sign understanding” (2012, p. 643).

According to a 2002 study, Olkun & Toluk found that Turkish textbooks included an overrepresentation of problem types for addition and subtraction word problems. The most prevalent types were $a + b = c$ and $a - b = c$ while other types were underrepresented or not present at all. Findings from this study suggest that teachers can acknowledge the limitations of textbooks in regards to nonstandard equations and word problems, and they should be provided with resources and training to expose elementary students to all equation and problem types, as discussed in the literature.

In some countries, such as India, there are no teachers' editions of the textbook available. Teachers have no guide or suggestions for how to use the textbook in their instruction. There are no tips for addressing students' misconceptions or re-teaching, if students do not understand the initial instruction. Current teachers are provided little to no background knowledge about the equal sign and the misconceptions that students may develop. Simply using the textbook as a sole instructional resources is not enough to address students' conceptual understanding of the equal sign, especially if non-standard equations are underrepresented in the text.

Other Instructional Materials and Methods

To develop conceptual understanding, teachers can use the Concrete - Representational - Abstract (CRA) framework. In this multi-sensory approach, physical manipulatives are used at the concrete level (counters, base-ten blocks, algebra tiles, and geoboards); drawings, pictures, and virtual manipulatives are tools used at the semi-concrete or representational stage; and at the abstract level, students use mathematical notation (numbers, symbols, and variables) (Witzel, 2005; Witzel, Mercer, & Miller, 2003; Strickland & Maccini, 2010). Concrete manipulatives should be accompanied with verbal explanation, and later transitioned to a representational

drawing and the abstract numbers so that students understand the connection between each stage (Maccini & Gagnon, 2000).

To develop a relational understanding of the abstract, the numerals and symbols of mathematics, including the equal sign, teachers can make use of a pan balance or a seesaw (Van de Walle, Karp, & Bay-Williams, 2016). Teachers can use a concrete pan balance with unifix cubes or counters to help students see and feel the meaning of equality, and transfer this knowledge into a representational drawing. There are also virtual manipulatives available from the National Council of Teachers of Mathematics (NCTM) for pan balance equations (NCTM, 2017). Once students have developed the conceptual understanding of the equal sign through concrete and representational methods, they can move on to using the abstract symbols, such as “=” (Van de Walle, Karp, & Bay-Williams, 2016).

Intervention Studies

In order to develop students’ algebraic thinking and relational understanding of the equal sign, Molina & Ambrose (2006; 2008) implemented a study in which students solve and discuss true/false activities and open number sentences. Working with eighteen third grade students in California (USA) over five sessions during the school day, the researchers found that the majority of students’ developed a new understanding of the equal sign, rather than just a stimulus for the answer, when non-standard equations were explicitly taught and classroom discussions emphasized noticing patterns. Furthermore, the understanding of the equal sign was intact two months following the original study. Students maintained their relational understanding of the equal sign from hearing how their peers explained their interpretations, but the researchers noted that developing a robust understanding of the equals sign can take considerable time (Molina & Ambrose, 2006).

A similar study by Molina, Castro, & Castro (2009) revealed that 25 8-year old Spanish students that received instruction with non-standard equations in six one-hour in-class sessions, over a period of one year with true/false sentences and missing-number number sentences, challenged students to reconsider their interpretation of the equal sign. Students justified their answers and articulated their strategies during whole-group discussions. Finally, students constructed their own number sentences with operations on both sides of the equal sign. During this study, students showed “operational, non-stable and advanced” understanding of the equal sign (Molina, Castro, & Castro, 2009, p. 359). At the conclusion of the study, almost half of the students showed an advanced understanding and four to five students’ understanding varied and was slightly unstable or inconsistent. The instability was most evident in instances “when the sentences involved computations which required a higher cognitive demand ...It was then when they altered the structure of the sentence or ignored some of the terms” (Molina, Castro, & Castro, p. 363). Results of these studies suggest that teachers must intentionally expose students to non-standard equations, as well as be aware of the cognitive demand of the operations and students’ sense of structure and patterns.

In another study, Stephens and colleagues (2013) examined students’ understanding of the equal sign in grades 3 to 5 in Massachusetts (USA). This study used tasks that encourage a relational understanding of the equal sign and a focus on equation structure - rather than having students rush to compute their answer (Stephens, Knuth, Blanton, Isler, Murphy Gardiner, & Marum, 2013). Tasks, such as true/false sentences and missing-number number sentences, provide an entryway into discussions about algebraic structure and the meaning of the equal sign for students at the very beginning of their early algebra experiences. The findings from this study suggest that students’ conceptions of the equal sign and equation structure require ongoing attention during the elementary and middle school years. Developing sophisticated understandings of the equal sign is not an easy task (Stephens, Knuth, Blanton, Isler, Murphy Gardiner, & Marum, 2013).

Booth (2011) found that while instructional techniques, such as worked examples (both correct and incorrect) and self-explanation prompts have been encouraged in the field of cognitive science and development, these strategies have failed to be incorporated in textbooks and classroom practice. Adapting textbooks to include non-standard equations is only part of the solution; the way teachers facilitate discussion about the meaning of the equal sign is also important. Some students will need explicit instruction to develop relational understanding of the equal sign, as well as review over time to maintain their skills. Simply telling students what the equal sign means does not develop their understanding.

Teacher Preparation

Another consideration in equal sign misconceptions is the way pre-service teachers are trained in math methods. Capraro et al. (2007) examined six math methods textbooks and found a wide range of strategies presented to pre-service teachers for explaining equivalence to their students. The majority of the textbooks did not mention the misconceptions that elementary students can form about the equal sign (see Reys et al. (2003) and Van de Walle (2004) for exceptions).

Conceptual Framework

The overarching theoretical framework for the study is rooted in constructivism and the importance of misconceptions (Piaget, 1970; Olivier, 1989). According to constructivism, a student learns because of an interaction between existing ideas and new ideas, as well as experiences. Students organize and structure knowledge based on units of interrelated ideas and concepts, called schemas. As students attempt to integrate new knowledge with their existing schemas, they may overgeneralize their previous knowledge, such as the equal sign in an equation in the standard format, to a new domain of knowledge, like a non-standard equation. When students incorporate their new knowledge, they may apply it to previous knowledge in a way that makes sense to them, but is mathematically incorrect. When teachers interpret students' errors as their rational and meaningful way to cope with new mathematical ideas and understanding, rather than the student making a silly or stupid mistake, they can use the errors as an opportunity to learn (Olivier, 1989). Teachers can create a classroom environment that normalizes errors as part of the learning process and engage the students in mathematical discourse about the errors and misconceptions in order to ensure students have deep conceptual understanding, while correctly connecting new knowledge to their previous knowledge (Olivier, 1989).

Misconceptions will never be entirely avoided, but teachers can intervene before the misconception becomes deeply rooted. First, teachers must understand why their students are making errors or how they have developed misconceptions before they can address them and develop interventions to promote true understanding (Olivier, 1989; Harbour et al., 2016). By using formative assessment tools, such as a screener, teachers can begin to discover the root of their students' misconceptions and errors. Teachers must first gather evidence on a mathematical concept or skill, like the equal sign, and then use the information to shape and guide their instruction (Harbour et al., 2016). Formative assessment is one way that teachers can collect evidence about students' thinking about the equal sign, adjust their instruction, and help students develop deep conceptual understanding about the equal sign before moving onto more advanced computation.

Methodology

This paper considers two different studies that were conducted in Jordan and India. After briefly describing each study separately, we synthesize the results of both studies in the discussion section below.

Setting and Participants (Jordan)

A total of 1,486 students (males = 674, females = 812) from Jordan participated in the pilot test for the Early Grade Mathematics Assessment (EGMA) Relational Reasoning subtask (RTI International, 2013). The pilot test was conducted by RTI International (an independent, non-profit research institute focused on social and laboratory sciences, engineering, and international development) and included 151 schools, representing all regions of Jordan. All students were in Grades 2-3, and the average age of the participants was 8 years, 4 months. The pilot test was conducted in 2014.

Procedures (Jordan)

The EGMA Relational Reasoning subtask is an orally and individually administered instrument that assesses students' knowledge of equivalence, decomposition, and multiplicative thinking. The items for the EGMA Relational Reasoning pilot test were divided among four forms, with 20 items on each form (including five anchor items) (RTI International, 2013). Between 340 – 400 students took each form. Each student was administered one form by a trained test assessor. The test assessor administered each item using a stimulus sheet

and recorded students' responses on iPads. See Perry (2016) for additional information about the EGMA Relational Reasoning pilot test and accompanying validity evidence.

Items for this study include visual and symbolic items that assess equivalence. Symbolic items include an equation with a missing value (e.g., $9 + 6 = \square + 10$). Visual items required students to watch as the assessor acted out a situation with balls and a bag. For the visual items, students were not shown any numbers or symbolic notation.

Analysis (Jordan)

For this investigation, students' responses on items with non-standard equations were analyzed. Quantitative data about students' correct and incorrect responses were collected through the assessment.

Setting and Participants (India)

Breaking Through Dyslexia (BTD), a non-profit educational organization in Kolkata, recruited 185 second standard (grade) students in private primary schools (males = 102, females = 83). The average age of the participants was 7 years, 0 months. Students were from six private, English medium schools in metropolitan Kolkata. Students completed the screener between mid-June to the end of July 2015.

Procedures (India)

Depending on the space available at the school, students completed the screener in small groups of 6-10 students at a time, during school hours. Students had unlimited time to complete the screener, which consisted of ten questions, using paper and pencil. The screener was constructed by the researcher, based on the NCERT (2006a) Syllabus for Classes at the Elementary Level. The questions were also adapted from several sources (see the Appendix in Eichhorn (2016) for the sources for individual questions).

Analysis (India)

For this investigation, students' responses on items with non-standard equations, including composition and decomposition tasks, were analyzed. Quantitative data about students' correct or incorrect answers were collected through the assessment, but qualitative data was also collected through observation and work sample analysis to determine the nature of their responses and their strategies used to find the answers. While the results of each study were first analyzed separately, the researchers then compared the results from Jordan and India together to find commonalities in students' responses.

Results

Jordan

Students' responses on the EGMA Relational Reasoning items (see Table 1) reveal that students in Jordan exhibit many of the misconceptions identified in previous studies in the United States (e.g., Behr et al., 1980; Carpenter et al., 2003, Denmark et al., 1976). For example, students' responses indicate that many saw the equal sign as a "do something" symbol, found the total of all of the numbers, or extended the problem (see above for detailed explanations of these misconceptions).

While many of the misconceptions mirror those identified in previous studies, additional errors and misconceptions emerged that demonstrate a lack of understanding about the equal sign and equality of expressions. Four additional misconceptions occurred on multiple items: same number, proximity, subset of equation, and next number in counting sequence. With "same number," students identified a number closest to the unknown value as the unknown value (e.g., $9 + 6 = \square + 10$; Response: $\square = 10$). With "proximity," students only attended to the unknown and the number next to the equal sign (e.g., $9 + 6 = \square + 10$; Response: $\square = 6$). With "subset of equation," students only attend to a portion of the equation.

Table 1. Jordanian students' responses on EGMA non-standard equations

Example	Response	Percentage	Interpretation
$9 + 6 = \square + 10$	—	43%	No response
	5*	22 %	Correct
	15	8 %	Do something
	25	3 %	Total/extend the problem
	6	3 %	Proximity to equal sign
	10	3 %	Same number
$4 + 1 = \square + 2$	—	28%	No response
	3*	24 %	Correct
	5	14 %	Do something
	7	6 %	Total/extend the problem
	1	13 %	Proximity to equal sign
	2	7 %	Same number
$17 = 10 + \square$	7*	49 %	Correct
	—	30%	No response
	5	3 %	Calculation error
	27	3 %	Total
	10	2 %	Same number
	11	2 %	Next number in counting sequence
$22 + 8 - 8 = \square$	—	45%	No response
	22*	25 %	Correct
	0	4 %	Subset of equation
	6	2 %	Calculation error
	8	2 %	Proximity to the equal sign
	10	2 %	Calculation error
$16 = 10 + 4 + \square$	2*	38 %	Correct
	—	28%	No response
	6	8 %	Subset of equation
	4	4 %	Same number
	5	4 %	Next number in counting sequence
	14	3 %	Subset of equation
Read by assessor: "Watch what I am doing. First I am putting 5 balls into the bag. Now I am putting another 4 balls into the bag. Next I take out 4 balls. How many balls are there in the bag now?"	5*	84%	Correct
	—	4%	No response
	9	3%	First action
	4	2%	Last number
	1	2%	Subset of equation
	3	1%	Calculation error
	8	1%	Calculation error

For example, some students responded that unknown value in the item $22 + 8 - 8 = \square$ was 0, since $8 - 8 = 0$. A few students also gave responses that indicated that they found the unknown value by determining the "next number in the counting sequence" (e.g., $16 = 10 + 4 + \square$; Response: $\square = 5$). Students performed better on items modelled visually with a bag and balls compared to items that included symbolic notation.

India

Three items from the ten-question number sense screener administered in Kolkata, India were analysed for the purposes of this comparative study. In Table 2, correct answers have an asterisk and are presented first, while incorrect answers follow. The most common misconceptions/errors are shaded in grey.

Table 2. Indian students' responses on screener questions with non-standard equations

Example	Response	Percentage	Interpretation
$\square + \square = 7$	6 + 1; 1 + 6*	23 %	Correct
	5 + 2; 2 + 5*	13 %	Correct
	4 + 3; 3 + 4*	25 %	Correct
	7 + 0; 0 + 7*	1 %	Correct
	5 + 6	18.4 %	Next numbers in ascending counting sequence
		3.8 %	No response – left blank
	24 + _	4 %	Incorporated numbers from preceding problem
	14 + 7	1 %	Total/extend the problem (backwards?)
$18 = \square + \square$	9 + 7; 1 + 7	1 %	Same number
	10 + 8; 8 + 10*	19 %	Correct
	9 + 9*	11 %	Correct
	17 + 1; 1 + 17*	10 %	Correct
	16 + 2; 2 + 16*	5 %	Correct
	0 + 18; 18 + 0*	0.5 %	Correct
	19 + 20	30%	Next numbers in ascending counting sequence
		3 %	No response – left blank
1 + 8	1 %	Separating digits – disregarding place value	
Application: $8 = \square + \square$	4 + 4*	27 %	Correct – only fair share combination
	4 + 4 and 5 + 3 or 3 + 5*	22%	Correct – at least one additional combination
	4 + 4 and 2 + 6 or 6 + 2*	9 %	Correct – at least one additional combination
	4 + 4 and 7 + 1 or 1 + 7*	2.7 %	Correct – at least one additional combination
		9 %	No response – left blank
	8 + another one digit number	8 %	Same number
8 + 8	2 %	Same number	

In summary, 62% of students correctly identified 2 addends in the missing addend question, $\square + \square = 7$, while 46% of students identified missing addends in the non-standard form, $18 = \square + \square$. Therefore, students in the Indian sample performed better on decomposing and composing tasks that followed the standard equation format ($a + b = c$) such as $\square + \square = 7$, as compared to non-standard equations ($c = a + b$) like $18 = \square + \square$. When decomposing 18, Indian students tended to use fair share methods and making a ten. For students that answered the missing addends examples incorrectly, the most common error (18.4% and 30% of the sample) involved disregarding the addition symbol and equal sign and creating a list of ascending numerals. This misconception among Indian students was not explicitly mentioned in the literature and was surprising to the research team. Errors were similar across the six private schools in the sample. Some Indian students did not show flexible understanding of numbers. While 61% of students correctly identified at least one way to decompose 8 in a real-life scenario (putting 8 marbles in 2 bowls - one red and one blue), 27% were not able to name combinations other than $4 + 4$ for the sum of 8 in the non-standard equation. Nearly 40% of students were incorrect, with 9% of students leaving the answer blank. However, Indian students performed better on non-standard equations and decomposing tasks when they were in embedded in a real-life context, as opposed to solely presented in the symbolic format.

Discussion

Overall, students from Jordan and India struggled with non-standard equations. Students have had limited exposure to multiple types of equations, and when faced with a novel situation, many did not attempt to solve

the problems. In both countries, students were more likely to attempt the problem when it involved a visual task or was embedded in a real-life scenario and they could draw their response.

Students also ignored or disregarded parts of the equations. Students in India have been overexposed to the task of ascending and descending numbers and many neglected to notice the symbols of the addition sign and the equal sign in the examples with missing addends. Students in Jordan ignored the digit to the extreme right ($6+9 = \square +4$, student would ignore the 4). This misconception reveals that the students lack conceptual understanding of the full equation, or the balance / relationship between the numbers.

Similar to the misconceptions mentioned in the prior international literature, students in Jordan exhibited the “do something” misconception when presented with examples such as $9 + 6 = \square + 10$ and answering “15,” or answering “5” for $4 + 1 = \square + 2$. Students in Jordan also found the total, or extended the problem, when they answered “27” for $17 = 10 + \square$. However, in many cases, students did not respond to many of the items.

In both Jordan and India, many students struggled with composing and decomposing tasks. On the EGMA, 30% of students in Jordan did not respond to the question $17 = 10 + \square$ and 28% of students did not respond to $16 - 10 + 4 = \square$. In India, 30% of students wrote ascending numbers when asked for both addends in $18 = \square + \square$. Although between 40 - 50% of students in both countries were able to identify the correct response to the above questions, some students clearly still struggle with non-standard equations and a relational understanding of the equality symbol.

Because textbooks may account for some equal sign misunderstanding, a textbook from each setting was analyzed for the types of equations that were present with the equal sign (Capraro et al., 2011). In the first 50 instances with the equal sign in the textbook from Jordan, there were 42 standard equations, 3 non-standard (including 2 vertical), and 5 standard equations with missing addends (decomposition) (Jordan Ministry of Education, 2017).

According to the Indian first grade math textbook published by National Council of Education Research and Training (NCERT), students were asked to identify both unknown addends when the sum is given (e.g. $\square + \square = 7$) on one page in the entire textbook, which consisted of 8 examples (NCERT, 2006b, p. 60). However, the format is still in the standard equation ($a + b = c$) format. The entire textbook featured 54 addition problems in the standard equation format with the equal sign, with 40 presented horizontally and 14 presented vertically.

From this textbook review of national government-approved/sponsored textbooks in each country, the standard equation is over-represented in the primary instructional resource used by teachers. When students are overly exposed to the standard equation, they may form misconceptions about the equal sign. Also, if textbooks focus on the symbolic, or abstract, notation of the equation, before anchoring students’ conceptual skills in the concrete and representational stages, students can easily misunderstand the meaning of the equal sign.

Conclusion and Recommendations

Teachers can use a screening tool or assessment as a first step in collecting baseline information regarding their students’ math abilities with the equal sign, as well as begin to understand the variability of number sense in their students. Through the constructivist lens, we view errors as students’ attempt to construct their math knowledge. Misconceptions will never be entirely avoided. However, when teachers establish a classroom environment where errors are seen as an opportunity to learn and grow in our understanding of math, students may respond more positively to math and have less anxiety while engaging with mathematical content (Olivier, 1989). They can also begin to adopt a growth mindset and view mistakes as opportunities for your brain to grow (Dweck, 2006; Boaler, 2015). By using the screener, or other screening tools, a teacher uses students’ errors to change his/her instruction, rather than attributing student performance solely to their teaching. Universal screening can be seen as assessment *for* learning more about students’ current levels of understanding and helping teachers understand how they adjust their instruction, not an assessment *of* how much the students have learned (Boaler, 2015).

Students will benefit from exposure to nonstandard types of equations, as well as concrete and representational experiences, to develop a relational and conceptual understanding of the equal sign to support their mathematical thinking.

Instructional Materials

Teachers can introduce nonstandard equations and composing/decomposing tasks to foster greater conceptual understanding of mathematical symbols, such as the equal sign, by using many instructional materials. Since teachers often use the class textbook as a guide for instruction, textbooks can list examples of multiple types of equation formats, including those with operations on both sides of the equation. However, teachers will also need access to concrete materials and representational drawings to build conceptual understanding of the abstract symbols, such as the equal sign. Concrete materials, such as a pan balance with real-life objects or counters, can help students visualize the meaning behind the symbol “=.” Teachers can draw representations on the board and encourage students to draw their own representations on slates or in their notebooks.

One example of an instructional program that uses nonstandard equations is *NumberSense*, which is used widely in South Africa and has informed the design of instructional materials in Jordan (Brombacher & Associates, 2011). In developing the *NumberSense* workbooks, the author team at Brombacher & Associates were very deliberate in wanting to avoid students developing a misconception with regard to the meaning of the equal sign. The author team wanted to ensure that students experience the equal sign in a relational way, that is as a signifier of equivalence and not as an instruction to perform an operation.

The *NumberSense* workbooks are a series of workbooks developed for Grade 1 to 7 students. In each lesson in grade 1 there is a counting routine, a manipulating number activity and a problem for students to solve. The teacher will have used the word equal many times during both the manipulating number and problem activities before the equal sign is used in the workbook. In the manipulating number activities, the teacher (at first with the support of a number line) will have dealt with questions such as: What is 5 plus 2?; What must I add to 5 to get 7?; and, What number must I add 3 to get 8? In other words, it is expected that students will already have an awareness of different ways of structuring equivalence statements long before they are confronted with the written form of these statements.

The first time the equal sign is used in the workbooks is in the second quarter of the Grade 1 program (Brombacher & Associates, 2011, p. 8) shown in Figure 1. A see-saw is used as a metaphor for equivalence (balanced) and students are introduced to a balanced see-saw and told “The two sides are equal”, next they are introduced to an unbalanced see-saw and told “The two sides are not equal” with a discussion about how to make the sides equal.

For the next few pages the addition number sentence always appears in conjunction with an unbalanced see-saw and the student is expected to complete the number sentence as a way of describing how to balance the see-saw. After a few pages of seeing the number sentence in conjunction with the unbalanced see-saw, the student is then exposed to number sentences on their own. The expectation is that the student can both reference see-saw metaphor to solve the number sentences, but also to start seeing the number sentences as a written form of the questions that form part of the daily manipulating number activity. Throughout the workbook, there are different number sentence structures, and hence different equal sign positions. This was done in an effort to avoid students developing the misconception that the equal sign is a signifier of an action to be performed and second, to support students developing flexible number manipulation skills - that is, being aware that numbers can be broken down and recombined in different ways.

When the *NumberSense* workbooks were piloted in South Africa, early grade teachers had mixed reactions to the deliberate design decisions about the different number sentence structures and hence the placement of the equal sign. Some teachers resisted the change to non-standard equations. Schools and teachers implement the *NumberSense* workbooks in different ways. Some use them as a supplementary exercise program while others use them as their mathematics program. Quite apart from how teachers implement the workbooks, few teachers allow the workbooks to guide the teaching trajectory. In most cases we find that teachers who are new to the program and its approach still feel a strong need to tell or lecture students on what to do before they allow students to work on the workbook activities. South African teachers have voiced concern about the students “finding the workbooks confusing” and cite the number sentences as a particular example of activities that students struggle with. Careful analysis of the situation reveals that teachers will typically introduce/tell/lecture the “equals concept” with the equal sign and unknown in a “standard” position, namely: $a + b = \square$. Despite the efforts of the workbooks to introduce these concepts in more robust ways, teachers’ dominant teaching habits/practices can still get in the way. That said, as teachers gain experience with the program, its approach and philosophy they increasingly allow the materials to lead and students to use the materials with less explicit guidance.

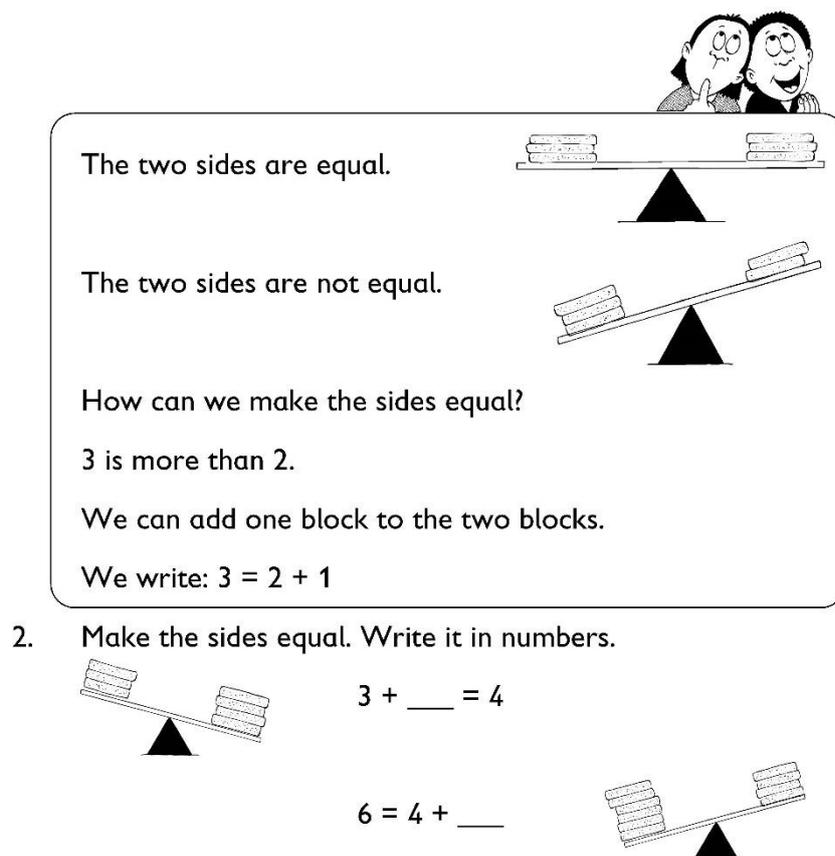


Figure 1. Introduction of the equal sign in grade 1 *NumberSense* workbooks (Brombacher & Associates, 2011, p. 8)

Overall, teachers and textbook creators can use many non-standard contexts to develop relational understanding of the equal sign with their students. Students in this study benefited from visual tasks and real-life scenarios that they could draw to show their understanding of equality. When teachers spend more time in the concrete and representational stages of equal sign instruction, students will develop greater conceptual understanding of the abstract symbol, “=”. A pan balance and representations of a see-saw are powerful instructional tools that teachers can use to reinforce the concept of equality to young students.

In addition to students being exposed to nonstandard equations in textbooks, teachers must facilitate discussion in order to develop students’ ability to analyze expressions (Molina & Ambrose, 2006; 2008). Through discussion, teachers elicit students’ current understanding and call their attention to the equation structure and patterns (Stephens, et al. 2013). Students will require repeated exposure over multiple years to develop and maintain a relational understanding of the equal sign, as opposed to a quick intervention. Consistent instruction over time, with similar mathematical language, materials, and facilitated discussions by teachers, will help students develop a deep and meaningful understanding of the equal sign. Since knowledge of the equal sign is critical to students’ success in mathematics, additional research should be conducted to determine how misconceptions about the equal sign are developed around the world, how they can be prevented, and which factors may be specific to low-resource countries that can impact students’ understanding of the equal sign.

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Author Information

Melinda S. Eichhorn

Gordon College

Wenham, MA

USA

Contact e-mail: Melinda.eichhorn@gordon.edu

Lindsey E. Perry

Southern Methodist University

Dallas, TX

USA

Aarnout Brombacher

Brombacher and Associates

Cape Town

South Africa
