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A Phenomenological Perspective to Bilingual Students' Word Problems Solving Behaviours

Sibel Telli<sup>1,3</sup>, Renata Rasch<sup>2,3</sup>, Wolfgang Schnotz<sup>2,3</sup>

- <sup>1</sup> Canakkale Onsekiz Mart University
- <sup>2</sup> University of Koblenz Landau
- <sup>3</sup> DFG Graduate School

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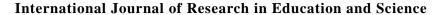
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## A Phenomenological Perspective to Bilingual Students' Word Problems Solving Behaviours

Sibel Telli, Renata Rasch, Wolfgang Schnotz

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#### **Abstract**

This study describes the problem solving behaviour of bilingual (German - Turkish) primary school students, their approaches and preferences while solving the word problems. Four different types of mathematical word problems (combinations, movement, subtraction and addiction) were given successively to four pupils (one girl and three boys) and their individual strategies during solution process were explored. After examining the students' answers, four theme, namely language, effect of daily experience, recontextualizing the problem and the mathematical answer were listed. Generally, the students struggle to use the representations effectively with their problem solving strategies. This study would be valuable with giving detailed information about bilingual students problems solving strategies under free language environment and results would be helpful to researchers to conduct an intervention study that specially targeting to improve the problems solving strategies of bilingual children with an emphasize to rehearsal their reading skills and representation repertoire.

## Introduction

Rapid economic and social changes in today's world bring the demands on education systems to develop the key humans' competencies for world interdependent changes, while keeping the importance of disciplinary knowledge. These demands on education systems and learners are evolved fast to develop the problem solving skills and critical thinking capacities of students to understand the technological and non-routine environment around themselves (Csapó, & Funke, 2017). Thus, not surprisingly, developing mathematical thinking is one of the key tasks for mathematics instruction in all curriculum and at the heart of these discussions and research results emphasize the importance of problem solving from the variety of aspects for the mathematics education (e.g. Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016, Grégoire, 2016; Schoenfeld, 1992). Moreover, many researchers have pointed out that word problems are a crucial part of the mathematics curriculum of the primary school (Fuchs et al., 2006; Verschaffel, Corte & Lasure, 1994) and effective in developing pupils' problem solving competences (Rasch 2001). However, research results revealed that school children have difficulties with word problems (Dewolf, Dooren & Verschaffel, 2017; Verschaffel & De Corte, 1993).

Problem solving's definition is a general term corresponding different research fields with several forms (e.g. Greiff, Holt and Funke, 2013). From the mathematical standpoint with the Polya's (1945) definition, it is as finding a way around a difficulty or an obstacle a solution that is unknown. According to Schoenfeld (1992), problems as routine exercises with a structure are organized to provide practice on a particular mathematical technique that typically follow routine while non-routine is not. Teaching problem solving skill in mathematics education is often related to the pioneering work of Polya (1945) which gives four main general stages in problem solving process that can be used to tackle (especially non-routine) problems. First stage is the *Understanding the problem*, in which problem solver is expected to understand the problem in general. Second stage is the *developing a solution plan* that mostly depends on how the solver understood and interpreted the problem at first stage and formulates the strategies for the solution. Third stage is the *implementing the plan*. The solver has the privilege of a number of strategies to select from his listed strategies. His individual election is based on the solver's own disposition. Lastly, the stage is *the evaluating the solution* in which of the tentative solution is verified and tested with the accuracy and efficiency. In this study, Polya's model is followed to analyze the German Turkish students problem-solving strategies

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The research outcomes pointed out the important role of language in the mathematics' classroom interaction, learning or teaching mathematics and mathematics conceptualization (e.g. Radford & Barwell, 2016). While

factors contributing to this are varied, many researchers identified that performance on mathematical word problems and language proficiency were moderately correlated; and children's text comprehension skills and mathematical word problem-solving performance were interrelated and it is among the powerful factors that affect the problem solving performance in different age groups (Paetsch, Felbrich & Stanat, 2015; Pimperton & Nation, 2010; Prevoo, Malda, Mesman & van IJzendoorn, 2015; Kyttälä & Björn, 2014).

Similarly, among bilinguals, success in solving word problems in mathematics is influenced by linguistic factors (Ambrose & Molina, 2014; Riordain & O'Donoghue, 2009). In this respect, many questions have been raised regarding how a bilingual's knowledge is represented in two languages. There are conflicting views about the learning of mathematics in second language at different levels of education according to research findings that demonstrated the challenges and opportunities for second language learners face in mathematics programs (e.g. Durand-Guerrier, Kazima, Libbrecht, Ngansop, Salekhova, Tuktamyshov & Winsløw, 2016; Barton, Chan, King, Neville-Barton & Sneddon, 2005, Mulat & Achavi, 2009). Additionally, bilingual children raised bilingually might never be in a purely monolingual situation for any long time and usually come from different cultural groups and their language use is naturally interrupted by moments of flexible use of languages, code switching (Moschkovich 2007). According to Lestor and Kroll (1993) social-cultural context are among the factors influencing problem solving performance (the other four are knowledge acquisition and utilization, control, beliefs and affective domain, which includes individual feelings, attitudes and emotions). Hence, students' real-word knowledge and life experiences have also effect on solving arithmetic problems (Voyer, 2010).

In the literature, the bilingual mathematical learning is discussed in many aspects, such as non-linguistic and linguistic communication of bilinguals with touchscreen, dynamic technology for exploring calculus concepts (Ng, 2016). However, there appears to be relatively little discussion of students' behaviour in solving word problems with attention to describing the individual differences (Boonen, van der Schoot, van Wesel, de Vries & Jolles, 2013; Pape, 2004). That's why, describing this process is important to understand these students' behaviour and strategies in the problem solving process.

Turkish is one of the biggest language minorities in Germany while mixing and code-switching between Turkish and German is the normal way of speaking for many second- and third-generation in everyday life (Auer, 2011).

The study reported in this article examines four German Turkish students' mathematical problem solving strategies and the relationships of problem-solving processes.

Reasoning along these lines, the following research questions were explored.

- What do bilingual primary school students while solving mathematical word problems?
- What are their approaches and preferences to solve the word problems?

## Method

## **Material and Methods**

## Sample

The sample involved four students (one girl and three boys) who have Turkish parents, their home language is Turkish and school language is German from one school. They and their parents were born in Germany. In addition to obtaining parental consent from both parents, each student was invited to participate in the study in person and was explained that they can leave the study anytime they wished with no questions asked. They were selected by their teacher according to their interest in mathematic class in a continuum line of the primary school grades, which means up to Grade 4 in German education system. Students were evaluated as average or slightly above average according to their teachers. The study was conducted at the beginning of second semester and student was at the middle of the grade. All students' names are pseudonyms.

## Measures

Word problems-Five different mathematical word problems were chosen from a task selection book (Rasch, 2001) by the research group. To follow the students' strategies at the different types of word problems:

combinations, movement, subtraction and addiction were selected. Firstly, the selected questions were translated into Turkish and back translated by one bilingual teacher. Secondly, the translated and back translated versions of the word problems were compared with the original German version by two other bilingual primary school teachers who worked independently for the face validity (Appendix A).

Two selected questions from four were slightly adapted, for example, names were replaced with the common girls' and boys' names at the students' school. Some words were changed for their familiarity in their social context such as steam boat (which is not much familiar to these students) to train (main transport mean in their city). Teachers participated in the translation process were born in Germany or Turkey, and had their life and education partially in both countries and were fluent in both languages at the professional level.

Task based Interviews- Task based Interviews were carried out to understand students' solution process and their answers after each task solution process. Students were asked the following four questions "Have you solved a similar problem before?", "What is difficult for you in the question?", "If one of your friends asked your help to solve this problem, how can you explain your solution" and "How can you prove your solution is correct if your friend claimed your solution was wrong"

*Video records*- Video data is mainly used to investigate the students' behaviour during solution process. This data source is used only for confirmation and to refresh the researchers' memory.

#### **Procedure**

In this study is a phenomenological research (Englander, 2012) with data from task based interviews, students' written solution for the word problems and video records to have in deep understanding the problem solving behaviour of bilingual primary school students. Considering majority of bilinguals are rarely equally fluent in both languages (Bialystok, 2009), this study provided a flexible setting and freedom to use the two languages to handle the possible language difficulties they might have. A copy of each word problem was presented and read aloud to the child by the native speaker in that language (German or Turkish) right after the word problem was given to the child in the written form in that language. Students had a space to work on and then were asked to solve the problems and make written recordings of any working out used in this process. Students received the questions and language for the questions in Latin Square Design and axillary material for their calculation. They chose the language for the interview and they were free to change it for a question anytime. A native speaker in the chosen language communicated with the student, administered the question and carried the interview.

Reading the problems to students ensured that the students understood each problem said but students were in no other way assisted. Following the completion of each problem, each student was asked if they had encountered a similar problem before, were asked to verbally explain their solution process. Each session was videotaped and observational notes were taken. Students were not given a time limit to solve the problem. The sessions ranged between 21 minutes to 53 minutes for the students. They were informed that the session was not an examination and their performance would have no effect on their math grades. They were told that only their solution methods were of interest. Through the problem solving process and interviews, students were free to verbalize their thoughts or to keep silent and they did not have any guidance for their behaviour. Additionally, students were asked to evaluate the difficulty of the each word problem on a five point scale represented by smiley faces right after the question was read by a native speaker and then at the end of their solution process.

#### **Analyses**

Analyses were performed on interview data and the four word problems answered by four students. Thus, the sixteen solutions of the students were examined. The focus of the analysis was on understanding the factors affecting the students' solution process and their problem solving behaviours. For each question, all student solutions were compared and then the factors leading the students' solutions were discussed and categorized. The results were verified with the raw data.

## **Results and Discussion**

Fatma (2<sup>nd</sup> Grade)

#### Problem - Train

Mutti, Vati und Yasemin fahren mit dem Zug. Für Kinder kostet es nur die Hälfte. Sie bezahlen insgesamt 30 €. Wie viel kostet die Karte für einen Erwachsenen und wie viel kostet sie für ein Kind?



Fatma's mathematical answer to the train-problem consisted of "6€ Awasenen" when it should have been Erwachsenen (Adult) and "1€ Kind" (Child). She was too shy in the interview and had challenge to bring forward any arguments for her solution. Although she said that she had encountered a similar task before, afterwards she said that she did not work with such a task before.

#### Problem - Ice Cream

Yasemin, Lisa ve Deniz dondurma almak istiyor. İki top dondurma almak için paraları var. Dondurmacı 3 çeşit dondurma verebiliyor: çikolatalı, vanilyalı ve çilekli. Yasemin kendisine kaç çeşit dondurma alabilir? Kaç tane farklı olanağı vardır?



This time Fatma responded the question "Have you ever solved a similar task before? with "No". In the interview she said that she had difficulties counting the different kinds of ice cream. Her mathematical answer to the word problem was just "5€". She did not show or explain her way of calculating. But she told about her experiences in an ice cream shop. She said that the man gave her three balls ice-cream and she gave him 5€.

#### Problem - Snail

20 metre derinlikteki kuyuda bulunan salyangoz (sümüklü böcek) kuyunun dışındaki çimene çıkmak istiyor. Her gün 5 metre tırmanıyor ve geceleri uyurkende 2 metre geriye kayıyor. Kaç günde kuyunun ağzındaki çimenlere ulaşır?

Fatma answered the third question similar to the previous one. She suddenly came up with a number as the answer. She explained her solution as follows: "Because it takes the snail a long time to get to the surface." The time concept might be too abstract for this age or she needs to reconceptualise the context. She had just the image of a snail crawling up a fountain and in her imagination it needed several days to get out of it considering the difficulty of sliding back while sleeping. Then she gave the answer as sixteen days that she said it was long enough for snail to climb out.

#### Problem - Burglar

Zwei Räuber entdecken einen vergrabenen Schatz, 2 Beutel Goldmünzen. Sie zählen die Münzen. In einem Beutel sind 34 Münzen, in dem anderen sind 52 Münzen. Sie wollen die Beute unter sich gerecht teilen. Wie viele Münzen müssen sie aus dem volleren Beutel herausnehmen und in den anderen füllen, damit in beiden Beuteln gleich viele Münzen sind?

92

Fatma answered the last question just with the number "92" without calculation or any other work shown. But most probably she attempted to sum up the numbers given in the task and she made an operational mistake. She did not explain her way of getting to the final answer. Since she was not willing to go any more with the task and stopped. She said she had never seen such a problem before.

Alp (2nd Grade)

Problem - Train

Mutti, Vati und Yasemin fahren mit dem Zug. Für Kinder kostet es nur die Hälfte. Sie bezahlen insgesamt 30 €. Wie viel kostet die Karte für einen Erwachsenen und wie viel kostet sie für ein Kind?

Go das hostet 30 € die Karle
lür ein Erwachsene leostet 50€
und lür ein Kind hostet 5€

Alp attempted to combine his solution with a drawing of a person with some coins. It took time for him to organize the information in the text and understood  $30 \in$  as a cost of one ticket instead of cost for all (father, mother and child). After this step he lost the connection with the text and concluded his answer as  $50 \in$  for an adult and  $5 \in$  for a child. He read the question, made a decision quickly with adding information to the text but he did not explain his strategy for the solution. In the interview, he said that he never worked on such task before and had difficulties with the question similar to Deniz (see 3.13,  $3^{rd}$  Grade). He did not translate the given situation to mathematical language. Most probably "altogether (insgesamt)" was complicated for him to form a mathematical interpretation.

## Problem - Ice Cream

Yasemin, Lisa ve Deniz dondurma almak istiyor. İki top dondurma almak için paraları var. Dondurmacı 3 çeşit dondurma verebiliyor: çikolatalı, vanilyalı ve çilekli. Yasemin kendisine kaç çeşit dondurma alabilir? Kaç tane farklı olanağı vardır?

Yasemin o disa ve Denis 3 tane dondurma almab istiyar. 3 Lorten. 3 möblich beilden.

Alp used a mixed way of two languages in his solution at the second question. He identified the three persons in the task and had an idea of three possibilities of having ice cream since three different kinds were given in the task. He did not give the possible combinations for these three kinds but organized the information necessary for the solution process with little difficulty.

Problem Snail

5 m nach unten. unter die Wisse stet ein

Alp drew a picture of a person with a spider, although neither a person nor a spider was mentioned in the task. At the same time these two pictures were about at the same level and static although question was asking updown movement. From this drawing we concluded that Alp could not catch the necessary information for his solution this time and his illustration was not helpful. His answer was: "5m down on the grass there is a spider" and missed the "up" movement one more time. In interview he did not give the reason how he came to this conclusion. Besides, he did not propose a way of his mathematical calculation to come to the solution. He might have gotten confused the words snail and spider, or he might not have been familiar with the word snail and renamed it with an animal familiar to him. However, it is also very common for young children to add some pictures and represent the text with possible details while trying to find out a solution (Rasch, 2001).

## Problem Burglar

Zwei Räuber entdecken einen vergrabenen Schatz, 2 Beutel Goldmünzen. Sie zählen die Münzen. In einem Beutel sind 34 Münzen, in dem anderen sind 52 Münzen. Sie wollen die Beute unter sich gerecht teilen. Wie viele Münzen müssen sie aus dem volleren Beutel herausnehmen und in den anderen füllen, damit in beiden Beuteln gleich viele Münzen sind?

34 Bentel

Sich gerecht teilen

Benteln geleich viele münzen

Alp's response to the question was in fact a search to understand and organize the information to solve the question. He transferred the number of bags as 34 instead of 2 and confused with number of coins (Münzen) in his notes although he recognized the structure of the problem and noted down "equal share, sacks will have equal gold pieces". He did not form a strategy and a mathematical answer. He said he did not know how to solve this problem. He said that he had not encountered such a task before and he did not write an answer.

## Deniz (3<sup>rd</sup> Grade)

#### Problem - Train

Mutti, Vati und Yasemin fahren mit dem Zug. Für Kinder kostet es nur die Hälfte. Sie bezahlen insgesamt 30 €. Wie viel kostet die Karte

für einen Erwachsenen und wie viel kostet sie für ein Kind? für ein Kind frontet es 3£ und für ein Erwachsenen 6 ± und sie braben nach 21 ± übrig.

Deniz answered to the train-question as follows: "A ticket for a child costs 3€ and for an adult it is 6€ and there are 21€ left." He calculated 30€-3€-6€=21€ in written text instead of showing his calculation in mathematical language. He caught the idea half for child and double cost for an adult which is one aspect of the problem. However, he did not transfer this idea to form a strategy to solve the problem. He subtracted a total cost that he estimated and missed one adult in the question during this process. He found 21€ which was an unsuccessful solution for this task. He found that 6€ was the double of 3€ for a solution and finding the resulting number. Deniz catched only one aspect of the question which is usually seen with primary school children (Rasch, 2001). He subtracted the values from the total cost separately and tried for a solution while showing the subtraction schemata that he learned in the mathematics classes. In the interview, he explained his solution over self-invented cases connected with friends. In these cases we noticed that his examples were concluded with money left in the pocket.

#### Problem - Ice Cream

Yasemin, Lisa ve Deniz dondurma almak istiyor. İki top dondurma almak için paraları var. Dondurmacı 3 çeşit dondurma verebiliyor: çikolatalı, vanilyalı ve çilekli. Yasemin kendisine kaç çeşit dondurma alabilir? Kas tapa farklı olanağı yardır?

alabilir? Kaç tane farklı olanağı vardır?

Liver will 2 Eis hugell baufen und

der Eismann gibt 3. Wie fiele hann

Liva und Deniz sich beurlen.

Liva und Deniz bann Lehoholade

vanile, erderere, und Apfelgeschmach

des ist rot oder grün

Deniz portrayed different behaviour when attempting to solve the second task, although at first it seemed similar to previous. We noticed that he rewrote the text and translated it into German. During this process he lost his concentration for solution process and meanwhile, he missed one person, Yasemin. In fact Yasemin was the person who would buy the ice cream. He understood the types of ice cream given in the question properly but added information about taste (apple-taste) and colour (red or green) which were not given in the task. This shows that he associated different kinds of ice cream with that task and the corresponding colours of strawberry and apple flavour. The mathematical solution of the word problem was totally missing although the question was administrated in German upon his request. He did not arrange several different kinds of flavours. In interview he tried to connect the question with life experiences, which was quite similar in the previous interview.

Problem - Snail

20 metre derinlikteki kuyuda bulunan salyangoz (sümüklü böcek) kuyunun dışındaki çimene çıkmak istiyor. Her gün 5 metre tırmanıyor ve geceleri uyurkende 2 metre geriye kayıyor. Kaç günde kuyunun ağzındaki cimenlere ulasır?

Die Sehnetbe brancht & Tagen bis er wiederauf die Wiese pommit.

Deniz answered the task in German as following. "It takes the snail 6 days to get to the surface (grass)." There was no calculation and he proposed first to calculate how long the snail was. During the interview in response to the question "How would you prove your solution is correct if yours friend claims your solution is wrong?", he said that "My friend should first look at how slow this snail was. If a snail was crawling very slowly, it would take a long time to come to the surface. My friend should know how slow a snail can come out of a fountain to check my solution." His willingness to learn the snails' length and speed were quite logical considering the effecting factors for a snail to come out of the foundation.

## Problem - Burglar

Zwei Räuber entdecken einen vergrabenen Schatz, 2 Beutel Goldmünzen. Sie zählen die Münzen. In einem Beutel sind 34 Münzen, in dem anderen sind 52 Münzen. Sie wollen die Beute unter sich gerecht teilen. Wie viele Münzen müssen sie aus dem volleren Beutel herausnehmen und in den anderen füllen, damit in beiden Beuteln gleich viele Münzen sind?

Deniz wrote "52-26=34" as answer. He did not use the correct numbers to operate with; however, he tried to work with the numbers given in the text. It seemed that language used in the context of task lead him to try subtraction for his solution process. He tried to find the difference between the given numbers by considering "take out (heraus nehmen)" in the text. In the interview he talked about "32 coins", although his solution was 34. He proposed to subtract till he reached the number 34 out of the bag with the 52 coins to get the solution.

#### Mehmet (4th Grade)

Problem - Train

Mutti, Vati und Yasemin fahren mit dem Zug. Für Kinder kostet es nur die Hälfte. Sie bezahlen insgesamt 30 €. Wie viel kostet die Karte für einen Erwachsenen und wie viel kostet sie für ein Kind?

2 Ervoldrene: 20€ 1 Kind; 10€ \$ 2 0+10 = 30€ Mehmet arguably did not use the efficient strategy to solve the question His answer was: "2 adults: 20€, 1 child: 10€, 20+10=30". He did not make any arithmetical operation mistake. His addition was correct. But he missed the idea half for child and double cost for an adult. He considers as three people and shared the cost equally.

#### Problem - Ice Cream

Yasemin, Lisa ve Deniz dondurma almak istiyor. İki top dondurma almak için paraları var. Dondurmacı 3 çeşit dondurma verebiliyor: çikolatalı, vanilyalı ve çilekli. Yasemin kendisine kaç çeşit dondurma alabilir? Kaç tane farklı olanağı vardır?

Mehmet solved the second question through making a list to document all the combinations that could be made with three types of ice-cream. He combined chocolate with strawberry, vanilla with strawberry and chocolate with vanilla. Although his strategy was enough for a successful solution, he missed the combination of the same kind of ice cream, which is very common mistake of primary school children (Rasch, 2001).

#### Problem - Snail

20 metre derinlikteki kuyuda bulunan salyangoz (sümüklü böcek) kuyunun dışındaki çimene çıkmak istiyor. Her gün 5 metre tırmanıyor ve geceleri uyurkende 2 metre geriye kayıyor. Kaç günde kuyunun ağzındaki çimenlere ulaşır?

Mehmet's calculation was "5-2+5-2+5-2+5-2+5=20 m." and his final answer was 5 days. He used "Vorwärtsarbeiten" a heuristic strategy to find the solution and calculate forward. Although his attempt to calculate back and forth movement of snail from the fountain began with right modelling (5-2), he could not improve his next attempt, and did not consider trying an alternative solution, such as a scale or line which may have assisted him. He recognized that the answer was not correct, but he said that he did not know what to do.

#### Problem - Burglar

Zwei Räuber entdecken einen vergrabenen Schatz, 2 Beutel Goldmünzen. Sie zählen die Münzen. In einem Beutel sind 34 Münzen, in dem anderen sind 52 Münzen. Sie wollen die Beute unter sich gerecht teilen. Wie viele Münzen müssen sie aus dem volleren Beutel herausnehmen und in den anderen füllen, damit in beiden Beuteln gleich viele Münzen sind?

He made an arithmetical operational mistake at this question and wrote a calculation: 52-34=22. He missed the carryover for a proper calculation. He made the same mistake with Deniz at the same question. He tried to find the difference between the given numbers by considering "take out (herausnehmen)" in the text.

#### **Task based Interview Answers**

With the aim to understand of the bilingual students' problem solving behaviour, each student was asked four questions for each of their solutions. The first question was about to learn their experience with word problems and the question is "Have you solved similar problem before?" The second grade students' answer for this question was "No". Students at grade 3 and 4 answered "Yes" and they said they solved some similar problems in their math classes.

The second question aimed to understand students' evaluation for the difficulty of the question and to understand the point what and why they found it difficult. Students in their answer pointed mainly the structure when they were asked "What is difficult for you in the question?" For example in the ice cream question they pointed the combination among the ice cream types, and back and forth movement in the question asking how many days would it take for the snail to get out of the fountain. Although, students recognized the structure which is essential to activate the existing mathematical knowledge for a solution (Pape, 2004), they did not plan and monitor their solution process and form a strategy for the solution which are the sub - steps of successful problem solving (Boonen, et all., 2013; Mayer, 1992).

The third question aimed to understand heurestic level of the students about their solutions and they were asked, "If one of your friends ask your help to solve this problem, how you can explain your solution?" To answer this question, generally, they did not set up a line to explain their activities. Students usually did not respond directly, murmured or they tried to have less eye contact to reconceptualize the task. All these behaviours might be the reason of (mathematical) anxiety because of not finding the solution, explanation or finding the questions difficult (Kyttälä & Björn, 2014; Trezise & Reeve, 2016).

Students' answer to the last interview question "How would you prove your solution if your friend said this solution was wrong?". We noticed that they did not check their answer and they did not identify a means of doing this. Looking back to their solutions (see 3.1), students went on certain key numbers and words from the problem which is very common way of unsuccessful problem-solvers and called as "a direct-translation strategy" instead of using strategy and model (Boonen, et. al, 2013; Hegarty, Mayer & Monk, 1995). That's why they did not answer the problem. Students failed to employ self-correcting mechanisms or monitoring their progress while working through the problems. Even when they were asked how they could check if their answer was correct, many respondents (except one willing to learn how slow the snail and length) did not identify a means for doing this and many responded with "I just think it's right". Generally speaking, these outcomes parallel previous research findings that describe the common problem solving behaviours of naïve solvers (e.g. Muir, Beswick & Williamson, 2008).

Lastly students were asked the difficulty of the question right after reading the question and after the solution process on a five point scale as self reflective evaluation for the question. They did not neither give consistent answers nor reasons for their argumentation. They were much emotional to answer or affected by their daily experiences. For example, they rated ice – cream question "easy" or "very easy" even after they could not find the solution.

Our purpose was to describe the problem solving behavior of bilingual primary school students on solving mathematical word problems and their approaches and preferences in the solution process. For this purpose four primary school students answered the four word problems and they were interviewed about their solutions. As for the first question aimed to find the answer was "What do bilingual primary school students while solving mathematical word problems?" After examining students' answers, four themes, namely language, effect of daily experience, recontextualizing the problem and the mathematical answer were listed.

The first defined theme is the *language*. The relation of language skills and problem solving process is well documented in the literature and the lack of knowledge and experience with the use of language was described among the difficulties related with the poor performances and modelling in problem solving (Lewis & Mayer, 1987; Fuchs, Fuchs, Compton, Hamlett & Wang, 2015). Specially for the second grade students (Fatma and Alp) in the study, the word problem might be early or they need to have a word problem repertoire and language skills for their modelling competencies. This age level found young for word problems for monolinguals and

their difficulties in linguistic inputs were reported in some studies (Boonen & Jolles, 2015). Reading and understanding the text that describes the task is an essential phase in word problem solving and that individual differences in literacy skills significantly affect word problem solving performance at various age students (Kyttälä & Björn, 2014). Research results pointed that balanced bilingualism or a strong language base at least for one language would be much helpful to understand the questions and problem solving process for the students (Marian & Fausey, 2006).

The second defined theme was the effect of daily experience to the solution process. The Ice Cream question is one of the examples for this group since the answer is highly likely under the influence of kids' daily life. For example, Fatma's answer was "5€" to this word problem depending on experience in ice cream shop for paying 5€ for three bowls of ice cream. In the case of, the student has ice cream in the market box or pay different amount to the combination of three types in the different shops this might have been misleading for the student. The child could state directly the amount paid in the real life cases even without feeling any need to calculate. This might have been among the reasons why Fatma gave up answering this question. She might have paid a certain amount of money for three types of ice-cream and she simply might have given up the calculation and go on with this amount that she usually paid or even without trying since the answer was already clear for her.

Recontextualizing the problem is defined as the third theme. Some of the difficulties might have been due to the nature of the question. Looking at these students' solutions, it might be discussible that questions might still suffer from not being really in this age group of students' life and experience. Although the questions were socially adopted for the students' life, (see, Appendix A), students at these ages actually do not have direct budget and they are not travelling alone (see limitation). For the given context to travel with train and calculate the amount that should be paid for the whole family is a pseudo subject for these students. Similarly, with the snail question, the child might not have wanted to find out how many meters the snail slides up or down. Maybe the child would like to work out, "How many meters am I able to spit farther than my friend?", this could be interesting or "How much faster he might be able to run than me?". Research outcomes pointed the importance of emotions for the problem solver's self-regulation and successful and unsuccessful solutions and the students' pleasure seems important for the solution process (Carlson & Bloom, 2005; Furinghetti & Morselli, 2009; Goldin, 2000; Hannula, 2006). Looking at the burglar question, students might have related with their life and interest with little adaptations like, "We have marbles or little cars and everybody should get the same number of pieces" for example. The reason to avoid the question might be not having enough curiosity stimulating and motivation towards the question and not feeling connected to the problem (Middleton & Spanias, 1999).

The last categorized theme is *the mathematical answer*. Students have problems to form mathematical modeling and usually formed their situation model wrongly or do not transfer properly. One particular student, Mehmet (at 4. Grade) used block of cubes for his calculation in snail, but he did not make it a systematic way. Then he stopped and counted how many he could.

Second question for the study aimed to find the answer *What is their approaches and preferences to solve the word problems?* Students preferred German for the interviews. Even though they understood and followed the questions in Turkish, they felt a need to confirm their understanding and chose their instructional language, German. In general students did not prefer the illustration at their responses for these questions (See 3.1). It was noticed that, they struggle to employ the proper strategies to solve the problem, like to make a list or draw a diagram. Only one student used person illustration with a spider but it was not helpful for a successful solution.

## **Conclusion**

This study aimed to describe German – Turkish bilingual students' problem solving behaviour and their preferences to answer the word problems in a way of applying two - language environments. Such flexibility intended to operate between the different knowledge types to be able to solve mathematical word problems and find out students' tendencies, preferences and difficulties. Research findings support the idea that representations are important in the problem-solving process in mathematics education and an adequate representation promotes to find the solution. Moreover students who are able to organize a problem with mental pictures might be able to understand the problem better (Boonen, van Wesel, Jolles & van der Schoot, 2014; Author(s), 2015; Thevenot & Oakhill, 2008). However, throughout our study, it was found out the students struggle to use the representations effectively with their problem solving strategies as well as the language(s). Solving a mathematical task depends on language, the linguistic representations of number words (different in German and Turkish) and word structures are important to understand the task (Daroczy, Wolska, Meurers & Nuerk, 2015; Göbel, Moeller, Pixner, Kaufmann & Nuerk, 2014; Helmreich, Zuber, Pixner, Kaufmann, Nuerk

& Moeller, 2011; Prior, Katz, Mahajna & Rubinsten, 2015; van Rinsveld, Brunner, Landerl, Schiltz & Ugen, 2015). The balanced language proficiency needs time and schooling, young students with unbalanced bilingualism would be supported by auxiliary material and training program, this would be helpful to improve their understanding, strategy for problems solving, enrich their repertoire (Saalbach, Eckstein, Andri, Hobi & Grabner, 2013; van Rinsveld, et al, 2015).

Some of the children's difficulties documented in this study are found to parallel the naïve students' struggles with the word problems in the literature, for example to understand and represent the relational elements of mathematic (Schoenfeld & Herrmann, 1982; Chi, Bassok, Lewis, Reimann & Glaser, 1989). However, explanatory conceptions such as "What a representation is?", "What it enables a learner to do that's important to understand", this social and cultural aspect and daily life of these kids should be also considered.

More focus on bilingual children and further (multidisciplinary) studies with early intervention programs focusing on problem solving behavior of individual cases might be helpful to see more and understand the bilingual students' problem solving process. This would be helpful to strengthen our understanding of the individual differences associated with learning and to handle pros and cons of bilingualism (Bialystok, 2009). By that way it would be also possible to understand what type of support these students need: more representational repertoire development or language (s) skills or both? Such a method or combined method(s) would be helpful to assess their representational processes and role of representations during the word problem solving, help to understand the language based difficulties, effect of daily life experience, social cultural issues on problem solving process. Additionally, testing (multimedia) instructional design, how picture and text balance will be arranged for the use of especially for the early age bilinguals to handle their unbalanced bilingualism, and what they learn or not learn from animation(s), how bilingual students who are living not only in between two language environments but also in between two cultures interpret the cultural interaction pattern? And how this is functional for them could be the research focus for the future studies. Outcomes of such research would be helpful for instructional material development and teaching in particular to the early age bilinguals who often have difficult time deciding what strategy to use or how to implement the chosen strategy and representation which is also cause of unsuccessful problem solving process (Hegarty & Kozhevnikov, 1999; Woodward & Montague, 2002; van Garderen & Montague; 2003).

This topic is open to deeper analyses to broaden the theoretical outlooks and to adopt methods which allow us to be sensitive to this complexity of bilingual students' cases. Therefore, more sophisticated measures should be used to examine these students' behaviors in word problem solving such as eye-tracking methods, retrospective verbal reporting and more on neuropsychological studies might be the topics for future educational research. First limitation of this study, although the students were selected as the representative of the age group in their school, it is still difficult to generalize results to a large population of German - Turkish bilinguals or bilinguals in general. Second limitation is that the number of mathematical word problems presented to the students was small and some of the problems may have promoted the use of one type of imagery more than the other type. Although the problems were designed for primary school grade, they may have been still too difficult for the bilinguals or object to misleading due to the students' daily experiences. Third limitation in this study, we do not know in which language pattern they mostly use at home and school but we know that they preferred German, their instructional language (Gabriel, Lilla, Zander & Hannover, 2014; Gebhardt, Rauch, Mang, Sälzer & Stanat, 2013). Studies testing the performance on reading and language measures would give more data to examine and compare the mathematical problem-solving performance of bilingual students. Lastly, although as possible as a friendly atmosphere tried to be provided through the study, the student might feel discomfort working with two adults.

This study would be valuable with giving detailed information about bilingual students problems solving behavior under free language environment and it would be helpful to researcher for a deep look for the results interpret and conduct an intervention study that specially targeting to improve the problems solving behavior of bilingual children with an emphasize to rehearsal their reading skills and representation repertoire. To conclude, for a deep-understanding of the bilingual child's perspective and problem solving behavior with word problems, their life experiences, socio-cultural background, how they perceive and interpret the cultural sensitive cases (e.g. two language and two cultural environments at the same time) would be considered. For these students more picture and their own life based interactive questions might be helpful to internalize the problem and to develop their modelling competencies. These are of course more the point for the bilinguals who might not have at the same level of language proficiency or for very young students their language level might not be at the level of expected or even below the monolinguals. More interaction and collaborative study to get into kids life will bring to overcome the difficulties and help them to develop their mathematical skills

## **Notes**

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## **Author Information**

### Sibel Telli

Canakkale Onsekiz Mart University Faculty of Education Anafartalar Campus 17100 Çanakkale Turkey

Corresponding author: sibeltelli@comu.edu.tr

Wolfgang Schnotz University of Koblenz-Landau Faculty of Psychology Fortstraße 7 76829 Landau Germany

### Renata Rasch

University of Koblenz Landau Department of Mathematics, Fortstraße 7 76829 Landau Germany

## **Appendix. Word Problems for the study** [Rasch 2001]

#### QUESTION-1 Train (Aufgabe zur Verhältnisverteilung)

#### Original

Mutti, Vati und Murks fahren mit dem Dampfer. Für Kinder kostet es nur dir Hälfte. Sie bezahlen insgesamt 10 DM. Wie viel kostet die Karte für einen Erwachsenen und wie viel kostet sie für ein Kind?

## Original-Translation to English

Mummy, Daddy and Murks take a steamboat. For children it is only half the price. Altogether they pay 10 DM. How much does a ticket cost for an adult and how much does it cost for a child?

#### Adopted-German

Mutti, Vati und Yasemin fahren mit dem Zug. Für Kinder kostet es nur die Hälfte. Sie bezahlen insgesamt 30 €. Wie viel kostet die Karte für einen Erwachsenen und wie viel kostet sie für ein Kind?

## Adopted-Turkish

Anne, baba ve Yasemin tren ile yolculuk yapıyorlar. Çocuklar sadece yarı fiyat ödüyorlar. Hepsi beraber 30 € ödediler. Yetişkinler için olan bilet ücreti ve bir çocuk için olan bilet ücreti ne kadardır?

#### Adopted version (Bilingual-German-Turkish) Translation to English

Mom, Dad and Yasemin take a train. For kids children it is only half the price. Altogether they have to pay 30€. How much does a ticket cost for an adult and how much does it cost for a child?

## QUESTION-2 Ice-Cream (Aufgabe mit kombinatorischem Hintergrund)

## Original

Streblinde, Quicki und Murks möchten sich ein Eis kaufen. Jedes Kind hat Geld für 2 Kugeln Eis. Der Eisverkäufer bietet 3 Sorten Eis an: Schoko, Vanille und Himbeereis. Was für ein Eis könnte sich Quicki kaufen? Finde verschiedene Möglichkeiten!

## Original-Translation to English

Streblinde, Quicki and Murks want to buy ice-cream. Every child has got money for two scoops of ice cream. The iceman offers three different kinds of ice cream: chocolate, vanilla and raspberry flavour. How many different kinds of ice cream may Quickie buy? Find all different possibilities!

#### Adopted-German

Yasemin, Lisa und Deniz möchte sich ein Eis kaufen. Sie haben Geld für 2 Kugeln Eis. Der Eisverkäufer bietet 3 Sorten Eis an: Schoko, Vanille und Erdbeereis. Was für ein Eis könnte sich Yasemin kaufen? Wie viele verschiedene Möglichkeiten gibt es!

## Adopted-Turkish

Yasemin, Lisa ve Deniz dondurma almak istiyor. İki top dondurma almak için paraları var. Dondurmacı 3 çeşit dondurma verebiliyor: çikolatalı, vanilyalı ve çilekli. Yasemin kendisine kaç çeşit dondurma alabilir? Kaç tane farklı olanağı vardır?

## Adopted version (Bilingual-German-Turkish)Translation to English

Yasemin, Lisa and Deniz want to buy ice cream. Every child has got money for two scoops of ice cream. The ice-cream man offers three different kinds of ice cream: chocolate, vanilla and strawberry flavour. How many different kinds of ice cream may Ceren buy? How many different possibilities are there?

#### **QUESTION-3 Snail** (Bewegungsaufgabe)

#### Original

Eine Schnecke in einem 20 m tiefen Brunnen will nach oben auf die Wise. Sie kriecht am Tage immer 5 m hoch und rutscht nachts im Schlaft wieder 2 m unten.

Am wievielten Tag erreicht sie den Brunnenrand?

## Original Translation to English

A snail deep down in a fountain of 20metres wants to climb on top of the edge up to the grass. Every day the snail climbs up 5m and during the night while sleeping it goes/slides back 2m. How many days does the snail need to get from the fountain to the grass? [Author (s),2001,pp:85]

A snail in a 20 meter deep well wants to go up to the meadow. She always crawls up 5 meters during day time and glides 2 meter down when she is asleep. On what day does it reach the well?

### **QUESTION-4 Burgla**r (Ausgleichsaufgabe)

#### Original

Zwei Räuber entdecken einen vergrabenen Schatz, 2 Beutel Goldmünzen. Sie zählen die Münzen. In einem Beutel sind 34 Münzen, in dem anderen sind 52 Münzen. Sie wollen die Beute unter sich gerecht teilen. Wie viele Münzen müssen sie aus dem volleren Beutel herausnehmen und in den anderen füllen, damit in beiden Beuteln gleich viele Münzen sind?

#### Original Translation to English

Two robbers discover a hidden treasure. Two packs of gold coins. They count the coins. In one of the bags there are 34 coins, in the other one there are 52 coins. They want to share the loot equally. How many coins have to be taken out of the more filled bag and put them into the other bag, to have the same number of coins in both bags? [Author (s), 2001, pp:34]

Two bandits discover a hidden treasure, 2 bags of gold coins. They count the coins. In one bag there are 34 coins, in the other there are 52 coins. They want to share the prey fairly. How many coins do they have to take off the fuller bag and put into the other bag, until the coins are equally distributed between the bags?