Comparison of Perimeters: Intuitive Interference in People Who Are Blind

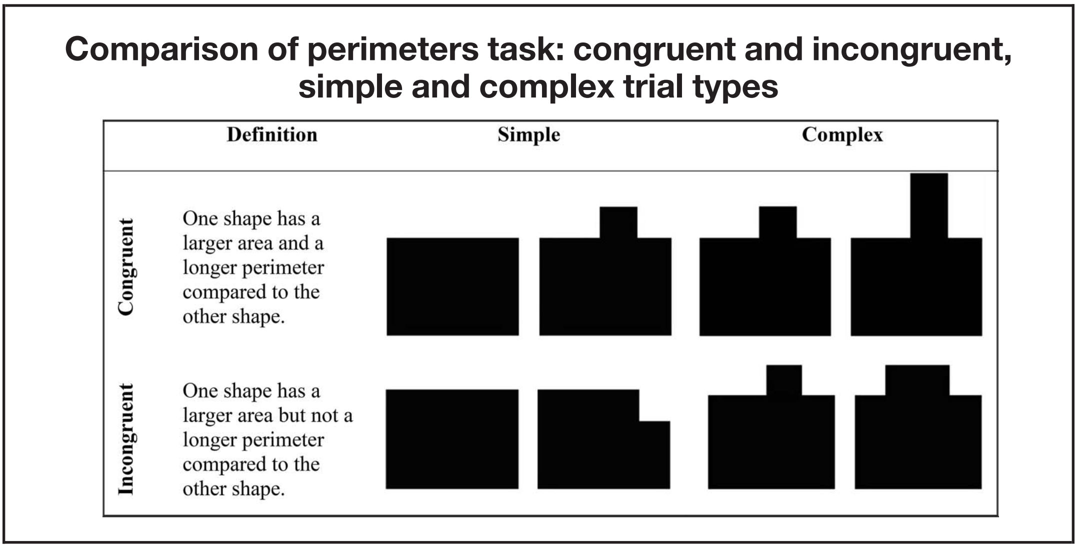
Orly Lahav and Reuven Babai

Structured abstract: *Introduction:* Difficulties in science and mathematics may stem from intuitive interference of irrelevant salient variables in a task. It has been suggested that such intuitive interference is based on immediate perceptual differences that are often visual. Studies performed with sighted participants have indicated that in the comparison-of-"perimeters" task, "area" was the irrelevant salient variable. Such studies have consistently shown that accuracy is higher and reaction times for correct responses are shorter in congruent trials (no interference of area), than in incongruent trials (interference of area). *Methods:* Fifteen participants who are blind completed a comparison-of-area and a comparison-of-perimeters tasks, each in a different session. In each comparison trial, the participants explored two tactile geometrical shapes using both hands. To collect the participants' responses and reaction times, the researchers used a Microsoft Excel Macro file designed for this study. *Results:* Findings demonstrated that for both rate of correct responses and their reaction times, the performance pattern resembled the one observed previously for sighted participants. In addition, reaction times that were observed there were about five times longer compared to those observed previously for sighted participants. *Discussion:* These findings indicated that interference of area in comparison to perimeters is evident for participants who are blind. They further suggest that, in mathematics, people who are blind experience interference that had previously been assumed to be tied to visual perception. *Implications for practitioners:* The study suggests that interference of salient irrelevant variables could cause difficulties in science and mathematics for students who are blind. This knowledge could guide educators and curriculum developers for these students in selecting appropriate learning and instruction methods. In addition, the current findings point to the additional amount of time needed to access graphical information tactilely compared to visually.

The current study focuses on a difficulty in solving geometry problems that are known to present a challenge to sighted people. We explored here if the same difficulty is experienced by people who are blind.

Intuitive interference in science and mathematics education

Solving science and mathematics problems poses a challenge for many students, as has been demonstrated in such national and international studies as the



Box 1

Trends in International Mathematics and Science Study (TIMSS) (Martin, Mullis, Foy, & Stanco, 2012; Mullis, Martin, Foy, & Arora, 2012). It has been suggested that salient variables (automatically or intuitively processed) that are not necessarily relevant for solving the problem could interfere with correct reasoning about the relevant variable in the problem. Such intuitive interference, stemming from the salient variables, may be the cause of difficulties in solving problems in science and mathematics (Stavy & Babai, 2008, 2010; Stavy & Tirosh, 2000). It was posited that such interference of salient variables stems from immediate perceptual differences that are often visual (Stavy & Tirosh, 1996). Recent studies of intuitive interference have included only participants with sight. Most of the studies employed a task in geometry as a model system to explore intuitive interference. This task will be applied in the current study as well.

Intuitive interference in the comparison-of-perimeters task

In the current study, we focus on a challenge encountered in comparing perimeters of two-dimensional geometrical shapes (that is, the total measurement of the lengths of the lines enclosing the regions of two-dimensional figures). Box 1 depicts congruent trials, in which the area of one geometrical shape is larger and its perimeter is longer compared to the other geometrical shape; and incongruent trials, in which the area of one geometrical shape is larger and its perimeter is identical as compared to the other geometrical shape. When second- to ninth-grade students who are sighted were shown the two geometrical shapes depicted as an example of an incongruent simple trial in Box 1, 70% and over from each grade level responded erroneously that the perimeter of the rectangle was longer since it "is larger" or "has more area" (Stavy & Tirosh, 2000). Apparently, when these sighted participants compared the perimeters of the shapes, they experienced interference caused by the salient, irrelevant variable "area"; that is, the two-dimensional surface enclosed within the specified boundaries of each of the two geometric shapes.

Evidence of similar interference has emerged in other research. For example, students aged 9 to 11 years expressed the belief that shapes with a larger area also had a longer perimeter in a study by Marchett, Medici, Vighi, and Zaccomer (2005). In a look at the beliefs of students and teachers about the relationship between area and perimeters, D'Amore and Fandiño Pinilla (2006) noted that some of the students erred in a comparison of perimeters of two given shapes, mistakenly choosing the shape with the larger area as having the longer perimeter.

To understand students' reasoning in science and mathematics, researchers have conducted reaction time studies to explore the reasoning that occurs while comparing perimeters (Babai, Levyadun, Stavy, & Tirosh, 2006; Stavy & Babai, 2008). In those studies, participants were asked to compare the perimeters of two geometrical shapes in congruent and incongruent conditions. In congruent trials, no interference of the salient, irrelevant variable area occurred, since the shape with a larger area also had a longer perimeter. In incongruent trials, there was interference by the irrelevant variable area, since the shape with a larger area did not have a longer perimeter (see Box 1).

Previous research has shown that accuracy was higher and reaction times for correct responses were shorter in congruent than in incongruent trials (Stavy & Babai, 2008, 2010). This finding was true among sighted schoolchildren, adolescents, and adults (Stavy & Babai, 2008, 2010; Stavy & Tirosh, 2000). In comparing the areas of the shapes, almost all of the responses were correct and were delivered relatively quickly in all conditions (Babai et al., 2006; Babai, Zilber, Stavy, & Tirosh, 2010). These findings confirmed that area is indeed the salient variable in this task and that participants find it difficult to ignore when comparing perimeters. In interpreting these findings, it was concluded that when processing the irrelevant salient variable area and processing the relevant variable perimeters result in the same conclusion, no interference is created and participants respond correctly and rapidly. When processing of area and perimeters results in two different conclusions (incongruent trials), a conflict is created. Surmounting this conflict and arriving at correct responses requires great time and effort (Babai et al., 2006, 2010).

Examining the performance in the comparison-of-perimeters task using simple and complex trials (see Box 1) showed that simple trials rendered the incongruent condition easier as compared to complex ones. In the congruent condition, no such difference between simple and complex trials emerged (Stavy & Babai, 2008).

Perception and cognitive abilities in people who are blind

People who are blind are able to compensate for their visual loss through perceptual (haptic, auditory, and olfactory) (Bishop, 2004) and conceptual information. Gibson (1962) presented tactile scanning as a modality parallel to visual scanning, and described the slow process of collecting information via tactile exploration compared to visual scanning. The delayed development of young children who are blind is generally built on a "developmental lag" theory (Warren, 1994), which means that children with severe vision loss are expected to acquire a variety of abilities at a later age than do children with sight. In research that compared the executive function of students with visual impairments to their sighted peers, highly significant differences in all domains of executive function were revealed, with the visually impaired students performing more poorly (Argyropoulos, Sideridis, Botsas, & Padeliadu, 2012; Tadicì, Pring, & Dale, 2009). The developmental lag usually closes up once enough opportunities to compensate for the loss of vision are available (Brambring, 2005; Hollins, 2000).

Mathematics learning ability in students who are blind

Since the language of mathematics is mostly visual, people who are blind must make a cognitive effort to collect mathematical information via auditory or tactile senses (Kapperman, Heinze, & Sticken, 2000; Thahane, Myburgh, & Poggenpoel, 2005). Klingenberg, Fosse, and Augestad (2012) examined mathematics education among Norwegian braille-reading students for 40 years (1967–2007); their findings showed that 57% of the participants had been educated in mathematics at the grade level appropriate for their age. This finding may result from equal access to mathematics learning materials, enabling the majority of students who are visually impaired to achieve mathematics tasks at a level similar to those of their sighted peers (Klingenberg et al., 2012; Van Scoy, McLaughlin, & Fullmer, 2005). But for those who did not achieve the grade level appropriate for their age, it might be that the developmental lag affected the understanding of particular mathematical concepts (Beal & Shaw, 2008). Their understanding could also be attributed to a lack of adequate adapted materials or resources, failure to develop appropriate compensatory skills, or insufficient conceptual development.

Students who are blind learn about area and perimeter measurement at school along with their peers with sight, but through tactile perception. This direct sensory experience enables the memory, which is based on kinesthetic ability, to easily retain information (Couvillon & Tait, 1982). Those who are adventitiously blind learn to collect information and compensate for their visual loss through other senses during their rehabilitation period. They learn and practice skills and relevant cognitive strategies within the tactile modality.

The current study

Previous studies have suggested that in the comparison-of-perimeters task, area is the salient variable and that sighted participants have difficulty in ignoring it when comparing perimeters of incongruent trials. These studies suggested that the interference could stem from the visual modality due to the ability of cognitive systems to detect the perceptual differences immediately (Stavy & Tirosh, 1996).

The goal of the current study was to explore whether participants who are blind and have the knowledge and the expertise to collect the geometrical shapes' properties through tactile modality experience such interference. We explored whether their pattern of performance would resemble that previously found for participants with sight (Stavy & Babai, 2008, 2010). Our specific research questions were:

1. Will rates of correct responses in the comparison-of-perimeters task be lower than those for the comparison-of-area task, and in the comparison-of-perimeters task will rates of correct responses in incongruent trials be lower than those for congruent ones?
2. Will differences in reaction times emerge between comparison-of-area and comparison-of-perimeters trials and between congruent and incongruent conditions?
3. Will differences in rates of correct responses and reaction times be observed according to participants' type of blindness, gender, and age performance?

Methods

Participants

The study included 15 persons who volunteered to participate. They were selected based on three criteria: total blindness, no additional disabilities, and ability to state definitions of the object's perimeter and area. They were aged 13 to 66 years ($M = 35$, $SD = 2.5$); 4 were adolescents (13–18 years old); and 11 were adults (23–66 years old). Average years of formal education was 13 years (ranging from 7 to 21 years, $SD = 2.5$). Five were female; 10 were male. All of them were totally blind; 9 were congenitally blind and 6 adventitiously blind. The participants were recruited using snowball sampling and were observed individually. To evaluate their demographics and prior knowledge, all participants were asked to complete a questionnaire (see below). The review board of the university ethics committee approved the research. At the first session, adult participants signed a consent form that was given to them as an audio file or in braille, and parental permission was obtained for the four adolescents.

Instrumentation

The research included one implementation tool and two data-collection tools, which are described in the following sections.

Simulated tasks

Comparison of areas and perimeters was assessed using 36 trials; in each, two geometrical shapes were presented. As depicted in Box 1, the trials were divided into two conditions: congruent, in which the area of one geometrical shape was larger and its perimeter was longer compared to the other geometrical shape (16 trials divided equally into simple and complex ones); and incongruent, in which the area of one geometrical shape was larger and its perimeter was identical to the other geometrical shape (16 trials divided equally into simple and complex ones). Since there was a difference between the areas in all the above trials, four filler trials (two identical geometrical shapes) were added to allow equal responses in the comparison-of-area task.

In the congruent and incongruent trials, the difference between the two geometrical shapes (addition or removal of one small square in one of the shapes) occurred either in the upper, lower, right, or left side of the shapes. In addition, in half of the congruent and incongruent trials, the geometrical shape with the larger area was presented on the right side; while in the other half, it was on the left side. The pairs of images were ordered randomly, and this order was retained in both tasks and across participants. All geometrical shapes were printed in a Swell-Form Graphics Machine, manufactured by Zychem, by using a standard print on Swell-Touch Paper. The sizes of the tactile shapes were designed according to the North American guidelines for tactile graphics for people who are blind (Miller et al., 2012). In the current study, all geometrical shapes were filled with the same texture properties; each shape was based on a 5.2 cm (2.0 in.) × 3.9 cm (1.5 in.) rectangle, and the added or removed square was 1.3 cm (.5 in.) × 1.3 cm (.5 in.) in size.

Questionnaire

The questionnaire aimed to gather demographics and prior knowledge about the participants. It contained seven items concerning the participant's age, age at vision loss, gender, years of formal education, geometrical shape and space curriculum at school, and ability to define perimeter and area.

Excel log

To collect the participants' responses during the research, we used a Microsoft Excel Macro file designed for this study. This file allowed us to collect the duration (in seconds, starting with one second) from the presentation of the trial to the participant's response (reaction time), and the participant's responses for each trial.

Data analysis

To evaluate performance, we used a quantitative analysis methodology with SPSS statistical software. Accuracy was measured by calculating the percentages of correct responses for each participant for each task, condition, and level of complexity. As recommended by Zar (1984), in order to normalize percentages data prior to parametric analysis, arcsin transformation was applied. Skewness and Kurtosis statistics were shown to be in the accepted range (± 2; one trial type had a Kurtosis of 2.5 within 2.2 of *SD*).

Reaction-time analysis included averaging reaction times for correct responses and reaction times for all responses for each participant for each task, condition, and level of complexity. As recommended by Zar (1984), in order to normalize these data, we applied logarithmic transformation. Skewness and Kurtosis statistics were shown to be in the accepted range (± 2).

A repeated-measure general linear model was carried out in order to detect significant differences. Significant interactions were followed by Tukey honest significant differences tests in order to reveal the source of the interaction.

Procedure

The data was collected in two sessions. In the first (which lasted for a duration of about 45 minutes), each participant signed an informed consent form, completed the questionnaire, and then completed one of the two comparison tasks: area or perimeters. In the second (about 35 minutes), the participant performed the other comparison task. Randomly, about half of the participants performed the perimeters task in the first session. Time between sessions was 7 to 10 days. Each comparison task started with a practice stage of two introduction trials and five self-practice trials.

In the comparison-of-area task, the participant was asked to judge whether the larger area was in the right shape or the left shape, or whether the two shapes had an equal area. Similar instructions were given with regard to the comparison-of-perimeters task. The researcher presented each trial, which was explored by a participant who used both hands.

Table 1
Rate of correct responses for the comparison-of-area and the comparison-of-perimeters tasks in congruent and incongruent simple and complex trials.

	% correct responses (*SD*)	
Condition	Area task	Perimeters task
Congruent	80.4 (19.6)	84.6 (20.7)
Simple	80.8 (21.6)	85.0 (18.4)
Complex	80.0 (22.6)	84.2 (26.5)
Incongruent	77.1 (22.2)	18.8 (20.9)
Simple	76.7 (29.1)	25.0 (33.1)
Complex	77.5 (20.7)	12.5 (24.1)

Results

We compared participants' accuracy of responses, reaction times for correct responses, and reaction times for all responses for both comparison-of-area and comparison-of-perimeters tasks in congruent and incongruent simple and complex trials.

Rate of correct responses

Mean accuracy and standard deviation (*SD*), for the comparison-of-area and comparison-of-perimeters tasks, in the two congruity conditions, and two levels of complexity are depicted in Table 1.

A $2 \times 2 \times 2$ repeated-measure ANOVA was conducted on rates-of-success data. The factors in the analysis were task (comparison of area and comparison of perimeters), congruity (congruent and incongruent), and complexity (simple and complex). Significant findings of these analyses are presented in Table 2. We found significant main effects of task and congruity. Overall, the success rate in the comparison-of-area task was higher than in the comparison-of-perimeters task (78.8% vs. 51.7%, respectively) and was higher in congruent than in incongruent trials (82.5% vs. 47.9%, respectively). A significant interaction of task-times congruity shows that although congruity had a significant effect in the comparison-of-perimeters task (84.6% in congruent as compared to 18.8% in incongruent, $p < 0.001$, by Tukey honest significant differences post hoc test), in the comparison-of-area task accuracy was similar for both congruent and incongruent trials. In line with that finding, analysis has shown that accuracy in the comparison-of-perimeters task was significantly lower than it was in the comparison-of-area task only for incongruent trials ($p < 0.001$, by Tukey honest significant differences post hoc test) and not for congruent ones. The results showed that level of complexity had no effect on accuracy of responses.

Table 2
Significant findings observed in a repeated-measure ANOVA performed on success rates.

Variable	*F*	*df*	*p*	η_p^2
Task	19.487	1, 14	= 0.001	0.582
Congruity	58.166	1, 14	< 0.001	0.806
Task × congruity	27.568	1, 14	< 0.001	0.663

Table 3
Reaction time for correct responses for the comparison-of-area and comparison-of-perimeters tasks in congruent and incongruent simple and complex trials.

	RTC in seconds (*SD*)	
Condition	Area task	Perimeters task
Congruent	11.3 (5.9)	10.0 (5.0)
Simple	11.8 (7.5)	9.5 (5.2)
Complex	10.9 (5.0)	10.4 (5.2)
Incongruent	11.5 (5.3)	14.5 (6.4)
Simple	10.4 (4.9)	15.2 (7.9) [9]*
Complex	12.7 (6.3)	15.3 (7.5) [5]*

RTC = reaction time for correct responses.
* Number of participants who provided at least one correct response.

Reaction times in comparison tasks

Mean reaction times for correct responses and *SD* for the comparison-of-area and comparison-of-perimeters tasks, in the two congruity conditions and two levels of complexity, are shown in Table 3.

A $2 \times 2 \times 2$ repeated-measure ANOVA was conducted on reaction times for correct responses data with the factors task (area and perimeter), congruity (congruent and incongruent), and complexity (simple and complex). Since the success rate of incongruent simple and complex trials was very low, only three participants were included in this analysis, since they were the only ones who provided correct responses to all trial types. This analysis revealed an interaction of task times congruity, $F(1, 2) = 13.610$, $\eta_p^2 = 0.872$, with a p value of 0.066. These findings suggest that reaction times for correct responses in the comparison-of-perimeters task may be longer in incongruent than in congruent trials (19.9 seconds vs. 9.9 seconds, respectively, in this subpopulation), while for the comparison-of-area task no difference between incongruent and congruent trials was observed (14.5 seconds vs. 12.4 seconds, respectively, in this subpopulation).

Because of the lower rate of correct responses in the comparison-of-perimeters task in incongruent trials, we further analyzed the reaction times for all responses (correct as well as incorrect ones) for each type of trial in order to explore differences among task, congruity, and complexity factors. A $2 \times 2 \times 2$ repeated-measure ANOVA revealed that for this analysis a main effect of complexity was found, $F(1, 14) = 6.220$, $p = 0.026$, $\eta_p^2 = 0.308$; simple trials were significantly shorter than complex ones (11.0 vs. 12.1 seconds, respectively). Moreover, an interaction of congruity times complexity was observed, $F(1, 14) = 8.743$, $p = 0.010$, $\eta_p^2 = 0.384$. A Tukey honest significant differences post hoc test indicated that incongruent complex trials were significantly longer (13.2 seconds) than incongruent simple, congruent simple, and congruent complex trials (11.0 seconds and $p < 0.005$ for all three).

Differences in rates of correct responses and reaction times

In our sample of participants, nine were congenitally blind and six were adventitiously blind, five were female and 10 were male, and four were adolescents and 11 were adults. To reveal if there were any differences in success rates and reaction times for all responses with respect to type of blindness, gender, or age group, six $2 \times 2 \times 2 \times 2$ repeated-measure ANOVAs were conducted for each analysis (type of blindness, gender, or age group, as between-subject factors); the factors were task, congruity, and complexity. No significant main effects and no interaction with either task, congruity, or complexity were observed for either accuracy of response or reaction time in any response for any of the three factors. Accuracy and reaction times for participants who were congenitally blind were similar to those of participants who were adventitiously blind: 67.0% and 62.5%, respectively, $F(1, 13) = 0.984$, $p = 0.339$, $\eta_p^2 = 0.070$; and 11.3 and 11.9 seconds, respectively, $F(1, 13) = 0.035$, $p = 0.855$, $\eta_p^2 = 0.003$. Accuracy and reaction times for men were similar to those for women: 65.2% and 65.3%, respectively, $F(1, 13) = 0.113$, $p = 0.742$, $\eta_p^2 = 0.009$; and 11.8 and 11.1 seconds, respectively, $F(1, 13) = 0.003$, $p = 0.958$, $\eta_p^2 = 0.000$. Accuracy and reaction times for adolescents were similar to those of adults: 68% and 64%, respectively, $F(1, 13) = 0.901$, $p = 0.360$, $\eta_p^2 = 0.065$; and 10.0 and 12.2 seconds, respectively, $F(1, 13) = 0.356$, $p = 0.561$, $\eta_p^2 = 0.027$.

Discussion

The aim of the current study was to explore, for the first time, whether participants who are blind experience intuitive interference in science and mathematics, focusing on the comparison-of-perimeters task. We studied whether their pattern of performance resembled that for sighted participants. The findings demonstrate that the performance patterns for both accuracy of responses and reaction times for correct responses in both area and perimeters tasks resemble those previously found in participants with sight. These findings indicate that people who are blind experience interference that has been assumed to rely on visual perception.

The findings show that the rate of correct responses is strikingly lower in incongruent than in congruent trials only in the comparison of perimeters, as has been previously found among participants with sight (Stavy & Babai, 2008, 2010). Although the participants in the current study were blind and perceived the information regarding the geometrical shapes tactilely and not visually, it is striking to note that interference of the area variable was evident.

Previous studies of sighted participants have suggested that overcoming interference is an effortful process, associated with longer reaction times (Babai et al., 2006; Stavy & Babai, 2008, 2010). The findings of the current study indicate overall longer reaction times for correct responses (more than five times longer), with broader ranges of reaction times (more than ten times higher *SD*), compared to that of participants with sight (Stavy & Babai, 2008, 2010). In addition, these findings suggest that overcoming the interference is also reflected in a longer reaction time for a correct response, since an interaction of task-times congruity was observed with a *p* value of 0.066. Difference in reaction times for correct responses between complex and simple in incongruent comparison-of-perimeters trials was found previously among participants with sight (Stavy & Babai, 2008). Here, analysis of reaction times for all responses reveals that in both tasks complex trials take longer than do simple trials, especially in the incongruent condition, suggesting that level of complexity leads to more processing in incongruent trials.

Potential limitations of this study may stem from the small number of participants. This did not influence the analysis of accuracy of responses, but it did prevent us from obtaining clear results with regard to reaction times for correct responses and between-subject factors analysis. Findings of the current study indicate that interference is evident in participants who are blind; future research could deepen our understanding regarding this phenomenon. In this study, we did not find differences in performance with respect to type of blindness, gender, or age. It will be valuable to examine if type of blindness has an effect on performance with a larger sample size. It is assumed that similar results will be obtained with older schoolchildren (sixth- to ninth-graders) who are blind; further research is needed to examine if relevant educational interventions (such as a whole class discussion or an explicit warning about possible interference of the variable area) would improve their performance (Babai et al., 2010; Babai, Shalev, & Stavy, 2015). It will also be important to examine this phenomenon in other topics in science and mathematics (Stavy & Tirosh, 2000). In addition, it will be interesting to investigate if brain activations in participants who are blind resemble the pattern of activations found in those with sight (Stavy & Babai, 2010).

Although it was assumed that the interference relies on visual perception, the findings of the current study indicate that people who are blind experience this interference. The study indicates that interference of salient irrelevant variables could be the cause of difficulties of students who are blind. Knowing the source of such mistakes could direct educators in selecting the appropriate learning and instruction methods. In addition, this information should be taken into account by curriculum developers for students with visual impairments. One of the direct implications of the current findings is the additional time needed to access graphical information tactilely compared to visually. Our findings indicate that at least five times more time is needed for participants who are blind to accomplish the task compared to those with sight. The findings that broader ranges of reaction times were measurable in people who are blind (as reflected by larger *SD*s) suggest an additional implication for teachers. They should be aware that skills for exploring perimeters and area are better developed in some students than in others, meaning that some will need more compensatory work, with an effect on the amount of exploration time.

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Orly Lahav, Ph.D., *senior researcher, Department of Mathematics, Science and Technology Education, the Constantiner School of Education, Tel Aviv University, P.O. Box 39040, Tel Aviv 6997801, Israel; e-mail: lahavo@post.tau.ac.il.* ***Reuven Babai Ph.D.,*** *senior researcher, Department of Mathematics, Science and Technology Education, the Constantiner School of Education and the Sagol School of Neuroscience, Tel Aviv University, Tel Aviv, Israel; e-mail: reuvenb@post.tau.ac.il.*

