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DEAF AND HARD OF HEARING STUDENTS' PROBLEM-SOLVING STRATEGIES WITH SIGNED ARITHMETIC STORY PROBLEMS

THE USE OF problem-solving strategies by 59 deaf and hard of hearing children, grades K–3, was investigated. The children were asked to solve 9 arithmetic story problems presented to them in American Sign Language. The researchers found that while the children used the same general types of strategies that are used by hearing children (i.e., modeling, counting, and fact-based strategies), they showed an overwhelming use of counting strategies for all types of problems and at all ages. This difference may have its roots in language or instruction (or in both), and calls attention to the need for conceptual rather than procedural mathematics instruction for deaf and hard of hearing students.

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Problem solving, defined here as the solving of story problems or what have traditionally been called word problems, is a valuable mathematical activity through which mathematics is learned and created, and by which mathematics understanding can be examined and measured. More than the simple use of a procedure, problem solving can be further defined as a sense-making process whereby a synthesis of knowledge and procedures is used as a means to devise a meaningful interpretation of a problem situation—the "story" in the story problem (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Successful problem solvers are inventive, practical, flexible, and reflective in their use of a variety of solution strategies (Baroody & Dowker, 2003; Franke & Carey, 1997; Hegarty, Mayer, & Monk, 1995; Schoenfeld, 1985). Problem solving thus becomes a "window" through which one can see not only what a solver knows about mathematics, but also, through its application, the depth and quality of that knowledge.

In the problem-solving process, the solver must first understand the problem situation. At the most basic level, this requires that the solver have full access to the problem—that is, that the problem be in a language and mode, and at a level of complexity, that match the skill and knowledge of the solver. The solver then devises a plan or solution strategy based on what she or he knows about the situation and about related mathematical concepts and procedures, and, finally, acts on that plan, thinking logically about its outcome. Skilled problem solvers, however, do more than simply translate the syntax of a problem (e.g., key words) directly into symbols and then operate on them—they analyze the semantics of a problem situation and, as part of the solution process, may decide to transform the problem situation into a form in which it will be easier to solve before they represent it mathematically. A *conceptual* rather than procedural understanding of mathematics allows solvers to draw on all of their knowledge and skills as they address the present situation. By contrast, those who have learned a set procedure that is attached (mentally or instructionally) to a specific problem type are stymied by any situation that does not follow the learned structure. These solvers tend to be rigid and unable to succeed at "true problem solving" (Hegarty et al., 1995; Kelly, Lang, & Pagliaro, 2003; Reed, 1999).

Researchers continue to find that many children have not been given the opportunity to develop the conceptual understanding of the mathematics one needs to be a successful problem solver. Instead, many have been taught and rely on rote procedures for solving story problems that are not based on sense making but on "surface-level analysis" (Kelly et al., 2003; Pagliaro, 1998b); such students scan the story problem for numbers and verbal cues (such as key words) that will "tell them what to do" (Garofalo, 1992; Hegarty et al., 1995; Wiest, 2003). Although these procedures may often result in a correct answer (and thus persist), they prevent the solver from making sense of the problem and from truly understanding the relationship between the situation and the mathematics that represents it (Parmar, Cawley, & Frazita, 1996). While these rote methods may provide limited success in school mathematics, a grade of "A" on a test, for example, they build a weak foundation for addressing more complex problems (Baroody & Hume, 1991) and are devastatingly inadequate for meaningful problem solving outside this setting, that is, in the real world.

Research on children's problem solving has shown that most (hearing) children in the early elementary grades (i.e., K–3) attend to the semantics of a story problem (the actions and relations depicted in the story situation) and use "intuitive analytic modeling skills" (Carpenter et al., 1993; De Corte & Verschaffel, 1987) to solve them. That is, young children tend to engage in mathematics in the same way they approach the world in general at this age. As stated by the National Council of Teachers of Mathematics (2000), "Problem solving is natural to young children because the world is new to them, and they exhibit curiosity, intelligence, and flexibility as they face new situations" (p. 116). However, research also has shown that as they get older, children seem to rely more on rote procedures for solving story problems (Garofalo, 1992).

Problem Types and Solution Strategies

Evidence for the sense-making aspect of children's problem solving, and the theoretical basis for the present investigation, is rooted in studies that have looked at the relationship between story problem types and the strategies young children use to solve them (Carpenter & Moser, 1984; De Corte & Verschaffel, 1987). In particular, when arithmetic story problems (i.e., those involving addition, subtraction, multiplication, and division) are categorized on the basis of their semantic structure, there are matches between these structures and how young children model the problem situations as they solve the problems. Described below is the problem type–solution strategy framework that has been used to describe and explain young (hearing) children's strategy development (Carpenter, 1985; Carpenter & Moser, 1984), and that serves as a tool for teachers to understand and successfully build on their students' thinking (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996).

Problem Types

Researchers in mathematics education agree in general on a framework of arithmetic story problem types that target the semantic structure of the problems, not the operations typically used to solve them. Table 1 shows a classification of typical story problems used in the early elementary grades. The classification is based on the problems' semantic structure of the situation (addition and subtraction as joining, separating, comparing, and part-whole; multiplication and division as grouping, partitioning, and measuring) and the position of the unknown value. There are a number of similar iterations of this scheme (e.g., Carpenter et al., 1999; Kintsch & Greeno, 1985). Though there are minor differences between the iterations (some have more categories), they share the critical dimensions of the semantic distinctions and the unknown quantity. For traditionally labeled addition and subtraction problems, the framework distinguishes the problems first by whether or not an action is involved in the situation. In action problems, there is some change in quantity (through either joining or separating) over time. In static or no-action problems, however, all the information is available at the start and no change occurs; the problem is presented more as a moment in time than as an event taking place over time. The second level of distinction is the placement of the unknown. In an action problem, the unknown can be the result, the change, or the start quantity. In Part-Whole problems, which are static problems, the unknown can be placed as the whole or one of the parts, while in Compare problems, the unknown is either the difference between the sets, the compared set, or the referent.

In traditional multiplication and division problems, the framework extends into "grouping," in which the total number altogether is unknown (traditional multiplication), "measurement division," in which the number of groups is unknown, and "partitive division," in which the number of items in each group is unknown (Carpenter et al., 1999).

Studies of young (hearing) children's solutions to such problems, when these children are given access verbally, show that successful problem solvers use strategies that reflect the structure of the problem situations (the stories), not necessarily the operations or procedures that adults and older children may use to find an answer. For example, while an adult may solve the Join Change Unknown problem in Table 1 with a subtraction operation ($11 - 3 = ?$), a young (hearing) child, paying attention to the joining or additive nature of the story (Angie getting more crayons than she already has), might solve the problem as $3 + ? = 11$ (adding on to 3 and stopping at 11). Although this may seem complicated and less efficient to an adult who might have a more mature understanding of mathematical operations, the young problem solver is using a solution that logically follows the storyline, and would be confused by the subtractive operation, as it "does not make sense."

The classification framework also reflects the relative difficulty of the problem types. In general, problem difficulty increases across the table (moving from Result Unknown, to Change/Part Unknown, to Start Unknown), and down, with action problems tending to be easier than no-action problems. The level of infer-

Table 1
Problem Type Framework

Operation	*Problem type*		*Unknown placement*		
Addition and subtraction	ACTION	Join	***Result*** Angie has 3 crayons. Jada gives her 8 more crayons. How many crayons does Angie have now?	***Change*** Angie has 3 crayons. Jada gives her some more crayons. Now Angie has 11 crayons. How many crayons did Jada give her?	***Start*** Angie has some crayons. Jada gives her 8 more crayons. Now Angie has 11 crayons. How many crayons did Angie have to start?
		Separate	***Result*** Angie has 11 crayons. She gives 3 to Jada. How many crayons does Angie have now?	***Change*** Angie has 11 crayons. She gives some to Jada. Now Angie has 8 crayons. How many crayons did Angie give to Jada?	***Start*** Angie has some crayons. She gives 3 to Jada. Now Angie has 8 crayons. How many crayons did Angie have to start?
	NO ACTION	Part-whole	***Whole*** Angie has 3 red crayons and 8 blue crayons. How many crayons does she have?	***Part*** Angie has 11 crayons. Three are red and the rest are blue. How many blue crayons does Angie have?	
		Compare	***Difference*** Angie has 8 crayons. Jada has 3 crayons. How many more crayons does Angie have than Jada?	***Compare quantity*** Jada has 3 crayons. Angie has 5 more crayons than Jada. How many crayons does Angie have?	***Referent*** Angie has 8 crayons. She has 5 more than Jada. How many crayons does Jada have?
Multiplication and division		Grouping	Multiplication (*total*) Angie has 3 packages of crayons. Each package has 6 crayons. How many crayons does Angie have?	Partitive division *(number in each group)* Angie has 18 crayons. She wants to put them into 3 packages with the same number in each package. How many crayons will be in each package?	Measurement division *(number of groups)* Angie has 18 crayons. She can put 6 crayons in each package. How many packages can she make?

ence involved in solving the problem affects its relative difficulty at both levels of distinction (Carpenter et al., 1993; De Corte & Verschaffel, 1987; LeBlanc & Weber-Russell, 1996). For example, a problem can be written so that the action or relation is more or less explicit, or comparisons can be more or less linked to setting up a one-to-one correspondence to solve the problems. Compare the following problem to the Compare Difference Unknown problem in Table 1: There are 8 birds and 3 worms. How many birds will not get a worm? This problem encourages the solver to use a matching solution, while the idea of matching to solve the problem in Table 1 is less explicit and thus requires the solver to infer or make use of information that is implied.

Solution Strategies

Studies in mathematics education of elementary school students show that when given the choice, children make use of three broad strategy types to solve arithmetic story problems such as those in Table 1 (Carpenter et al., 1999): (a) modeling, (b) counting, and (c) fact based. Each of these strategies is briefly described and contrasted below in regard to the implications for the types of problems children who use these strategies tend to be able to solve, and are exemplified in Table 2. A child who uses a modeling strategy would represent each quantity in the problem sequentially and act on the quantities according to the action or relation depicted in the problem, keeping track of only one quantity at a time. Because of this sequential approach, children who directly model problems are generally unable to solve problems with the "start" unknown, and may find problems with no action (e.g., Compare problems) more challenging, as the problem situation itself does not explicitly indicate a solution action.

In contrast to modeling, in which all the quantities and actions are represented, a child using a counting strategy would use one quantity within the problem as a starting point without physically representing it, and use the other quantity only to mark or keep track of the action taken on the first quantity. Counting strategies require some form of double counting to simultaneously keep track of how many counts have been made and when to stop the counting sequence. For example, in the counting strategy described for the Separate Result Unknown problem presented in Table 2, the solver maintains a backward counting sequence (10, 9, 8) while also conducting a forward counting sequence to keep track of the number of counts made (1, 2, 3) either mentally or by means of some physical marker (fingers or manipulatives). The use of counting strategies also involves a conception of number as parts and wholes. This conception supports a view of operations as inverses, so that, for example, a sep-

Table 2
Exemplification of Strategy Types

	Problem	
	Join Change Unknown Angie has 3 crayons. Jada gives her some more crayons. Now Angie has 11 crayons. How many crayons did Jada give her?	Separate Result Unknown Angie has 11 crayons. She gives 3 to Jada. How many crayons does Angie have now?
	Strategy type	
Modeling	The child counts out 3 cubes. S/he continues to add 1 more cube, continuing the count ("4, 5, 6 . . .") until s/he reaches 11. S/he then counts the number of cubes s/he has added and answers "8."	The child counts out 11 cubes, then from that removes 3 cubes. S/he counts the remaining cubes and answers "8."
Counting	The child starts with the number 3 as the beginning of her/his counting sequence and continues to count until the number 11 is reached, keeping track of how many counts s/he has made (e.g., pulling out one cube for each count or making a tally mark for each count). The child counts the number of cubes/marks and answers "8."	The child, starting with 11, marks 3 counts backward in the counting sequence ("10, 9, 8"). S/he answers "8," the last number in the sequence.
Fact: Recall	The child answers "8" because s/he knows the fact that 3 + 8 = 11.	The child answers "8" because s/he knows the fact that 11 – 3 = 8.
Fact: Derived	The child answers "8" because s/he knows that 3 + 7 = 10 and 1 more makes 11, and that 7 + 1 = 8.	The child answers "8' because s/he knows the fact that 10 – 3 = 7, and deduces then that 11 – 3 = 1 more than 7, therefore, 8.

arating problem may be solved by counting up as well as by counting down. An understanding of the inverse relationship of the addition and subtraction operations would then support reversing the action depicted in problems and thus allow one to more easily solve start unknown problems. Thus, children who use counting strategies show more sophisticated counting skills and a conceptual knowledge of number.

There are two types of fact-based strategies that children use, *recall,* where the fact is known, and *derived*, where the needed fact is indirectly deduced from the recall of other known facts. Derived facts are often based on a knowledge of doubles (e.g., "I know that 5 plus 5 equals 10, and 1 more equals 11, so 5 plus 6 equals 11") or facts related to 10 (e.g., "I know that 8 plus 3 equals 11 because I know that 8 plus 2 equals 10 and 1 more equals 11"). Children learn (not memorize) facts through problem solving such that a child who uses modeling or counting may, depending on the number combination in a given problem, use a fact-based strategy. Eventually, children will predominantly use fact-based strategies. Their ability to do so, however, depends on their expanded understanding of number relationships (Carpenter et al., 1999). These strategies, then, can be thought of as developmental, with some "variability in the ages at which children" use them (Carpenter et al., 1999, p. 109). Studies of young hearing children show that they generally move from modeling as they enter and move through kindergarten and first grade to counting during first grade, and finally to fact-based strategies, although not in such a clean and linear fashion. Children may use both modeling and counting strategies for some time (if allowed), often depending on the problem and its action. A few known basic facts such as $1 + 1 = 2$ or $5 + 5 = 10$ may be used early, but the consistent use of recall and derived fact strategies develops over time (Carpenter et al., 1993, 1999).

As described above, these three strategy types differ in the counting skill and number knowledge that support their use. The use of counting strategies requires more sophisticated counting skills than the counting required when children model problems. The strategy types can also be described as moving from less to more abstract, with modeling being the least abstract and fact based being the most. In general, studies show that young hearing children tend to use less abstract strategies before more abstract strategies, depending on their understanding of the given problem, their underlying conceptual knowledge of the mathematics entailed in the strategy, their related comfort or confidence in using the strategy itself, and the efficiency of a particular strategy (Carpenter et al., 1999).

Research on Problem Solving and Strategy Use Among Deaf and Hard of Hearing Students

Studies of similarly aged deaf and hard of hearing students' problem solving have shown similar strategy type use, though occurring at later ages. This suggests the possibility of a delay in these students' problem-solving performance (Chien, 1993; Frostad, 1999; Secada, 1984). Results of these studies, however, were based on presented stimuli that were either stripped of context (i.e., computation problems) or presented in contexts that were potentially limiting. Problems given in picture form, for example, cannot completely illustrate the nature of the problem situation (e.g., showing action), and therefore must assume a shared experience or accurate inference.

The majority of studies involving deaf and hard of hearing students solving problems "in context" have presented the problems in written language (Kidd & Lamb, 1993; Kidd, Madsen, & Lamb, 1993; Serrano Pau, 1995). Not surprisingly, given the great difficulties deaf and hard of hearing students have with understanding the written mode of languages such as English (Allen, 1995; Traxler, 2000), these studies found that the problem-solving performance of deaf and hard of hearing students was significantly behind that of hearing students, with a high positive correlation between success in problem solving and reading proficiency (Kelly & Gaustad, 2007; Kelly, Lang, Mousley, & Davis, 2003; Kelly & Mousley, 2001; Serrano Pau, 1995). Subsequent research, then, has focused primarily on English-language issues and reading strategies, not on the mathematical processes deaf and hard of hearing students use when solving story problems.

An investigation by Frostad and Ahlberg (1999) of contextual problem solving (presented via an animated computer game) by Norwegian deaf and hard of hearing children, ages 6–10 years, revealed three ways in which the children addressed this task: (a) by focusing solely on the numbers in the problem, (b) by approaching the problem as a "take away" circumstance, and (c) by approaching the problem as a part-whole association. While the latter two ways are conceptually oriented and logically appropriate, with the children focusing on the situation and the relationship of the quantities within that situation, the first is not conceptually based, and is therefore less effective and less reliable. Frostad and Ahlberg described this approach as one in which the children "simply picked the available numbers and combined them rather coincidentally according to well-known procedures" (p. 288). Here, the children were not making use of their understanding of the problem situation or of the concepts of mathematics. The procedures and strategies they employed were not used purposefully as conceptually linked tools, but, rather, often indiscriminately, as hopeful solutions to problems.

Investigations of deaf and hard of hearing children's solution strategies when they are presented with computation problems (e.g., 6 + 3 = ?) in sign language (Chien, 1993; Frostad, 1999; Secada, 1984) suggest that the production of the counting string in sign may influence the children's abilities with counting and number. In many signed languages, the counting sequence is a series of systematic rules that follow a simple pattern to a certain point of change. In American Sign Language (ASL), for example, the counting sequence for numbers 1–5 is such that a finger is "added" to a particular handshape: the number 1 being signed with the index finger extended only; 2 having the index finger and the middle finger extended; 3 having the index and middle fingers, as well as the thumb, extended; 4 having all fingers except the thumb extended; and 5 adding the thumb so that all fingers are extended, all with palm facing in. At the sign for 6, however, the base handshape completely changes, with the palm facing out and the thumb touching the pinky finger. The series 7–9 basically starts again, with 7 signed palm out but with the thumb touching the ring finger, 8 with the thumb touching the middle finger, etc. The base handshape then changes again at 10, with the thumb only out and shaken slightly. Other signed languages present numbers similarly, though the major changes may take place at different terms (Frostad, 1999; Leybaert & Van Cutsem, 2002; Nunes & Moreno, 1998). In addition, most signed languages have some level of embedded cardinality in their number signs. In ASL, for example, the signs for numbers 1–5 inherently show the number of elements within the set being represented. The sign for 2 has two fingers raised; the sign for 5 has five fingers raised. It is this ease of moving one finger over, or raising an additional finger for each number, and this inherent cardinality, that researchers have found to possibly influence counting, and thereby, problem solving.

Secada (1984) found that while deaf and hard of hearing children between the ages of 3 and 8 years who were native signers of ASL were more likely than their hearing peers to correctly state the next number in a counting sequence, they were less likely to correctly perform a rote counting exercise, because they misconstructed the signs (i.e., had difficulty physically forming them) or did not (yet) understand the production rules. Likewise, Leybaert and Van Cutsem (2002) found that deaf and hard of hearing 3-to-7-year-old signers of Belgium French Sign Language were more accurate than matched hearing children in counting objects and creating sets, but showed significant delays in sign-counting production, with errors mostly occurring at the point of major change. Nunes and Moreno (1998) found similar confusion with the British Sign Language counting system among young deaf and hard of hearing children as these children attempted to use a signed algorithm (double counting) to solve mathematical problems. Frostad (1999), however, suggests that the counting system in sign language (in this case, Norwegian Sign Language) could have a positive impact on deaf and hard of hearing children's success in mathematics problem solving, acting as an "extra 'aid' to the production system" and affording these children a "richer repertoire of strategies" (p. 146). Specifically, Frostad points to the cardinality found in the signing of lower numbers such as 1–5. He does caution, however, that the ease and efficiency of counting in sign language may facilitate a procedural understanding, as opposed to the more desirable and useful conceptual knowledge of mathematics concepts and skills, if not properly supported with a true understanding of number. For a more complete discussion of language and mathematics, see Pagliaro (2010).

No known study to date has focused on the problem-solving strategies or strategy development of deaf and hard of hearing students when they are given full, linguistic access to contextually based arithmetic story problems. In light of the importance of problem solving in the mathematics curriculum and the historically poor achievement of deaf and hard of hearing students in mathematics, specifically in problem solving (Allen, 1985; Traxler, 2000), the present study investigated the mathematics problem-solving success and strategy use of primary-level (grades K–3) deaf and hard of hearing students when presented with arithmetic (addition, subtraction, multiplication, and division) story problems in ASL. The study was specifically designed to parallel the studies by Carpenter and colleagues outlined above (Carpenter, 1985; Carpenter et al., 1993, 1999, 1989; Carpenter & Mosher, 1984) in sample age, procedure, and content so as to structure a comparison of sorts between results. Four research questions guided the study:

1. What strategies do primary-level deaf and hard of hearing students who are ASL signers use to solve arithmetic story problems when these problems are presented in ASL?
2. What is the progression of strategy use exhibited among primary-level deaf and hard of hearing students who are ASL signers across grade levels when these students are given arithmetic story problems in ASL?
3. How does strategy use differ between more and less successful primary-level deaf and hard of hearing students who are ASL signers when these students are given arithmetic story problems in ASL?
4. How does the strategy use of primary-level deaf and hard of hearing students who are ASL signers differ from the strategy use found among hearing students in parallel studies?

Methodology

To answer the research questions, we designed a descriptive study using structured interviews for data collection, with frequency counts and some parametric statistics for data analysis.

Sample

A total of 232 K–3 children from 9 schools for the deaf and hard of hearing across the United States participated in the present study. The schools were selected to maximize the number of children who would likely have good comprehension of ASL. An initial list of schools were chosen from the reference issue of the *American Annals of the Deaf* on the basis of their use of ASL as a primary language of communication and their ranking by number of students in grades K–3. A letter of interest was sent to the top 22 schools on this list. Of these, 16 (73%) expressed interest in the study. Visits were arranged to 14 of the 16 interested schools. The purpose of the visits was twofold: (a) for the researchers to observe and informally evaluate student sign communication skills so as to maximize the number of student participants, and (b) for the schools' teachers and administrators to understand the responsibilities of participation, including scheduling. Based on information gathered during these visits, 9 schools best met our criteria for inclusion in the sample with respect to communication, diversity (e.g., geographically, racially), and the presence of a significant number of students in grades K–3 (a total of 260 students), and were thus invited to participate in the study. These 9 schools were situated in both urban and rural settings in the northeastern, middle Atlantic, midwestern, and southwestern regions of the United States.

After parent consent was obtained, all participating students were administered three receptivity tasks from the American Sign Language Assessment Instrument (ASLAI; Hoffmeister, 1999). At the time of the present study, there were no standardized instruments to formally measure development of ASL parallel to those instruments for spoken and written English. The ASLAI, however, is such a measure under development at the Center for the Study of Communication and the Deaf at Boston University. Consisting of subtests that assess age-related receptive and expressive skills in ASL, the ASLAI has been used in several large-scale research studies, for example, Schick, J. deVilliers, P. deVilliers, and Hoffmeister (2002), and P. deVilliers, Blackwell, and Hoffmeister (1988) (as cited in Hoffmeister, 2009). The three receptive subtests of the ASLAI used in the present study were Synonyms, Antonyms, and Plurals and Arrangement. These subtests each demonstrate at least "acceptable" internal consistency ratings. Their coefficient alphas are Synonyms .87, Antonyms .81, Plurals and Arrangement .70. Their Guttman split-half coefficients are Synonyms .87, Antonyms .81, Plurals and Arrangement .69 (Hoffmeister, 2009).

Based on the results of these subtests, data on 59 of the 232 children who participated in the study were selected for analysis. These children had scored at or above the mean for deaf children of deaf parents (i.e., native signers) in each of their respective age groups (ages 5, 6, 7, 8, and 9 years) in the larger sample. Thus, the 59 children were at or above the level of the average native signer of their age in ASL comprehension in the present sample. The children ranged in age from 5 to 9 years (n = 11, 14, 15, 15, and 4, respectively), with a mean age of 7.4 years, and spanned grade levels kindergarten through three (n = 9, 16, 18, and 16 respectively). (Children's mathematics levels as indicated by their teachers were highly correlated with grade level, $p < .000$; therefore, grade level was used for all analyses.) The subsample was split nearly equally between male and female (29 and 30, respectively) and spanned the four hearing-loss levels (mild: 3, moderate: 5, severe: 18, profound: 33). Thirty-four children had at least one deaf parent, and 21 had no deaf parents; parental hearing status for 4 children was not known (2) or not reported (2). Seven schools were represented in the subsample; no students from two of the nine schools in the full sample achieved the ASL receptivity score necessary to be included in the present analysis.

Procedure

Children's problem-solving strategies were elicited in a one-on-one interview conducted by a Deaf adult proficient in ASL. During the interview, the children were shown as many as nine addition, subtraction, multiplication, and division story problems. The problems were representative of typical story problems that children in their age group are exposed to in school. The order of the problems provided for what we thought (based on studies of hearing children) would be an increase in difficulty while still giving the children some success along the way. Given the age levels and relative attention span of our sample, as well as what we knew from the studies of hearing children, we felt that these nine problem types provided the best access to mathematics thinking while also giving us a broad spectrum from which to see problem difficulty.

The problems were presented by a Deaf signer in ASL on videotape. The written-English translations of the nine problems along with an English gloss of their ASL versions (see Baker-Shenk & Cokely, 1991, for gloss key) are shown in Figure 1 (numbers in brackets refer to Set B, as explained below). The problems are comparable to those used in the studies by Carpenter and colleagues referenced above involving hearing children (Carpenter et al., 1993; Carpenter & Moser, 1984). A panel of four Deaf, native signers of ASL who had experience with children in schools translated the problems from written English to ASL in such a way that three factors were true of the signed versions: (a) They would be appropriate for primary-level children; (b) they would maintain the mathematical structure of the original problems; (c) they would follow the linguistic rules of ASL. Attention to the mathematical structure of the original problem in translation is critical given the well-defined set of problem types described in the mathematics education literature and results from a study by Ansell and Pagliaro (2001), which showed possible changes in problem type depending on how a story problem was signed. An interpreter certified by the Registry of Interpreters for the Deaf translated the signed problems back to English, confirming the translations. Two versions of each problem that differed only in the number combination used were recorded, Set A having numbers 1–20 and Set B having numbers 1–10. So that none of the children would be encumbered by numbers that were out of their counting range, each child was assigned either Set A or Set B depending on that child's knowledge of number as assessed through a series of related tasks. Rote counting skill was assessed by first having the children count "as high as they could" beginning with 1, then by having them continue to count upward from 14. Cardinal understanding was assessed by having the children count a set of eight cubes which were subsequently covered with a sheet of paper. The children were then asked how many cubes were hidden. If a child could not count up to 20 correctly and efficiently, or could not correctly state that there were eight cubes hidden, that child was given Set B. All children were at least able to rote count up to 10, and thus all were assigned to either Set A ($n = 32$) or Set B ($n = 27$).

Upon completion of the number knowledge tasks, the problem-solving interview was administered. The interviewer explained this task as a series of stories on videotape that contained a question for the child to answer. Throughout, the interviewer periodically reminded the child that she or he could use the available materials (counting cubes of two different colors, paper and markers), their fingers, or "just thinking" in order to answer the questions. The child could watch the videotape of the signed story as many times as she or he wanted. The interviewer did not participate in any way in the child's problem solving. If the child could not remember a detail of the story, the interviewer asked the child if she or he wanted to see the video again. As the child solved each story problem, the interviewer marked the child's strategy choice on a coding sheet. If a child's solution strategy was not obvious to the interviewer, the child was asked to explain his or her actions. The same response by the interviewer was given regardless of whether the child's response was correct or incorrect. The interviews ranged in length from 20 to 45 minutes and were videotaped.

Coding and Analysis

Solutions to each of the nine problems were coded for correctness and strategy use. Coding of the strategies included general type (modeling, counting, fact based), specific strategy within the types, correctness, and viability. A viable strategy is one that is appropriate to solve a specific problem and is free of systematic error, such as those described in Table 2. (An example of a nonviable strategy for the Separate Result Unknown problem in Table 2 would be if a child added the two shown quantities.) If a child changed strategies while solving a particular problem, the last strategy used was coded so as to match the solution strategy with the final answer. Codings were originally done by the interviewer; we subsequently watched the interview videotapes to check and, as necessary, change the codings. Because there was a very strong correlation in the data between strategy viability and correct answer

Figure 1

Problems in English and English Gloss, in Order Given

Problem 1: Join Result Unknown (JRU)

English:

Aaron had 3 [2] cars. Jumal gave him 8 [6] more cars. How many cars does Aaron have altogether?

Gloss:

___t____ ____t___________________

INDEX-lf BOY NAME A-A-R-O-N INDEX-lf HAVE THREE CAR THREE-lf- - - - -

- -

__t_______________________________

INDEX-rt BOY J-U-M-A-L INDEX-rt HAVE EIGHT CAR EIGHT

- -

_____puff ck____________

INDEX-rt rt-GIVE-TO-lf rt-EIGHT-lf

___________________whq_______________________________

ALTOGETHER-lf HOW-MANY CAR INDEX-lf HOW-MANY

Problem 2: Separate Result Unknown (SRU)

English:

There were 11 [6] children on the playground. 7 [4] children went home. How many children were still on the playground?

Gloss:

___t_______ ____________rh-q_____________ ______nod______________

YOU KNOW SCHOOL OUTSIDE PLAY 5↓-CL-ctr 'area' THAT-ctr 5↓-CL-ctr 'area'

_______mm/puff ck________

ELEVEN CHILDREN PLAY (2h)5↓-CLwg-(ctr) 'mixed play'

SEVEN GO-AWAY HOME

____ whq__________

(2h)HOW-MANY CHILDREN STILL (2h)5↓-CLwg-(ctr) 'mixed play' (2h)HOW-MANY

Figure 1
(Continued)

Problem 3: Part Unknown (PU)

English:

Megan has 13 [9] balloons. 8 [6] are red and the rest are blue. How many blue balloons does Megan have?

Gloss:

GIRL INDEX-lf NAME M-E-G-A-N INDEX-lf HAVE THIRTEEN BALLOON+ THIRTEEN

S-CL-lf "grab and hold strings of balloons; look up"---

nod
INDEX-sweep-lf 'up at balloons' SOME RED SOME BLUE

_t__
INDEX++ "counting balloons in air; look up" RED EIGHT

____________whq____
HOW-MANY BLUE INDEX-sweep-lf 'up at balloons' HOW-MANY

Problem 4: Whole Unknown (WU)

English:

There are 4 [2] girls and 9 [7] boys playing soccer. How many children are playing soccer?

Gloss:

_____________t_________ _________mm________
CHILDREN PLAY SOCCER (2h)5-CL↓wg-ctr 'mingle'

__t__
GIRL FOUR-lf/ctr - - - - - - - - -- - - - - - - - - - - - -

__t__ ___nod___
BOY NINE-rt/ctr

________________________whq______________________
HOW-MANY CHILDREN PLAY SOCCER HOW-MANY

Figure 1
(Continued)

Problem 5: Compare Difference Unknown (CDU)

English:

Rachel built a tower 8 [4] blocks high. Pat built a tower 14 [7] blocks high. How much higher is Pat's tower than Rachel's?

Gloss:

```
 _t__                         ___t___                                  ___________mm___________
GIRL  NAME  R-A-C-H-E-L  INDEX-rt   PLAY   (2h)RECT-CL++ "blocks"    [(2h)alt.o→-CL-rt  "build tower"

______mm_________
(2h)B→-CL-rt "tower"
                         EIGHT-rt

          _t__                                                        ______prsd lips____
INDEX-lf  GIRL  NAME  P-A-T    (2h)alt.o→-CL-lf "build tower"    (2h)B→-CL-lf "tower"     FOURTEEN-lf ---

-----------------
EIGHT-rt lower       B:-CL-rt lower ---------------------------------------------------------------------------
                                           _________________________whq______________
                     B:-CL-lf higher       HOW-MANY   higher-1→↕-CL-lower    HOW-MANY
```

Problem 6: Join Change Unknown (JCU)

English:

Bob wants 15 [8] worms. He has found 9 [5] already. How many more worms does he need to find?

Gloss:

```
                         ___t__                     ___nod__
BOY  NAME  B-O-B  INDEX-lf  WANT*  WORM    FIFTEEN-lf higher--------------------------------------------------

-----------------------------------------------
______t_____
FIND  FINISH    NINE-rt lower-----------

------------------------------------------------------------               FIFTEEN-lf higher----------------------
_________________________________________________________whq________________________________
HOW-MANY NEED  B:CL↓ wg "move up to 15"    EQUAL                        HOW-MANY
```

Figure 1
(Continued)

Problem 7: Multiplication (MLT)

English:

Kelly has 3 [3] bags of candy. There are 4 [2] candies in each bag. How many candies does Kelly have?

Gloss:

```
__t_
GIRL   NAME   K-E-L-L-Y  HAVE   THREE   (2h)C-CL 'bag'     (2h)C-CL 'bag'++-rt-ctr-lf

__t___
CANDY    FOUR++-rt-ctr-lf

_____________________________whq____________________
ALTOGETHER    (2h)HOW-MANY  CANDY  (2h)HOW-MANY
```

Problem 8: Partitive Division (Share)

English:

Jake had 12 [6] cookies to sell. He put the cookies into 4 [2] bags with the same number of cookies in each bag. How many cookies were there in each bag?

Gloss:

```
__t_                        __t____                                      _rhq_
BOY   NAME  J-A-K-E INDEX-lf  MAKE  TWELVE  COOKIE   FOR    SELL     TWELVE-lf

___t_                  ______t_____      ______mm_____________________
BAG                    COOKIE+-lf     PUT+++-rt "into each finger of 4-HS"
        FOUR-rt-----------------------------------------------------------------------------------------------------

UNDERSTAND  MUST    SAME-rt "across fingers of 4-HS"
-------------------------------------------------------------------------

______________________________________whq_____________
HOW-MANY   EACH+++-rt "onto fingers of 4-HS"    HOW-MANY
----------------------------------------------------------------------------------
```

Figure 1
(Continued)

Problem 9: Measurement Division (MS)

English:

Paul had 15 [8] caterpillars. He put 3 [2] caterpillars in each jar. How many jars did he put caterpillars in?

Gloss:

__t_
BOY NAME P-A-U-L INDEX-lf HAVE FIFTEEN CATAPILLAR FIFTEEN

__t_ ______mm__________
JAR+ C-CL+++ "jars in a row"
B↑-CL--------------------

_________t____ ______________mm____ ____________________________whq______
CATAPILLAR-lf THREE++ "in row of jars" (2h)HOW-MANY JAR+ (2h)HOW-MANY
B↑-CL-------------------------

Note. The signer was left-handed.

(r = .935, p < .01), and because our focus is on strategy use, success is described here by the viability of the strategies employed by the children. The use of viability provides a more accurate indication of success with the problems, as an error in counting, for example, would not count against a successful strategy. Level of success is thus described by the percentage of participants across and within grade level and problem type who used viable strategies and, within viable strategies, with respect to the three general strategy types (modeling, counting, and fact based). The relative frequency of the different strategy types across problem type was also compared across grade level so that we might investigate the progression or development of strategy type use. Finally, the relative frequency of strategy types was determined for more and less successful problem solvers. Based on the mean number of viable strategies used (4.4), the sample was further split into two groups—those who were relatively more successful at solving the problems (i.e., those who had a viable strategy on more than four of the problems, n = 29) and those who were relatively less successful (i.e., those who had a viable strategy on four or fewer of the problems, n = 30). Both groups included children from all grade levels K–3, and with both deaf and hearing parents. (See Table 3 for demographic information by grade level.)

Table 3
Demographics of the Sample, by Grade Level (N = 59)

	At least one parent Deaf		*Gender*		*Student age (years)*					*Hearing loss*			
Grade level	*Deaf only*	*Deaf or hard of hearing*	*F*	*M*	*5*	*6*	*7*	*8*	*9*	*Mild*	*Moderate*	*Severe*	*Profound*
K	4	4	4	5	9	—	—	—	—	—	1	3	5
1	9	10	4	12	2	12	2	—	—	—	2	7	7
2	12	12	14	4	—	2	10	6	—	2	—	3	13
3	8	11	8	8	—	—	3	9	4	1	2	5	8
Total	33	37	30	29	11	14	15	15	4	3	5	18	33

Results

Strategies

The strategy types used by the primary-level deaf and hard of hearing students to solve arithmetic story problems presented to them in ASL are presented here in the order of problem difficulty as determined by the children's success (the percentage of viable strategies per problem). Problems with a higher percentage of viable strategies were considered to be easier than problems with a lower percentage of viable strategies. (For details on the determination of relative problem difficulty, see Ansell & Pagliaro, 2006.)

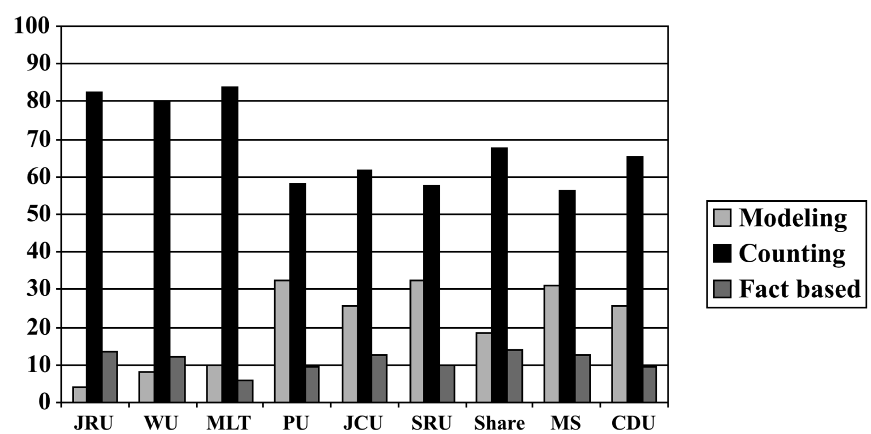
As was the case in prior studies, all three strategy types (modeling, counting, and fact based) were used by the children in the present study. Figure 2 shows the distribution of strategy types by problem difficulty. For each problem, all codable strategies, both viable and nonviable, are represented. Two important pieces of information are revealed in these results. First, on *all* problems, counting strategies were predominately used while fact-based strategies were used minimally, regardless of problem type or difficulty. Second, modeling strategies were more frequently used on more difficult problems—an inverse relationship to the use of counting strategies. For example, for the easiest problem, Join Result Unknown (JRU), about 82% of the strategies used were counting strategies. Modeling strategies accounted for only about 4% of the strategies used. In contrast, the strategy distribution for the more difficult Separate Result Unknown (SRU) problem, while still showing a majority of counting strategies (approximately 58%), included about 33% modeling strategies. The percentage of fact-based strategies was similar for the two problems: 14% and 10%, respectively.

In order to understand the relationship between modeling and problem difficulty, and in particular to determine if the problems were more difficult because of the use of modeling, we calculated the percentage of viable strategies within each strategy type. Figure 3 shows the distribution of viable strategies for modeling and counting only. (The low percentage of fact-based strategies and the coding criteria for viability make a reliability comparison of this strategy type meaningless.) The results show that while counting strategies are still prominent, they are less reliable, that is, less likely to be appropriate or viable for solving the problem. This is especially the case with the increase in problem difficulty as shown by the lower percentage of viable counting strategies in the more difficult problems. For example, in the JRU problem, half the modeling strategies used were viable, while 95% of the counting strategies were viable. This means that for this problem-and likewise for the Whole-Unknown (WU) problem—counting strategies were more reliable, that is, more likely to lead to success, than modeling strategies. In contrast, in the SRU problem, just over 69% of the modeling strategies were viable, as compared to 52% of the counting strategies. Thus, for this more difficult problem, modeling strategies were more reliable. In fact, across all problems (and in each except the two easiest problems, JRU and WU), modeling strategies were more reliable than counting strategies (about 69% of modeling strategies were viable, compared to about 62% of counting strategies), although modeling strategies were used far less often (20%) than counting strategies (about 69%). (It should be noted that ASLAI receptive scores showed a moderate positive correlation with total viable strategies for the nine problems, $r = .641$, $n = 59$, $p = .000$.)

A closer investigation of the counting strategies showed that the majority (65.8%) were "count-on" strategies. When using a count-on strategy, the child would take one of the quantities in the problem and continue the counting sequence by 1's from that point based on the other quantity or quantities in the problem. This strategy is appropriate for problems in which the solution is the sum of

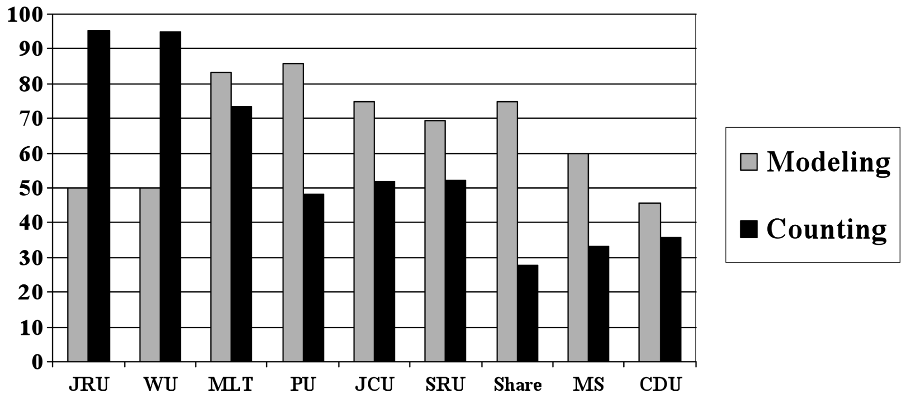
Figure 2

Distribution (by Percentage) of Strategy Type Use (Viable and Nonviable), by Problem Difficulty (Left to Right)



Notes. JRU, Join Result Unknown. WU, Whole Unknown. MLT, Multiplication. PU, Part Unknown. JCU, Join Change Unknown. SRU, Separate Result Unknown. Share, Partitive Division. MS, Measurement Division. CDU, Compare Difference Unknown.

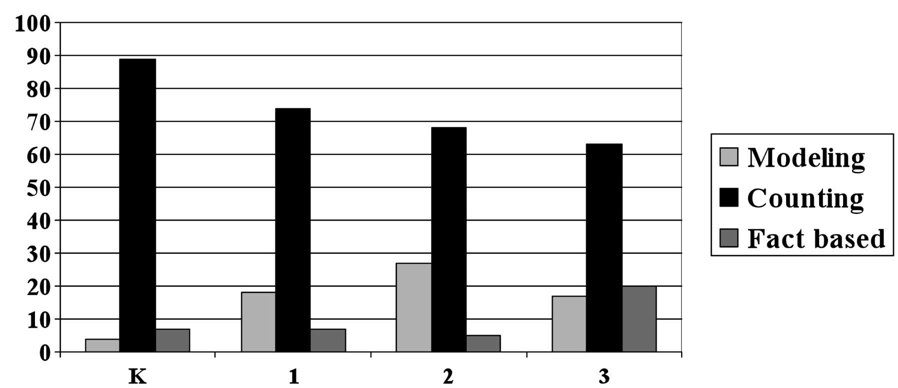
Figure 3
Distribution (by Percentage) of Viable Strategy Type Use, by Problem Difficulty



Notes. JRU, Join Result Unknown. WU, Whole Unknown. MLT, Multiplication. PU, Part Unknown. JCU, Join Change Unknown. SRU, Separate Result Unknown. Share, Partitive Division. MS, Measurement Division. CDU, Compare Difference Unknown.

numbers—for example, JRU, WU, and multiplication (MLT). (It should be noted that for the multiplication problem, a distinction was made between the count-on strategy that students used inappropriately—i.e., simply adding the two numbers given in the problem—and the count-on strategy that is appropriate where one number is repeatedly added on to (by 1's) a set number of times as indicated by the other number). In the present study, however, many children used a count-on strategy regardless of problem type, even when it was inappropriate to do so. For example, children used this strategy appropriately in the JRU problem (see Figure 1 for story problem). They would take the 8 and continue counting up three times (in reference to the other quantity in the problem), stopping at and answering "11." For the SRU problem, however, a count-on strategy is not appropriate; even so, more than a third (34.8%) of the counting strategies used in this problem were count-on. In this case, these children found the sum of the two numbers given in the problem, rather than the difference between them. In fact, count-on strategies made up 45% of the counting strategies used across the problems for which it is not an appropriate strategy.

Figure 4
Distribution (by Percentage) of Strategy Type Use (Viable and Nonviable), by Grade Level



Progression of Strategy Use

The progression of strategy use was determined by analyzing the distribution of strategy types by grade level. As shown in Figure 4, as grade level increased, the use of modeling strategies increased from kindergarten to second grade, while the use of counting strategies decreased by grade level, up to and including third grade. (The decrease in the use of modeling strategies by third graders can be explained by the increase in the use of fact-based strategies among these children.) Chi-square analyses show the distribution of strategy type across grade levels to be statistically different (p = .000) for all grade-level comparisons except the comparison between grades 1 and 2, which was significant at p = .011, even though a similar pattern is evident—that is, counting strategies were used predominately, fact-based strategies were used sparingly, and modeling strategies were used moderately (and especially in more difficult problems).

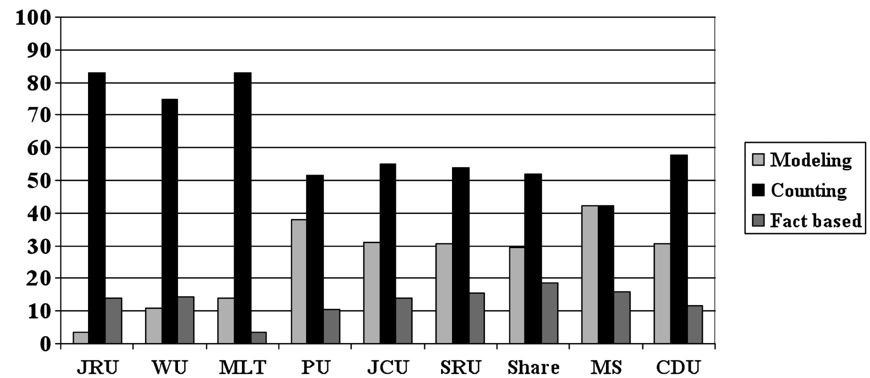
Strategy Use Difference: More Successful and Less Successful Problem Solvers

In order to obtain a more detailed understanding of strategy use among deaf and hard of hearing children, we analyzed the subsample on the basis of their success in problem solving (again, here, "success" is defined by solution viability), trying to discern whether those who were relatively successful were unique in terms of background variable or strategy use. The subsample analyses of background variables showed that the two groups were not significantly different in any way (e.g., parental hearing status, level of hearing loss), with two exceptions: (a) The more successful problem solvers were older than the less successful problem solvers by a mean of 1.1 years (p = .000), though the complete range of ages (5–9 years) was represented in both groups; and (b) the more successful problem solvers scored significantly higher than the less successful problem solvers on the ASL receptivity task (M = 32.38 vs. M = 26.83, respectively; p = .000).

With regard to strategy use, Figures 5 and 6 show that both groups used all three strategy types, with counting strategies again predominating and fact-based strategies used sparingly. (Since success was based on viability, both viable and nonviable strategies were included in this analysis.) A striking difference between the strategy use of these two groups, however, is that the more successful problem solvers made significantly more use of modeling strategies than the less successful problem solvers. Overall, the more successful problem solvers were about twice as likely to use modeling strategies as the less successful problems solvers (see Table 5). This was particularly true on the more difficult problems, as can be seen by comparing Figures 5 and 6, which show the distribution of strategy type use by problem among the more successful and less successful problem solvers, respectively.

Figure 5

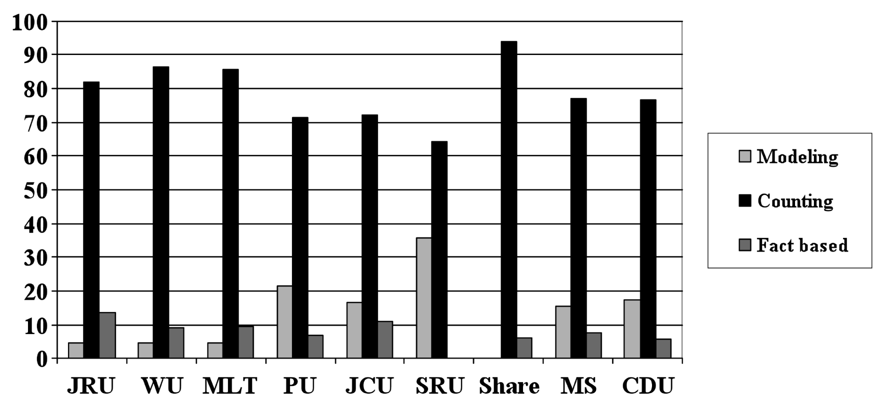
Distribution (by Percentage) of Strategy Type Use (Viable and Nonviable) by More Successful Problem Solvers



Notes. JRU, Join Result Unknown. WU, Whole Unknown. MLT, Multiplication. PU, Part Unknown. JCU, Join Change Unknown. SRU, Separate Result Unknown. Share, Partitive Division. MS, Measurement Division. CDU, Compare Difference Unknown.

Figure 6

Distribution (by Percentage) of Strategy Type Use (Viable and Nonviable) by Less Successful Problem Solvers



Notes. JRU, Join Result Unknown. WU, Whole Unknown. MLT, Multiplication. PU, Part Unknown. JCU, Join Change Unknown. SRU, Separate Result Unknown. Share, Partitive Division. MS, Measurement Division. CDU, Compare Difference Unknown.

Discussion

Much as was found in previous studies (Frostad, 1999; Kelly & Mousley, 2001), our results indicate that deaf and hard of hearing students make use of the same general types of solution strategies as hearing students do. However, our findings raise questions as to whether the data indicate a delay in strategy type use among these students, as concluded in previous studies, or show evidence of the existence of an entirely different pattern of development. A delay implies that the same pattern of development exists for deaf and hard of hearing students as for hearing students, but at a later time. Thus, if the developmental sequence for strategy types used by deaf and hard of hearing children were similar to that used by hearing children, then there would be an increase in the more abstract strategy types, with an increase in grade level; that is, the younger students would use more modeling strategies while the older students would use more counting and fact-based strategies. Further, if there existed a delay in strategy type use among deaf and hard of hearing students, the data should show a late onset of strategy type use, with the more abstract strategy types (counting and fact-based) appearing later, at the second- or even third-grade level, and perhaps not at all in the earlier grades (kindergarten and first grade). As shown in Table 4, the data on strategy type use by grade level do not support either case. Results from the present study show that deaf and hard of hearing children who are users of ASL not only use an overwhelming number of counting strategies for all types of problems, but also at all ages, including very early on.

We originally expected that, like hearing children, the deaf and hard of hearing children would use more concrete strategies (e.g., modeling) at the younger grades and shift to more abstract strategies (e.g., counting) as they progressed through the grade levels. But this was not the case: Counting strategies, while representing the majority of strategies used at each grade level, decreased in use as grade level increased. Conversely, the use of modeling strategies increased with grade level. Fact-based strategies were used minimally (less than 10%) across grades K–2, reaching an appreciable proportion (20%) in grade 3. Thus, while studies of young hearing children show them to have a general progression of strategy type use from modeling to counting to fact based (Carpenter, 1985), data from the present study suggest that deaf and hard of hearing children who are ASL users may possess a different progression in which counting strategies are among the first to occur and continue throughout the lower elementary grades. The progression then would perhaps be something like this: counting to modeling/counting to fact based/counting.

We suggest two hypotheses to explain such a difference. First, it could be the result of a linguistic influence. Aspects of ASL, and signing in general, such as the ease of use of the counting system, the benefits of its manual nature, and its inherent cardinality, may naturally lead children to counting strategies. As Frostad (1999) suggests, the ease of use of the counting system may be related to counting strategies being used as the default strategy by these deaf and hard of hearing children.

Table 4
Frequency of Strategy Type Use, by Grade Level

Strategy type	*Distribution by grade level (% in parentheses)* *K*	*1*	*2*	*3*
Modeling	1 (3.7)	16 (17.8)	40 (27.0)	23 (17.2)
Counting	24 (88.9)	67 (74.4)	100 (67.6)	84 (62.7)
Fact based	2 (7.4)	7 (7.8)	8 (5.4)	27 (20.1)
Total	27 (100.0)	90 (100.0)	148 (100.0)	134 (100.0)

Table 5
Distribution (by Percentage) of Strategy Type Use by More Successful and Less Successful Problem Solvers

Problem solvers	*Strategy type* *Modeling*	*Counting*	*Fact based*
More successful	24.8	62.4	12.8
Less successful	12.1	79.6	8.3

In addition, because ASL is manual, the language allows children to hold a separate number on each hand; thus, children can simultaneously manipulate two separate counting sequences to aid them in their solution strategies. For example, in Problem 2 (see Figure 1), where the situation involved taking 7 from 11, some of the children who used a double counting strategy started with ELEVEN on one hand and SEVEN on the other, then proceeded to count back by one step on each hand until the hand that originally indicated SEVEN had counted down to zero (ELEVEN/SEVEN, TEN/SIX, NINE/FIVE, etc.). They then looked to the other hand (initially ELEVEN, now FOUR) for the remaining quantity and the answer. By contrast, children using an oral language, such as English, would need to remember their place in each sequence as they alternated between the two, counting up or down. For those using ASL, their place in the sequence would always be present, and the counting fluency (that Frostad, 1999, observed) may have helped them to maintain the sequence, making their procedure more efficient and perhaps more effective.

Finally, the children in the present study made use of the cardinality aspect in some of the number signs in ASL. In these cases, children "toggled" between using the number sign as a label for the set and using the raised fingers within the sign as manipulatives or elements of the set. For example, in Problem 1 (see Figure 1), where the sum of the two quantities (8 and 3) is unknown, most children put out the sign EIGHT on one hand and the sign THREE on the other. They then made use of the three raised fingers on the THREE sign as markers to keep track of the number of counts needed to increment the sequence on the other hand (9, 10, 11). The use of this aspect was seen across problems, particularly when one of the given quantities included the numbers 1–5 for which the cardinality of the number is explicit in its sign. For those problems in which both quantities were greater than 5, some children separated one of the quantities (whole) into two smaller quantities (parts) to facilitate the use of toggling. For example, in Problem 2 (see Figure 1), some children split the quantity 7 into parts, first putting out the sign FOUR and counting back from 11 on each finger, or toggling (10, 9, 8, 7), then putting out the sign THREE and continuing to count back, marking the count on each finger (6, 5, 4). Use of the cardinality of the number signs ensured the correct number of markers and gave the children a more efficient procedure with less chance of error or miscount. Indeed, it may be that, at least at the earlier grade levels, using the cardinality found within the number system of ASL may actually form a hybrid "modeling/counting" strategy of sorts whereby the fingers can represent the number as well as markers or counters, thus reducing the risk of error.

We caution those who would interpret this hypothesis to mean that they should teach these aspects of language as procedures for solving a problem (whether a computation or a story problem), however. If the child does not have a conceptual understanding of number (that is, does not truly understand the concept, for example, that the number of fingers raised actually represents the number of elements in the set for signs 1–5, but not for the signs 6–9), the child will erroneously use the procedure regardless of cardinality within the number sign. We did see some evidence of this in our data. One boy, for example, wished to add 6 and 9 for Problem 3, Set B (see Figure 1). Upon holding up the signs for each number on his hands, he proceeded to count on from 6 the number of fingers raised in the sign for 9 (i.e., 3), getting 9 as the answer. The child was satisfied with his answer, giving no indication that he had done anything incorrect, nor that his answer made no sense mathematically (the sum of two nonzero numbers would be the value of one of the numbers) or in the context of the problem situation. He seemed simply to follow a procedure—a rule—that he had learned. His actions indicated no distinction between the number sign (label) and what it signified (the quantity). This behavior indicates a procedural orientation to problem solving similar to those described in the studies cited above (Frostad & Ahlberg, 1999; Garofalo, 1992; Hegarty et al., 1995; Parmar et al., 1996; Wiest, 2003).

Our second hypothesis to account for the overwhelming and early use of counting strategies relates to instruction. Studies reveal a very traditional approach to mathematics instruction within the education of deaf and hard of hearing students that includes emphasis on direct instruction, memorization, and practice exercises over conceptually based learning, in which true problem solving fosters both the development and use of higher-order cognitive functions and critical thinking skills (Kelly et al., 2003; Pagliaro, 1998b, 2010; Pagliaro & Ansell, 2002; Pagliaro & Kritzer, 2005). It may be, then, that educators are encouraging, if not outwardly teaching, the use of procedures for all mathematics (computation as well as problem solving) without first establishing a conceptual understanding of mathematics concepts such as number, as Frostad (1999) cautioned against. Teachers, for example, may have students make use of the cardinality in ASL for numbers 1–5 as a quick and easy means of adding or subtracting. Because these adults understand the separation between the sign or label for the number and the elements in the set represented by the raised fingers, and can move easily between the two, they may assume that their students understand in the same way. However, the students may not, seeing only the labels and a procedure that says "take one sign and continue to count (up or down) on the extended fingers of the other hand"; hence, they likely will continue to use the procedure on quantities, and with signs that do not show cardinality.

In addition, research has shown that most teachers within the education of deaf and hard of hearing students have not been adequately prepared in mathematics and/or mathematics education (Dietz, 1995; Kelly et al., 2003; Kluwin & Moores, 1985; Pagliaro, 1998a, 2010; Pagliaro & Kritzer, 2005). Therefore, even if they are otherwise high-quality teachers, they may not be aware of the need for true problem solving nor have the confidence or knowledge base necessary to guide students in their development of mathematics concepts and problem solving. With the best intentions, they may believe that they are advancing students' achievement by encouraging the use of counting strategies such as, for example, the count-on strategy, when in reality this may do more harm than good, as its effect may be to skip the benefits of the more concrete modeling and limit the choice of strategies the child has when faced with a problem to solve.

Finally, the fact that deaf and hard of hearing students already show a gap in their mathematics performance in the early elementary grades (Kritzer, 2009; Traxler, 2000) may worsen the bias in favor of a procedural approach to teaching mathematics. Teachers facing the pressure to move students through the curriculum and "close the gap" may not feel that they have the time to allow students to explore various solution strategies on their own as recommended by the National Council of Teachers of Mathematics (2000) and the *National Action Plan for Mathematics Education Reform for the Deaf* (Dietz, 1995), and as suggested in the studies by Carpenter and colleagues (Carpenter & Moser, 1984; Carpenter et al., 1993, 1999, 1989).

We propose that all children, especially those who are deaf or hard of hearing, be taught in a way that develops a conceptual understanding of mathematics and of problem solving in particular. Students should discover and work with various strategies, particularly more concrete strategies (modeling), in order to develop this understanding—even if that modeling comes in the form of the sign counting system—making *appropriate* use of cardinality, for example. This suggestion is supported in the comparison of more and less successful problem solvers. While both groups still made extensive use of counting strategies, the more successful problem solvers varied their strategy use, choosing, for example, to use a modeling strategy on more difficult problems. In fact, one child explained that she was "not good at counting back" as she reached for the available cubes and proceeded to solve the problem by modeling. This child was very proficient at the count-on strategy but knew her limitations with counting backward; thus, not feeling confident with this counting strategy, she chose the more reliable (for her) modeling strategy. In contrast, many of the less successful problem solvers, when faced with a more difficult problem, tended to forge ahead with the count-on strategy regardless of its appropriateness. These students did not seem to recognize the value of modeling in problem solving, an approach that research suggests leads to viable solutions across a broad range of problems for hearing children as young as kindergarten age (Carpenter et al., 1993).

We suggest that teachers use care not to push students to bypass more concrete strategies for more abstract ones and/or limit students by imposing one strategy or strategy type, but, rather, allow students the opportunity and the time to experiment with various strategies and strategy types so that when faced with a difficult problem, they have a "menu" of approaches from which to choose the more appropriate and reliable. While this may not be a short-term solution to the problem of time pressure, in the long term, conceptual understanding and the ability to solve problems successfully may prove to be the factor that lessens or even eliminates the gap in achievement.

Recommendations for Further Study

Given the small number of studies in this area and their various limitations, there is critical need for further investigations of problem solving among deaf and hard of hearing students. This includes studies of the linguistic influence of a visual language on mathematics instruction and learning, and of the instructional practices that lead to problem-solving success. It is important that future research go beyond the narrow population considered in the present study as well. Participants in our study were from schools for deaf and hard of hearing students, which enroll approximately 20% of these students in the United States (Gallaudet Research Institute, 2008). In addition, we focused on gathering students from only those schools that used ASL as the primary language of instruction. While this may limit the study's implications, it was necessary in order to allow these students full access to the problems with no language barrier. Additional studies, however, are needed to investigate the problem-solving strategies of primary-level deaf and hard of hearing students who are in other school settings and do not primarily use ASL, as results may vary.

Finally, it is imperative that professionals focus on all deaf and hard of hearing children as unique learners, documenting what they do and how they do it. Possible differences in a deaf or hard of hearing child's experiences, language, and/or cognitive organization may dictate a differentiated approach to instruction that does not follow the progression of a traditional, general education curriculum. Knowing how deaf and hard of hearing students solve problems, regardless of their communication preferences or educational placement, provides insight into their understanding of mathematics concepts as well as their thought processes. This, in turn, may have implications for other academic areas.

Note

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