

Collective but Diverse: Preschool Teachers Networking to Develop Toddler Mathematics

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Received: 3 March 2017 / Accepted: 20 September 2017
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This article focuses on professional development among teachers within a Swedish national network on toddler mathematics education. The activities within this network can be understood as participant-oriented collective professional development based on a knowledge of practice approach. An inventory of toddler mathematics was performed within this network through observations of authentic mathematical activities. The inventory made diversity in the participating teachers' perceptions of toddler mathematics visible; different activities were labelled similarly and opposite, similar activities were labelled differently. The inventory made clear the need for theoretical influence and discussions in practical inquiry. The addition of theoretical notions made previous invisible diversity among the network members visible, and strengthened the opportunities for further collective professional development. Diversity becoming visible contributed to the knowledge of practice approach, as well as the development of the professional language of the participating teachers.

Keywords: preschool teachers • knowledge of practice approach • collective professional development • toddler mathematics • teacher knowledge

Introduction

This article concerns collective professional development in relation to the idea of teaching mathematics to toddlers, as it is perceived and developed within a Swedish national network on toddler mathematics education. The focus is directed toward understanding professional development among teachers within this network; it was shown to be collective but at the same time diverse. A common interest and networking over a longer period of time would presumably result in a common concept of mathematics education for the youngest children. Our study shows indeed a collective concept of what mathematics include in early education, but the striking result is the wide diversity in the participating teachers' perception of toddler mathematics. However, this diversity does not necessarily imply a failure; rather, it opens up possibilities to further strengthen collective professional development.

Background

The importance of high quality education in mathematics for children under the age of six is emphasised internationally, as early mathematical development has been shown to be important for later mathematical achievement (Claessens, Duncan, & Engel, 2006; Simpson & Linder, 2014). The number of very young children (aged one to three) who attend preschool is increasing in the Nordic countries (Reikerås, Løge, & Knivsberg, 2012), and now close to 90% of all Swedish two-year-olds attend preschool (National Agency for Education, 2014). Sweden has a long tradition of early childhood education and care, where preschool has been part of the formal educational system since 1998. Also, Swedish preschools have a long tradition of social pedagogy (Bennett & Tayler, 2006) where care, socialisation and learning constitute a coherent whole. However, even though the youngest children attending preschool are increasing in number in this formal education system, children younger than three years old are seldom included in studies on mathematical development (Reikerås, Løge, & Knivsberg, 2012), which is why knowledge in the area is limited.

In addition to the increasing demand for preschool teachers due to increased enrolment among the youngest children, there has been a corresponding increased emphasis on mathematics education in preschools (see revisions made to the national curriculum for preschool in Sweden, National Agency for Education, 2011). As part of this movement a network was established in 2010 for teachers, researchers and teacher-educators working with toddler mathematics. There are approximately 30 active members of the network who meet twice a year. The authors of this article have been part of this group for several years. It is the active preschool teachers of the network that are the focus of our attention in this article. Additionally, because they work with the youngest children in preschool, these teachers accordingly have a special interest in the teaching of mathematics.

The activities in this network can be understood as collective professional development based on the interest and initiative of the network members. There is no network leader, but instead the content of the meetings is collectively decided. Literature is read, and experiences with toddler mathematics are shared and discussed at the meetings. In addition to meetings, the participants have jointly produced a book on toddler mathematics (Franzen & Björklund, 2015) and participated in a national mathematics education conference.

At the spring meeting in 2016, those members actively working in child groups were asked by the authors of this article to observe and document activities in which toddlers encounter mathematics. The purpose was to make an inventory of toddler mathematics in preschool. Even though the national preschool curriculum includes several mathematics-related goals, they are not goals for children to attain, but rather they provide direction for content and activities. Further, based on the curriculum, each preschool has freedom to choose the approaches most appropriate for its own setting. Thus, the aim of the inventory was to discover the kinds of activities in which toddlers were observed encountering mathematics in preschool, and to determine what mathematical content they experienced in these activities.

The documentations were collected and analysed during autumn 2016. During the analysis, it became obvious that what had appeared as a relatively commonly held view on toddler mathematics was really not that unanimous. The network members seemed to have quite different perceptions of toddler mathematics, or at least they used different expressions when labelling and describing toddler mathematics. In this article, we will elaborate on this diversity in relation to collective professional development; of specific interest are:

- what differences can be found in these preschool teachers' perceptions of toddler mathematics? and

- what does this diversity imply for collective professional development within similar boundless organizations of teachers?

The article starts with a section focused on professional development. After this, the methodology, including theoretical foundation, of the study is presented. Finally, the results, discussion and implications are presented.

Professional Development

The two main types of pedagogues working in Swedish preschools are preschool teachers and child-minders. A child-minder possesses an upper secondary school education, while to become a preschool teacher one must complete a three-and-a-half-year university programme in preschool teacher education. The preschool teacher profession is mostly characterised as “educational generalists”, without specialisation in any particular subject, but preschool teachers educated after 2001 have studied mathematics education. Studies have shown that there is positive impact on children’s’ mathematical development when teachers working with the youngest children attend professional development geared towards mathematics content and pedagogy (Simpson & Linder, 2014).

Professional development is an umbrella term for several different types of activities and settings of which professional networks are one (Sowder, 2007; Avalos, 2011; Simpson & Linder, 2014). The word *professional* has been used in relation to education in Sweden since the mid-1980s to clarify the difference between political issues and professional issues. Previously, teachers had been the implementers of political decisions, but when the school system was decentralized teachers could choose and argue for different teaching approaches (Carlgren, 1994). Even though the Swedish curriculum provides direction for content and activities in preschool, there are no central regulations for how preschool teachers are to arrange activities, and there are no goals for children to attain at a certain age or level. With only vague descriptions of content and without well-defined methods, the teacher plays a central role in the enactment of curriculum, which can be seen as a “forced autonomy” (Skott, 2004).

Professional development of teachers has increased, maybe as a result of studies showing how instructional improvement and increased student achievement are dependent on the professional development of teachers (Sowder, 2007; Avalos, 2011; Simpson & Linder, 2014). However, there are different views on how such professional development is best organized. Regardless of organisation, professional development has in common that it focuses on teacher learning and on transforming that learning into a benefit for student learning (Avalos, 2011). Three different ways of organising professional development include *knowledge for practice*, *knowledge in practice* and *knowledge of practice* (Cochran-Smith & Lytle, 1999). Knowledge for practice is when a learner, through formal professional development, is to acquire knowledge already known by others. An example of this kind of professional development is that of university studies. Knowledge in practice is when teaching is seen as a trade to learn through teaching. Then, professional development becomes an issue of more teaching and reflecting on one’s own teaching. Knowledge of practice refers to when teachers use their own school and their own classroom to investigate learning, knowledge and theories. In Sweden, this latter approach has increased in recent years. This increase is based in research showing that if groups of teachers work together over time to develop their educational practice, then there is greater chance of impact on classroom practice. Such professional development programs may involve reflection in and on practice, as well as reflection on external sources (Clarke & Hollingsworth, 2002). This increase is also visible in research on professional development, where significant scholarship has occurred in regard to teacher co-learning as well as cooperation between teachers and researchers (Avalos, 2011).



The network on toddler mathematics can be understood as collective professional development based on a knowledge of practice approach. More specifically, the network can be considered as a knowledge of practice approach labelled as practical inquiry (Sowder, 2007). In practical inquiry, no external visions are to be implemented. Instead, teachers reflect on their own practice with a focus on questions they raise themselves. Practical inquiry, as in having no external vision to be implemented, is in line with the Swedish curriculum, providing no central regulations for how to arrange activities and no goals for children to attain at a certain age or level. Based on these conditions, the network members have worked together over time to develop their educational understanding and practice of toddler mathematics. The primary activity at the network meetings is exchange and discussion of experiences where the members show and discuss activities from their preschools. Even though some of the members are researchers, their role is not to educate or facilitate but instead all members have a joint responsibility in the development of toddler mathematics.

Research overviews by Sowder (2007) and Avalos (2011) identify principles for successful professional development, several of which apply to the network on toddler mathematics; it is the teachers who have identified their learning needs and the process to be used; the network activities are related to the members' teaching; the activities relate to individual needs but are organised around collective learning; the activities are continuous and ongoing; there is a combination of tools for learning and reflection, and as the members are from different preschools the network also includes external sources. One challenge of a knowledge of practice approach is, however, that teachers sometimes re-interpreted new guidelines and their own teaching in a way that enable them to avoid changes, confirming earlier established practice (Morgan, 2009). Such re-interpretations may result in seemingly large consensus among participants, even if they actually teach very differently. One scenario is teachers using different words to describe "the same kind" of mathematics teaching, another scenario is teachers using similar words to describe "different kinds" of mathematics teaching. These two scenarios will be used further on when presenting differences found in the network members' perceptions of toddler mathematics.

Methodology

At the network meeting in spring 2016, the members were asked to document activities in which toddlers encountered mathematics. The intention of this inventory was to create an empirical basis for developing the knowledge within the network of *the concept of* mathematics content for learning. Through authentic observations, we would find out in what kinds of activities toddlers were observed encountering mathematics in preschool, and what mathematical content they experienced in these activities. Thus, the question to be investigated was not *if* these teachers taught mathematics, but instead *what* mathematics toddlers were offered to learn and *how* this learning of mathematics was orchestrated. At the network meeting the inventory was presented verbally, and afterwards the information was also e-mailed to the network-members along with a form including questions and a schema.

Empirical Data

For the autumn 2016 meeting, the network members who wanted to (participation was of course voluntary) were to document activities in which toddlers encountered mathematics. This is the type of selection Flyvberg (2006) call information-oriented but deviant, in that the informants are selected due to their outspoken interest in mathematics education, not randomly selected. The observations were documented on a pre-prepared form where the "mathematical activities" were to be described. This pre-prepared form can be considered a questionnaire, using both open-

ended and close-ended questions with the aim of identifying both consistency and meaning (Patton, 2002). The first question was to “describe the situation”. This question was supported by the help-questions: Who was present? What mathematical content? What happened? How did the situation start? Was the situation spontaneous or planned? If the situation was planned, on which grounds? After this, a question followed about the teacher’s as well as the children’s actions in the situation. What did the teacher do and say? What did the child/children do and say? One question was also asked about how common a situation like the one described was for this/these child/children. After describing the activity, the activity was to be labelled based on Bernstein’s (1999) notions of *vertical and horizontal discourses* and Claesson, Engel, and Curran’s (2014) notions of *basic and advanced content* (to be further explained in the next section). These notions were introduced and discussed at the network meeting in spring 2016 and described in writing in the above-mentioned e-mail sent to all members.

Vertical and Horizontal Discourse versus Advanced and Basic Content

In order to create a comprehensive inventory of the mathematics toddlers encounter in preschool, we needed a theoretical point of departure that would help us discern and frame the observed activities. Since our interest was directed towards both the questions of what and how mathematics is made a content for learning in preschool, no single model could be found. Instead, we were inspired by discourses used by Claesson et al. and Bernstein: The notions of *vertical and horizontal discourses* (Bernstein, 1999), together with Claesson et al.’s (2014) notions of *basic and advanced content*, were used to label the documented activities. Bernstein’s notions were used to label *how* mathematics was made a content for learning, while Claesson’s notions were used to label *what* – that is, the content for learning.

Bernstein used vertical and horizontal discourse to distinguish between different kinds of knowledge. A vertical discourse is characterised by its coherence of content, hierarchically interconnected procedures, specialised language, systematically organised activities and by a focus on general knowledge. A horizontal discourse is characterised by its location within communities, high relevance to the situation, every-day language, segmentally organised, and maximised encounters with persons and habits. These two discourses were used to distinguish between activities, using everyday situations as a starting point and activities having mathematical concepts as a starting point. Based on everyday situations or mathematical concepts being the starting point, vertical and horizontal discourse can be understood as two extremes. In the extreme horizontal discourse, it is sufficient that mathematics content is part of everyday activities and routines, with no need to make it explicit for the children. In the extreme vertical discourse, mathematics is the starting point with no need for applications. Thus, every day is the starting point in the horizontal discourse and mathematics is the starting point in the vertical discourse. If these two extremes are visualised on an axis (Figure 1), there will be a gradual shift on the line, and somewhere in the middle there is a shift concerning everyday life or mathematics being the starting point for the design of preschool mathematics education.

As mentioned, the Swedish preschool curriculum includes several mathematics-related goals. Examples of these are that preschool should strive to ensure that each child “develop their understanding of space, shapes, location and direction, and the basic properties of sets, quantity, order and number concepts, also for measurement, time and change” (National Agency for Education, 2011, p. 10). However, these are not goals for children to attain, and there are no explanations of which shapes, which quantities or which number concepts children are to develop their understanding of. Thus, Claesson et al.’s (2014) notions of basic and advanced mathematics were used to label the level of difficulty of the mathematics content in the documented activities. In Claesson et al.’s (2014) study, the mathematical content is characterised as basic or advanced depending on the skills and knowledge of the majority of the children in a

particular group. If the majority has already mastered a specific content, it is labelled as basic, even though there might be children not yet familiar with the content. Advanced mathematics is, on the other hand, new content for the majority of the children in the group. In the inventory presented here, the notions of basic and advanced mathematics were not connected to groups of children but to the children involved in each activity. In the instructions to the preschool teachers, this was expressed as “what basic or advanced content must be related to each child, and here you, who know the children, are the expert.”

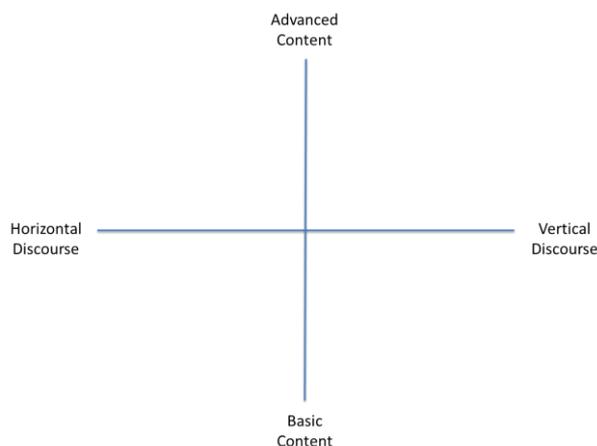


Figure 1. The figure visualises vertical and horizontal discourse, as well as basic and advanced content extremes. On the axis there will be a gradual shift on the line. Somewhere in the middle, there is a shift concerning everyday life or mathematics being the starting point for the design of preschool mathematics education as well as for the content in the activities being basic or advanced in relation to the child/children involved.

Figure 1, with the four notions, horizontal and vertical discourse, basic and advanced content, were used by the teachers to label the documentation in the inventory. This gave us the opportunity to find out in what kinds of activities toddlers encountered mathematics in preschool, and to find out what kind of mathematics content they experienced in these situations. The two extremes of basic and advanced content can be understood as differences in the mathematics content for learning, while the two extremes of horizontal and vertical discourse can be understood as differences in orchestration or how mathematics is made part of preschool education.

Analysis

The results presented in the next section are based on an analysis of 39 documentations that were gathered within the network. One of these documentations was eliminated since the activity described had not been labelled according to the guidelines. The other 38 documentations were analysed using content analysis. Through a content analysis we aimed to find patterns and themes in seemingly random information with the purpose of understanding the message from the author of the documentation within the frames and context of what the document allows (Bryman, 2008; Patton, 2002). The close-ended questions were categorised deductively, while the open-ended questions were analysed inductively, and together these analyses were compiled in a schema. First, each documentation was assigned a number in this schema. It was noted whether the activity was a planned activity or a spontaneous activity, based on the teacher’s self-reported

categorisation. After this, we gave each activity a very short description, making it possible to sort the activities; for example, *reading book*, *circle time* or *sand-play*. Then, we noted which mathematical content was part of the activity; for example, *numbers*, *space* or *measurement*. Finally, we noted whether the activity was labelled within a horizontal or vertical discourse, as well as whether it contained advanced or basic mathematics.

As mentioned, the intention with the inventory was initially not to compare, but as we made the schema, we noticed patterns of differences that we wanted to investigate more thoroughly. Based on these insights, we started to compare situations that were labelled similarly. For example, we compared activities labelled within a vertical discourse containing advanced mathematics. In what ways were these activities similar? Further, we compared activities that were labelled differently. In what ways were they dissimilar? We found that different words sometimes were used to describe “the same kind” of mathematics teaching, and that same words sometimes were used to describe a “different kind” of mathematics teaching. Examples of this will be presented in the next section, and after that the diversity will be discussed in relation to collective professional development.

What Differences can be found in these Preschool Teachers’ Perceptions of Toddler Mathematics?

As mentioned, the aim with the inventory was to find out in what kinds of activities toddlers are observed encountering mathematics in preschool and to find out what mathematical contents they experienced in these activities. In Figure 2 below, each documentation is illustrated by the number it was assigned in the above-described schema.

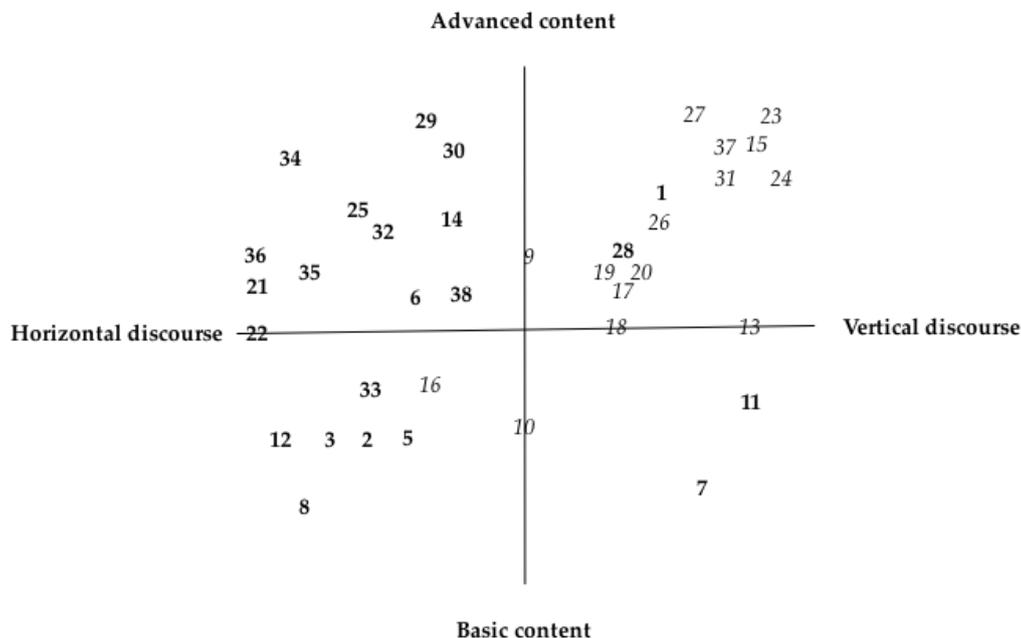


Figure 2. Overview of the documentations based on how they were labelled by the teachers regarding vertical and horizontal discourses, as well as basic and advanced content. Italic numbers imply planned activities, while bold numbers imply spontaneous activities.

When analysing the documentations, it was noted whether an activity was labelled by the teachers as a planned activity or a spontaneous activity. In Figure 2, the italic numbers are planned activities, while the bold numbers are spontaneous activities. There is an even distribution between planned and spontaneous activities (17 planned and 20 spontaneous). However, these are not evenly distributed in the figure. As can be seen, there are more planned activities in the vertical/advanced quadrant and almost only spontaneous activities on the horizontal side of the figure. Based on the meaning of horizontal and vertical discourse, this is not surprising, since everyday occurrences used as departure points for mathematics education are closely related to spontaneous teaching situations, however not exclusively. Based on the figure, it seems like the teachers, when they plan toddler mathematics, plan activities with advanced rather than basic content. This seems logical, since when planning activities it make sense to focus on content not yet mastered by the child, and based on the children being toddlers there is a lot of “new” mathematical content for them to explore.

Based on the figure, one could argue that the toddlers encounter mathematics in very different situations with mostly advanced content. This leads to another question: what is the outcome if we compare activities labelled as similar or as different? Below we offer four examples of activities labelled as similar or as different in regard to the notions in Figure 1. The first example illustrates the teachers’ use of the same words to describe “different kinds” of mathematics teaching while the other three examples illustrate the use of different words to describe the “same kind” of mathematics teaching. After this, in the last section, we will focus on what these illustrated diversities imply for collective professional development within similar boundless organisations of teachers, as with the network focused on here.

Example 1: The Use of the Same Words to Describe “Different Kinds” of Mathematics Teaching – Shapes

Activities 15 and 23 will be used to illustrate how the same words are sometimes used to describe quite different activities. These two activities were categorised as advanced within a vertical discourse (Figure 3).

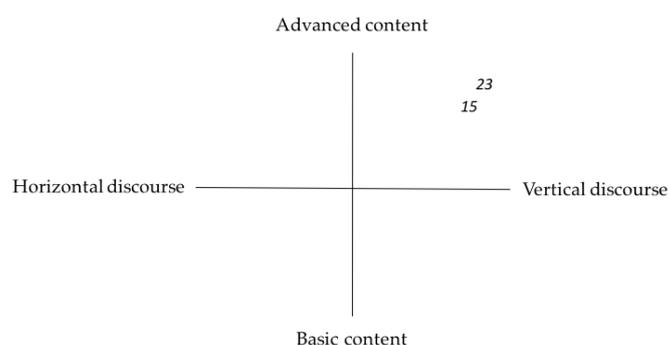


Figure 3. Activities 15 and 23 as they were categorised by the preschool teachers.

One teacher and three children aged two to three were involved in activity 15. Before the activity took place, the teacher cut paper into various geometric shapes. The shapes were triangles, circles, squares and rectangles. The cut-out figures were of different colors and different sizes. In the described activity, the children were to make a collage of these cut-out figures. The children liked to glue, and they had done it several times before. While the children were making their collages,

the teacher named the figures they chose. She also talked about what the figures looked like and compared them with objects in the room.

Activity 23 is a circle time that involved one teacher and several children aged two-and-a-half to three-and-a-half. During this circle-time, the teacher introduced the concept of the cube. She did this by using a chant about a cube. After the chant, the children together with the teacher counted the six sides on a cube, and the children were to explore a cube with light inside. Then, the children searched for objects in the room shaped as cubes. Finally, the children gathered again, and they compared the objects they had found, discussing whether the objects were considered to be cubes or not.

At a first glimpse, these activities look very similar. They were planned by the teacher and they involved geometrical shapes. However, in activity 15 the task for the children was to glue the cut-out shapes. The children enjoyed gluing and had done it several times before. However, it was only the teacher who talked, and the children were not asked to collect any material to compare with the figures. It seems that there were parallel activities in the activity, the children gluing and the teacher talking about the shapes. The activity being described as advanced seems to be based mainly on the cut-out figures, while it can be questioned whether these figures really made up the content of learning in the activity. The same activity could have been done with cut-out flowers and, if so, the activity would probably not have been described as advanced biology just because the teacher named the parts. In activity 23 the children explored a cube, and they were to find objects shaped as cubes, which emphasises the specific features of the geometrical figure. The teacher wrote about “introducing” the concept of cube, indicating that this was a not a familiar concept for the children. Thus, the cube was made the content of learning in the situation, and the children did not seem to have prior experiences of cubes, at least not from preschool. Without saying that one activity is better than the other, there are differences in the level of opportunity to learn new mathematical content in them, and they differ more than would be expected based on how they were labelled by the teachers.

Example 2: The Use of Different Words to Describe the “Same Kind” of Mathematics Teaching – Sorting

Activities 6, 11 and 37 will be used as a first example of how different words are used to describe the “same kind” of mathematics teaching. Activity 6 is placed in the horizontal/advanced quadrant, activity 11 in the vertical/basic quadrant and finally activity 37 in the vertical/advanced quadrant (Figure 4). Thus, one could expect the activities to be quite different regarding both content and design.

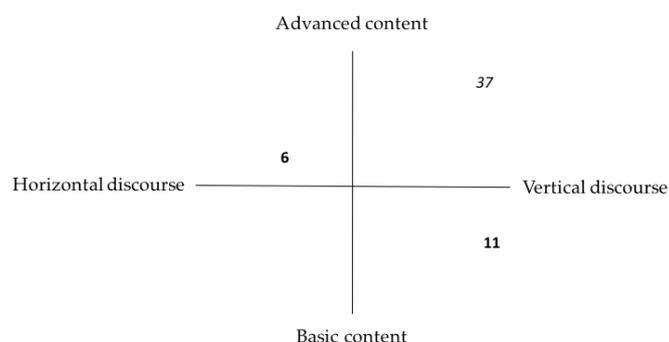


Figure 4. Activity 6, 11 and 37 as they were categorised by the preschool teachers.

One teacher and one child were involved in activity 6. The age of the child was not mentioned, but based on the focus of the network it ought to be between one and three. The situation is described as this child putting objects in a row. He started with cars and then continued with other objects. The mathematics is described as “putting things in a row,” where no explicit “rule” for how to organise the objects in the row is described. The teacher did not give any further instructions and did not introduce further material.

Two children were involved in activity 11. Their ages were not identified, but they ought to be between one and three. The children were sitting together exploring a material intended for sorting by color. The material consisted of clothespins and small boxes in different colors, where the clothespins were to be placed on the box of the same color. The material was always available for the children to use at this preschool. One of the children made pairs based on color, while the other was not focused on color but on the function of the clothespins. The teacher added more material in more colors to make the activity more advanced for the child making pairs based on color.

One teacher and one child aged two years and ten months were involved in activity 37. The activity was planned by the teacher based on the child’s interest in sorting activities. The teacher gave the child a box with small plastic bears in different sizes and colors. The child was asked to sort the bears and did so. According to the teacher, this child had sorted this material similar several times before. When the child picked up the bears, she named their colors at the same time; for example “blue, yellow, red and green,” The teacher confirmed what the child was saying.

These three activities were labelled differently (Figure 4), but when considering how they are described by the teachers, they seem quite similar. In all three situations children were sorting. In activity 6 no mathematical content was specifically identified, and the teacher did not add to the situation. Still, the content was described as advanced. In the other two activities the sorting was based on color. In neither of these situations were the children challenged to explore new concepts; rather, they seemed to have explored them before, and the teachers were generally adding more of the same into the activity. The content was, in other words, described as advanced in two of the situations even though no further challenges were added.

Example 3: The Use of Different Words to Describe the “Same Kind” of Mathematics Teaching – Filling Containers

Activities 3 and 35 will be used as a second example of how different words were used to describe the “same kind” mathematics teaching. The two activities were characterised within a horizontal discourse, but activity 3 is labelled as containing basic content, while activity 35 are labelled as containing advanced content (Figure 5).

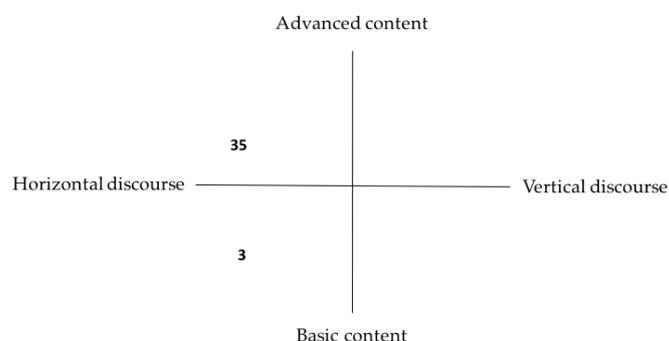


Figure 5. Activity 3 and 35 as they were categorised by the preschool teachers.

Activity 3 involved a group of children playing in a sand-box. One of the children, aged two, was new at the preschool. The other children were making sand cakes for a bakery. They talk about the shapes of their sand cakes and that they needed to make many sand cakes for the bakery. The new boy observed the other children and started to fill a mug with sand. Then he emptied the mug. He continued to fill the mug and empty it several times. As he did this, the teacher said “full” and “empty”.

Activity 35 describes one child, aged two years and 10 months, sitting in a sand-box. In the documentation, it is written that the child was “sitting alone in the sand-box and working with volume training.” The child had a mug that she filled with sand and then she emptied the mug. She continued to fill the mug and empty it several times and then said, “I have many.”

Thus, the action of the children in the two activities was described in similar ways, but activity 3 was labelled as containing basic content, while activity 35 was labelled as containing advanced content. However, in activity 3 the teacher added new notions in saying “full” and “empty” as the child filled and emptied the mug. In activity 35 the child did not seem to be exploring “volume” as described by the teacher, but instead focused on the amount of piles of sand. She said, “I have many,” but the teacher did not offer any further exploration of how many. The interpretations made by the teachers are thereby interesting to compare, directing attention towards the perspectives that inform their interpretations of “what is there to learn in an activity,” and “what is this child directing his/her awareness towards in this particular situation.” And, based on the previous knowledge of the children and the action of the teacher, maybe these two activities ought to have been labelled the opposite in regard to the content.

Example 4: The Use of Different Words to Describe the “Same Kind” of Mathematics Teaching – Counting

A third and final example of how different words are used to describe the “same kind” of mathematics teaching will be based on activities 7, 12 and 17. All three activities are about counting 1-5. Activity 7 is labelled in the vertical/basic quadrant, activity 12 in the horizontal/basic quadrant, and finally activity 17 in the vertical/advanced quadrant (Figure 6).

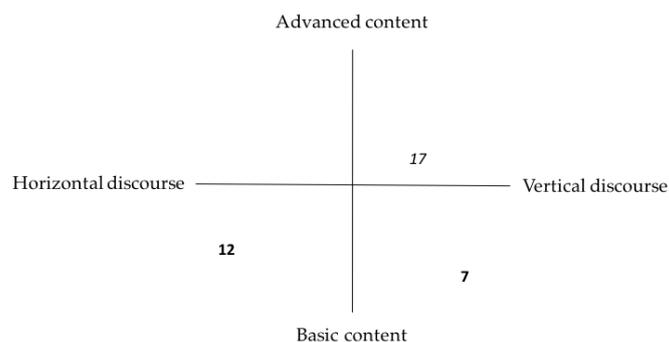


Figure 6. Activity 7, 12 and 17 as they were categorised by the preschool teachers.

Activity 7 involved a circle time with seven children and one teacher. The teacher told a story for the children. The story was about Mrs. Shark who hunted five fishes. The teacher used figures of fishes in paper to illustrate the story. One after another, the fish disappeared. The teacher asked the children about the quantity of the fish as she told the story. She wrote in her documentation that one of the children started to understand one and two, but when there were more fish, the children said “many”.

Activity 12 involved one teacher reading a book to three children aged one and two. The book was about a girl who got hurt and needed to put on patches. When the girl in the book is put a patch on her toe, one child took off her sock and started to count her toes but did not succeed. The other two children followed and started to count their toes. Then the teacher also took off her sock, and they counted their toes together with help from the teacher. The children concluded that they had equally many toes, but that the teacher's toes were bigger.

Activity 17 describes an activity occurring every day at this preschool. In the changing room, there is a ladder with five steps. The children use this ladder to climb up to the changing table. When they did this during this study, the teacher counted the steps with the children. This is something they do several times every week, and the teacher has noticed that sometimes children try to skip a number when they count to see if the teacher notices it.

Both the action of the children and the mathematics content are similarly described in these three examples, but activity 7 is characterized in the vertical/basic quadrant, activity 12 in the horizontal/basic quadrant and activity 17 in the vertical/advanced quadrant. Based on the documentations, the children in activity 17 seemed to be able to count to 5 but the content was labelled as advanced, perhaps due to the children's own challenging approach to the content (trying to fool the teacher). The same content was labelled as basic in activities 7 and 12. However, in activity 7 the children did not know the numbers above 2 before the activity why the content may be seen as advanced. Similarly, in activity 12 the children could not count to five by themselves but were challenged in comparing quantities of five and size, a content that was not consolidated from children's earlier experiences. Thus, activity 17 actually seems to contain the most basic content of the three in relation to the children involved.

What does this Diversity Imply for Collective Professional Development within Similar Boundless Organisations of Teachers?

The network on toddler mathematics education can be understood as collective professional development based on the interests and initiative of the network members. However, based on the inventory results above, showing how different words are used to describe the "same kind" of mathematics teaching, and how the same words are used to describe "different kinds" of mathematics teaching, one could argue that the network is not to be considered professional development in and of itself with regard to common grounds and joint professional language, at least not successfully so. Does the result indicate that professional development based on a knowledge of practice approach does not work?

The result shows diversity in the preschool teachers' perceptions of toddler mathematics: different activities are labelled as similar and opposite, similar activities are labelled as different. As mentioned, the initial aim with the inventory was to develop knowledge in the network of *which* mathematics toddlers are offered to learn and *how* this learning of mathematics is orchestrated. However, based on the results, we argue that the inventory instead ought to be used to increase the learning in the network further. This diversity that was made visible by the inventory is not to be seen as a failure, but as an opportunity for further collective professional development focused on mathematics and mathematics education. Before the inventory, this diversity had not been visible, not during the meetings, the writings for the book or the conference. It was when the teachers were to label their own teaching with theoretical notions that the diversity was made visible.

Thus, the result shows that collective professional development in the form of the described network does not automatically imply consensus. At the same time, the result shows that theoretical notions can be of value in creating professional development based on a knowledge

of practice approach collective and unified instead of collective but diverse. The diversity mainly appears in the ways in which the teachers label the content in the activities and less in how they label the design of the activities. The teachers were told to label the activities as basic and advanced in relation to the child/children involved in the activities. The documentations made visible that this was probably not always the case. Sometimes children already having knowledge of a mathematics content was very visible in a documentation. Still the content could be labelled as advanced. Other documentations described toddlers struggling with mathematics but the content was labelled as basic. Thus, sometimes it seems like the content was labelled based on other reasons than the involved children. This can become a good starting point for further professional development in the network. What is basic and what is advanced mathematics for individual toddlers? What does it mean to challenge toddler's mathematical development and what content is possible to introduce?

The diversity illustrated in this article is a phenomenon that we think is also common in other forms of professional development but seldom made visible. The diversity that became visible when the theoretical notions were included may have at least two explanations. One explanation is that the teachers perceived toddler mathematics differently, and because of that that they labeled the activities differently. Another explanation is that the teachers interpreted the theoretical notions differently, and because of that they label the activities differently. Whichever, theoretical notions being connected to experienced activities is a possible take-off for further professional development in the network. As mentioned, the network includes several of the principles for successful professional development identified in the research overview by Sowder (2007) and Avalos (2011); for example, it is the members who have identified their learning needs and the process to be used; the network activities are related to the members' teaching in preschool; the activities relate to the individual needs of the members but are organised around collective learning; the activities are continuous and ongoing; and, as the members are from different preschools, the network includes external influence. However, another principle for successful professional development is development of theoretical understanding, which up until the inventory had been less of a focus for the network. This is also emphasised in the Education Act in Sweden (2010:800), that all education ought to be based on scientific knowledge and documented experiences.

Thus, the result does not show that a network of teachers cannot be considered as collective professional development, or that professional development based on a knowledge of practice approach does not work. What the result show is the need for theoretical influence and discussions in practical inquiry. The intention with labelling activities with theoretical notions is not to learn the theoretical notions as such, but to make it possible for existing diversity to become visible, and by that to become an issue for discussion and learning. Further, the introduction of theoretical notions makes development of a joint professional language possible. For example, what does basic mathematics for toddlers imply? By comparing authentic examples, notions such as basic and advanced mathematics, as well as vertical and horizontal discourses, can gain significant meaning. Thus, making diversity visible contributes to our knowledge of professional development as well as the development of a professional language. A joint professional language can be considered important in a high quality, equivalent and research-based education for children under the age of three.

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