

# Sharpening the Focus on Numbers and Counting: Preschool Educators Differentiating Aspects of Mathematical Knowledge for Teaching

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This article reports a study of educators differentiating aspects of mathematical knowledge for teaching in preschool as part of a further-education programme. Eight Swedish preschool educators participated in focus group discussions about documentations from their own practice during a school year, to enhance their awareness of their mathematics education practice. The object of learning for the participants is more specifically how numbers and counting are made content for learning in preschool practice. The educators' authentic documentations are reflected upon in focus group meetings and further analysed by the researchers to find out what aspects the educators differentiate during the programme. Results show that educators' learning about content for children's learning in preschool is a complex process. This process involves several aspects necessary to discern for a professional development to occur, but the collaborative approach seems prosperous in that new aspects are brought to the fore, influencing their reflective practice.

**Keywords:** mathematical knowledge for teaching · numbers and counting · preschool educators · professional development · variation theory

## Background

The Swedish national curriculum for preschool and the political agenda highlights mathematics as a necessary content for learning in the early years. There is a growing body of research on mathematics education in the early years confirming that mathematics education in preschool is decisive for further learning and development of mathematics skills (Duncan et al., 2007; Östergren & Träff, 2013). The early childhood educator is in this respect a key person to provide experiences to young children that will lay the foundation for children's further development.

In order to enhance the current knowledge about preschool mathematics and in particular how numbers and counting are made objects for learning in early childhood mathematics education, a research project was launched<sup>1</sup> in Sweden, conducted in close collaboration between researchers and preschool educators. The current study is one part of this larger project, focusing specifically on educators' perceptions of numbers and counting as a content for learning in

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preschool. Here we present an analysis of the progression in the educators' perceptions as it is expressed in their documentations in texts, pictures and videos. Our study will contribute to the understanding of educators' mathematical knowledge for teaching (MKT) in that it offers a description of how numbers and counting are conceptualised as learning content among the participating preschool educators. This will provide deeper knowledge about professional development and what it means to teach mathematics in early childhood education.

## Context of the Study

Sweden has the same curriculum for all children enrolled in preschool from the age of 1 to 5 years. The national curriculum was revised in 2010, bringing in a stronger focus on mathematical content and competences, but very little is said in the guidelines about how to teach preschool mathematics. It is up to the educators to make informed choices of methods for teaching and objects for learning, in an effort to offer meaningful experiences of mathematical content such as shape and space, number, order, measurement and change (National Agency for Education, 2011).

Swedish preschool is included in the education system but not all educators have a teacher degree. There are many child minders employed in preschool and together with preschool teachers, they have a common responsibility to provide the children with a stimulating environment (National Agency for Education, 2011). Swedish preschool educators (teachers and child minders) are generalists with a broad competence and holistic responsibility to ensure that each child is provided with opportunities to develop socially, emotionally and cognitively (Pramling & Pramling Samuelsson, 2011). Specific content knowledge for teaching is in this respect very limited in the professional education programmes and depends on individual interests and studies. A recent evaluation of preschool in Sweden done by the Swedish Schools Inspectorate (2016) reveals a general lack of knowledge and implementation of teaching and instruction in preschool practice. The critique concerns missed opportunities to extend children's experiences in goal-oriented activities. One reason, based on an overview of preschool teachers' pedagogical awareness (Björklund & Barendregt, 2016) might be that a lack in content knowledge influences the approach to teaching specific contents (see also Ma, 1999). Mathematics is specified by the national curriculum as a knowledge area for preschool children, but a general or ambivalent approach to the subject may impact on how mathematics teaching is orchestrated.

A lot of effort have been made in recent years to improve the quality of early childhood education, not least due to the changes in the Education Act (The Ministry of Education, 2010) that states teaching is supposed to occur in all school forms, including preschool. The Education Act even provides guidance for how quality is to be ensured, through documentations. This is expressed in the curriculum in the following way: "The quality of the preschool shall be regularly and systematically documented, followed up, evaluated and developed. Evaluating the quality of the preschool and creating good conditions for learning requires that the child's learning and development be monitored, documented and analysed." (National Agency for Education, 2011, 14). The curriculum further points out the importance of using different forms of documentations and evaluation instruments to provide knowledge of the conditions of the learning practice and to make it possible to monitor children's learning and development in mathematics, among other knowledge areas. This documentation is supposed to be completed regularly and systematically to allow analyses of how the preschool is fulfilling the goals as intended in the curriculum.

The intention of the use of documentations is clear enough, but there are discrepancies in the ways policy documents and guidelines expect the documentation task to be conducted. Documentation as a concept is described in two ways in some texts (complementary to the curriculum) provided by the National Agency for Education: 'documentation' and 'pedagogical

documentation'. In the policy documents, the term documentation is used to point out that children's development and learning shall be followed up and analysed. The curriculum also says to use different kinds of documentations and evaluations. The complementary material that is to facilitate the implementation of the curriculum does however use the term 'pedagogical documentation', in other words not only as a tool for evaluating progress but as a basis for further pedagogical planning. How the educators understand and manage documentation in preschool practice can thus lead to different educational consequences.

Accordingly, all preschools are to document their practice and in the current project these documentations are a central point of departure for the educators' reflections, discussions and developing understanding of their pedagogical practice. In this study we consider these documentations to be expressions of the educators' perceptions of the contents for learning. The specific research question we are addressing is: How are numbers and counting perceived as learning content by preschool educators during a prolonged period of reflective actions?

## Professional Development in Early Childhood Mathematics Education

Educators' professional knowledge is generally considered composed of propositional, practical and personal knowledge that includes both content knowledge and pedagogical knowledge (Bertram & Christiansen, 2012). The profession constitutes 'teaching' as the primary activity, which may be defined as an act to enable someone to discern and develop awareness of phenomena that he or she has not been able to 'see' before (Pramling Samuelsson & Pramling, 2013). This implies that educators need to know more than just the facts of a content area. They need to know the underlying principles and the conceptual structure of the content and what facilitates children's learning or discerning of these principles and structures. Furthermore, teaching requires constant professional judgement and decision-making that is informed by both theoretical and methodological knowledge within the field. Difficulties in implementing mathematics teaching are according to Venkat and Askew (2012) primarily linked to limited pedagogical content knowledge regarding mathematical structures and how to present them to the learners.

Due to the heavy influence of policy and politics, the public and the education community, teaching is a profession in constant change (Hargreaves, 2000). Most countries have experienced similar changes in the teaching profession: earlier, there was emphasis on managing a group of learners so that they are likely to acquire knowledge, whereas the teaching practice of today to a large extent is assumed to address the individual needs of the students and to interact with them. The former approach consequently demands different skills than the latter. The former quite practical trade of teaching, meant that once the teacher had been trained and graduated, the teacher was left alone in her classroom to manage a large group of learners where disciplinary skills to organise the classroom situation were perhaps more important than subject knowledge (which were rarely questioned or reflected upon). "One learned to be a teacher through practical apprenticeship, and one improved as a teacher by individual trial-and-error" (Hargreaves, 2000, 156). This image of teaching had to be challenged, as it simplifies the teaching profession of today, which is heavily influenced by new technology, rapidly changing policy and increasing cultural diversity. Teacher individualism in such a time does not improve teaching practice. According to Hargreaves, key features for improvement are instead further education's closeness to the teaching practice and the teachers' sense of meaning for their practice. No teacher can manage to learn all new methods and technologies, as well as subjects, on their own to cope with uncertainty and new reforms. This calls for a collective approach in professional development, which in many ways is a necessity in contemporary education. The Swedish government has adopted this collective view and argues in the policy documents for education the need for collective

development processes among educators. This is apparent not least in recent drives for professional development through online courses and teaching guides. These drives aim at developing communities of practice, where members learn through a process of mutual engagement and enterprise and developing discourses (Kennedy, 2005). Learning in these communities is based on the interaction that takes place between the members. The added value (compared to individuals attending courses and mentoring programmes) is the combination of different individuals' knowledge and experiences that will contribute to a creation of new knowledge.

Professional development in contemporary education systems is a reoccurring issue, but the content of the development process is known to be crucial for its success. Timperley (2008, 13) holds it necessary that focus is directed not only on structure and on teaching methods, but rather on the question "What do we as teachers need to learn to promote the learning of our students?" Such development processes can only take place close to the teaching-learning practice. Studies of effectiveness of professional development show that programmes aiming at educators' enhanced skills to focus the responsiveness and learning outcomes of the students are most likely to succeed, rather than focusing on methods and knowledge of teaching in general.

As earlier mentioned, preschool educators in Sweden are educational generalists without specialisation in any particular academic subject. Still, they are obliged to teach mathematics and utilise the knowledge and experiences they have of mathematics education and young children's learning. It thereby becomes of interest to direct attention towards this presumably problematic feature of preschool teaching: if effective teaching (meaning that children will discover new ways of understanding and making use of mathematics) is to be facilitated, then educators' knowledge of the content should be highlighted as equally important as their general pedagogical skills. Ma's (1999) findings from classroom studies show that mathematical knowledge alone does not correlate with efficient teaching and students' learning outcomes, but profound mathematical knowledge (conceptual understanding of the structure and connections of mathematical content) opens up more opportunities to engage students and see opportunities to develop concepts that emerge in a situation. An intervention study of early childhood educators highlighted similar issues, whereas teaching material and methods were rarely used in conceptually developing ways if the educator did not recognise the underlying structure, for example an abacus and its base-ten structure, that may facilitate and reify arithmetic computation and understanding (Venkat & Askew, 2012). The present study can shed some light on the conditions for such development among preschool educators, through our inquiry of the educators' differentiating numbers and counting as content for learning in preschool.

If professional development is to occur, the content for learning has to be an essential part of the educators' practice and a source for challenges that the educators are interested in developing their knowledge about. Numbers and counting are in this sense relevant content to discuss with preschool educators as potential contents for learning. These notions are familiar to preschool practice, even though "counting" does not occur as a goal to strive towards in the national curriculum. Even so, studies show that counting is indeed a common activity or act with young children (Björklund & Barendregt, 2016). Numbers are mentioned in the curriculum in terms of "develop their understanding of ... the basic properties of sets, quantity, order and number concepts" (National Agency for Education, 2011). The relationship between numbers (or rather numerical magnitude) and arithmetic (counting) skills is furthermore unchallenged in the field of research on children's mathematics development, but these skills constitute the basis for further learning in different mathematical areas (Siegler & Braithwaite, 2017). Empirical studies give evidence for the necessity of a profound understanding of numbers to develop efficient strategies in arithmetic problem solving, which are founded in the early years before formal education begins (Neuman, 1987; 2013; Östergren & Träff, 2013). Consequently there are enough

reasons for choosing numbers and counting as relevant content in which to enhance educators' awareness. Even so, as research clearly shows, it is not enough for educators to have substantial knowledge of numbers and counting per se.

Mathematical knowledge for teaching (MKT) young children involves knowledge about how mathematical procedures work, what challenges children are likely to encounter and how to best define mathematical terms and notions to the children (Hill et al., 2008). Learning, and teaching, about numbers and counting is in this respect a complex matter, since number sense and arithmetic procedures rely on several basic ideas and principles. Number sense means knowledge and facility with numbers, knowledge and facility with operations, and applying this knowledge and facility with number and operations to a range of computational settings (McIntosh, Reys, & Reys, 1992). A large body of research shows that children have abilities to determine the quantity of small sets of numbers before they learn to count, in a process called 'subitising'. They also seem to have arithmetical expectations of their surrounding world and magnitude estimation abilities at a very early age (McCrink & Wynn, 2004; Wynn, 1998). But to develop this fundamental sense of number and magnitude into representations of numbers, such as number words and numerals, children need instruction and participation in a socio-cultural environment where it is necessary to communicate about quantities. To use numbers in numerical operations or arithmetic problem solving requires knowledge about numbers' cardinality and ordinality, meaning that a number word may determine the quantity of a set and number words are related to each other in orderly fashion (see Gelman & Gallistel, 1978). However, to operate with numbers in arithmetic problem solving, one also needs knowledge about the part-whole structure of numbers and the problem to be solved (what is the whole and what is a part to be added or subtracted) and knowing which strategies to use for which kind of operation (Baroody, Torbeyns, & Verschaffel, 2009). This complexity in learning to know and use numbers and counting strategies is an important aspect of preschool educators' mathematical knowledge for teaching, but as Klette (2007) concludes, this kind of conceptual insight to the content for learning is often taken for granted and is not problematised in the educational settings.

## Theoretical Framework

Our interest in this study is the educators' perceptions of numbers and counting as content for learning in preschool and more specifically how their perceptions change over a time of reflective activities. In other words, we identify how the educators learn to differentiate numbers and counting as a content for learning in the practice they themselves plan, enact and evaluate. We will use the theoretical framework of Variation theory of learning (Marton, 2015) to analyse the educators' perceptions and towards what aspects of numbers and counting as a content for learning they direct their attention.

Learning is in accordance with Variation theory (Marton, 2015; Marton & Booth, 1997) considered to be the process of differentiating more and more aspects of a specific phenomenon. When encountering a phenomenon, certain aspects are discerned, due to the person's earlier experiences of similar kinds or due to what is made possible to discern in the current situation. In other words, when a person sees a familiar phenomenon, he or she understands it based on earlier experiences, but if the phenomenon is presented in a way that a new aspect, not earlier recognised by the person, is made possible to discern, learning occurs since the person then changes his or her way of seeing the phenomenon.

In collective processes, such as focus groups where educators get together and share their experiences of similar situations or content for learning, the possibility is that different ways of understanding emerge in the discussion. These differences depend on the participants discerning different aspects of the same phenomenon. In variation theory terms, such occurrences open up

dimensions of variations, where the participants in interaction explore how aspects vary within the phenomenon and between phenomena. One example of this taken from the current project is one educator describing that many of her 5-year-olds enjoy counting on the counting sequence, up to 10 and further. Another educator reacts to this by saying “but even if they count up high, it doesn’t mean they understand the numbers they say”. These educators share their understanding of children’s mathematical knowledge and skills often through examples from their practice. In the collective sharing process, different interpretations bring aspects that may not have been considered before, as the first educator proudly encouraged her children to count on the sequence, but had not considered the possibility that the counting did not necessarily mean that the children could make use of their impressive skills to count high. In our study, these occurrences of differentiating new aspects of the learning object (numbers and counting as content for learning in preschool in this case) are of central interest as they are expressed in authentic documentations from their preschool practice. In accordance with the theoretical frame, once a new aspect has been discerned it is more likely it will be recognised in forthcoming similar situations. Placing their own perception in contrast to a different perception is necessary for such insights to appear, which are facilitated in collective discussions between practitioners working in similar practices.

Educators’ perceptions, or rather their knowledge of the content in their pedagogical work, has been conceptualised by Ball, Thames and Phelps (2008; see also Shulman, 1986) in terms of mathematical knowledge for teaching (MKT), which includes both the educator’s subject knowledge and pedagogical content knowledge. MKT refers to the relation between educator’s knowledge of the content itself (what does it mean to know numbers and how to count) and the children they aim to teach, including teaching methods (what have the children experienced as difficult before and how can the learning content be made understandable to them). Thus, it is not enough to have subject knowledge or knowledge of children’s learning respectively, there is a relation between the learner and the content to be learnt that has to be considered and respected in a teaching situation. MKT is of interest in our study, since mathematical knowledge for teaching is made the object of learning for the participating educators. We are thereby not directing any interest to their content knowledge or skills per se, but how they perceive numbers and counting as a content to be learnt by the preschool children with whom they work. There are several studies of teachers’ mathematical content knowledge in elementary school and some from Kindergarten and early mathematics education (Mathematics and Science Partnership (MSP), 2010) but rarely regarding preschool educators and the specific educational context that preschool, and in particular Swedish play-based and child-centred preschool, entails (see Oppermann, Anders & Hachfeld, 2016, for one rare example).

## Methods of Inquiry

A group of educators (n=8) from four different preschools in a municipality in the southwest of Sweden (named A, B, C and D) participated in focus group discussions during one preschool year (September–April). The focus groups were designed to be part of an in-service training programme aiming at facilitating collective professional development within the area of mathematics education in preschool. The participants have different educational backgrounds, both child minder and preschool teacher qualifications, and the number of years in preschool profession varies between less than 5 years and more than 20 years. One or two educators from each preschool attended the meetings. The focus group approach intended to encourage the participants to share their experiences and knowledge about numbers and counting as a content for learning in their daily preschool practice. The programme took a collective learning approach, meaning that the educators themselves were the driving force for their professional development,

supported by their documentations from their own work with numbers and counting. The educators' documentations of their pedagogical practice consisted of photo collages, video-films and written observations and were collected on a common web domain for the participants to enable sharing within the group and with the researchers. These were the basis for focus group discussions and the educators' forthcoming documentations. The researchers' role in the focus group process was not to guide the participants' learning, but to maintain focus on the learning object – numbers and counting as content for learning in preschool. Six focus group meetings were conducted during the programme, each meeting lasting for 2.5 hours.

The analysis in this particular study concerns the content for learning expressed in the documentations, and how the perceptions of this content evolves over a period of time. We will thereby describe and make inferences of how the educators perceive numbers and counting as learning content, and furthermore what children are offered to learn in preschool mathematics education, based on a content analysis of the documentations. Variation theory of learning provides the guiding principles for our analysis allowing us to describe such aspects that constitute the direction of the educators' perceptions.

### *Data Material and Analysis*

The curriculum for Swedish preschool states that pedagogical practice has to be documented for pedagogical purposes and assessment. How this is accomplished is an open question, but many preschools use photographs and video documentations that are used for reflection with both the children and the educators. These documentations are also meant to be used as tools for assessing them achieving their goals. In other words, if the preschool practice has offered the children sufficient opportunities to learn the goals mentioned in the curriculum. The documentations used for this study are in this respect authentic resources with a specific aim to document work with numbers and counting.

The data we have analysed are 40 documentations made by the eight participating educators, consisting of texts, pictures and videos. Not all preschools provided documentations for every meeting. The primary focus for the educators has been to contribute with documentations directed at numbers and counting, as it appears as content for learning in their daily pedagogical practice. These documentations have been the centre for discussions in six reoccurring focus group meetings during eight months. Audio-recordings and field notes from these have supported our interpretations of the visual, audio or graphical documentations. The documentations themselves are the main data source in this particular study, complemented by the recorded discussions where the educators express certain intentions or interpretations. The focus group design followed a pre-defined schema in that the initial meetings (and related documentations) were to concentrate on when and how children count and encounter numbers in the preschool practice. In the following session participants were also encouraged to direct attention towards interaction occurring in mathematical situations. In the second semester (meetings 4-6) these themes were repeated and national guidelines and support material for curriculum implementation were added. Altogether, these topics were used to retain focus on numbers and counting as content for learning in preschool.

Table 1 shows the documentations provided by the four preschools (A-D), which are used as data sources for analysis in the current study.

Table 1  
*Documentations Used as Data for Analysis*

Date for meeting	A	B	C	D
September	Introductory meeting			
October	Making a row of stones Comparing big cup – small cup Building with wooden bricks Turn-taking in game Setting the table	Necklace of pearls in different shapes Game of mushrooms with dots and numerals	Cubes with numerals 1-10 in a tower Marbles in a hand making dice patterns	Constructing patterns and 3-D structures with “Fischer Tip” material Decorating a picture frame with ten diamonds
November	Playing with balls, standing in a circle Cushions in geometrical shapes to build and play with Measuring pieces (in centimetres) put together in a long row	Counting pearls to make a picture of pearls following a model Circle time, counting children	Constructing tracks for marbles Making picture patterns of pearls Making seven fish of pearls for an aquarium	Playing and sorting plates with numbers as a jig-saw puzzle Discussing numerals on the clock on the wall
January	Measuring snow depth with ruler	The cup game, numerals 1-10		Creating letters and numerals of building sticks Building a tower, children comparing with their length

March		Hiding the end of a pearl string in a box – figuring out the number of the hidden pearls Pairing numerals with number of objects and pictures	Video of toddlers sorting shapes, sizes and counting
April	Task cards “finding n” in the woods Numeral shapes found in the woods	Making shapes with own bodies, comparing number of sides with picture model Video of children making shapes with their bodies Arithmetic-app on computer tablet together with corks and finger counting Playing Monopoly board game Weighing, counting and sorting different pearls	Counting number of letters on a board Discovering the shapes of boots Dividing bread evenly in baskets Sorting corks with numerals and letters Setting the table for the trolls, imaginary play

Since our aim was to study the process of differentiation over time, it is vital that the documentations are analysed chronologically, to discern any progression in differentiating aspects of numbers and counting as content for learning. The documentations do not reveal progress on an individual level, they are treated as a collective that includes educators with different background, experiences and motivation. Nevertheless, our analytical approach will provide a description of this specific group of educators and their learning about numbers and counting as content for learning in preschool.

Variation theory informs our analysis in directing attention to what aspects of ‘numbers and counting as learning content’ that are expressed in the educators’ documentations. Each documentation has been described in terms of what was prominent in the text, picture or video, supported by the educator’s own expressed intentions and interpretations from the focus group meetings. In accordance with the theory, those aspects made visible constitute the educators’ perception of the learning object. By interpreting the aspects made visible over time, we found out how different aspects appeared as central and new aspects made visible to the educators extended their perceptions of numbers and counting as learning content in preschool.

## Results

In general, considering the documentations as a collective of perceptions, there is a movement during the programme from a very diffuse and overarching focus that rather presents mathematics appearing in different situations, towards a greater awareness of numbers and counting, ending up in documented activities where children's learning processes are focused. The shift is an important result since it implies that the educators during a rather long period of time have differentiated aspects of numbers and counting as content for learning that were not present in their perceptions in the beginning of the project. We will in the following text present in more detail how this progress emerged and what aspects appeared central in the documentations.

The analysis reveals five aspects that were differentiated during the project time:

1. occurrences of numbers and counting in children's activities;
2. purpose for using counting and numbers;
3. children identifying numbers and numerals;
4. differences in children's knowledge and skills; and
5. learning the meaning of numbers.

These aspects appear as qualitatively different from each other but together they constitute a coherent understanding of the complex phenomenon the educators are learning about in the programme.

### *Occurrences of Numbers and Counting in Children's Activities*

Many of the initial documentations show activities and play situations where counting is assumed to happen, but the educators rather emphasise other aspects of mathematics, such as the children knowing which cup is bigger and sorting sticks in a long row encourages discussions of location (in front, behind). The documentations generally visualise situations where the children's knowledge of counting is expressed and used, but not made an object for inquiry of deeper meaning, such as in one documentation showing a table with a long row of stones, initiated by a child to make it easier to count the collected stones (see Figure 1).



*Figure 1.* A child has taken initiative to sort stones in a long row, to make them easier to count.

Numbers are not documented as a specific content for learning, nor expressed as part of the activities other than as a natural part of the counting procedures (e.g. as number words). The

educators differentiate counting activities as part of the activities they involve children in or that children initiate themselves. The aspect that the educators are discerning is mainly the *occurrence of counting* in children's play and activities, in line with the guidance from the focus group meetings, to document children's exploration of numbers and counting in planned and in spontaneous situations. These early documentations rarely include any processes of an explorative or instructive nature. They are catching the counting act in children's play. Figure 1 is nevertheless an example of emerging process documentation.

### *Purpose for Using Counting and Numbers*

The early documentations of numbers and counting as content for learning in the participating preschools have a distinct common theme: using counting as a strategy to solve tasks where you need to know 'how many items'. This is found in different documented activities, such as setting tables, making patterns and playing board games. Characteristic for these early documentations is the focus on counting as an act to solve tasks of enumeration, not arithmetic problem solving, in games and play.

One documentation shows the setting of the lunch table where counting helps the children decide the number of cutlery needed. Counting is, based on such documentations, discerned as a tool for the children solving the task. At the third meeting the educators' documentations show a more explicit focus on counting as a strategy to find out the cardinality of a set of items, in other words: to know how many there are. Documented acts are sorting and pattern making, in particular where the children have shown initiatives to count to complete their task.

Two preschools had documented measuring activities, one using informal measures (bricks measuring the height of the children) and one using a folding rule measuring the snow depth in the yard (see Figure 2).



Figure 2. Measuring snow depth requires tools and strategies such as counting the number of units on the ruler or measuring with a concrete reference point (shovel).

Numbers are mathematised in these documentations, meaning that the children discovered phenomena or problems where they needed numbers to make sense of the situation. Numbers (and numerals on the ruler) and their numerical meaning were brought forward as necessary for the children to use. The initial documentations did present similar activities where children counted, but the measuring activities highlight a different aspect of numbers and counting, where

counting a number of units (blocks or centimetres) has a different and more complex meaning than counting sets of items.

### *Children Identifying Numbers and Numerals*

Counting is perhaps an easier act to recognise and document as an occurrence in preschool, but during the discussions about the documentations another aspect is brought to the fore: children discovering numerals in their surroundings and taking initiatives to sort them in order and make meaning of them in different contexts. This aspect also frames activities that are directed towards children's existing knowledge and acts, rather than the process of learning and developing meaning.

One preschool shared, at the third meeting, documentations where numbers and numerals were differentiated as potential learning objects. The educator presented pictures of two children's play with plates with numerals fitting together as a jig-saw puzzle, enabling the plates to be ordered by the children in the number sequence. The other documentation showed a picture of the clock on the wall, in addition to written descriptions of the children telling the counting sequence found on the clock and then discussing at what time each child would leave the preschool and go home. These documentations show children exploring numerals and their meaning as parts of an ordered string of symbols. The order is important and children are observed discussing and elaborating on this meaning. Focus is here directed towards numbers and numerals and the meaning they constitute to the children in certain situations.

At the meeting that followed the clock and puzzle documentations, several documentations from the other preschools also included numbers and numerals as central features. Numbers were shown as objects included in the counting sequence in planned games. "The cup game" (see Figure 3) was played with the whole child group sitting in a half circle with a number line in front of them and cups with numerals 1-10 written on their bottoms (up-side-down on the floor). An item was hidden under one of the cups and one child was to guess under which cup the item was hidden. If the guess was wrong, the other children supported the guesser by saying she should look for a "higher" or "lower" number. The reasoning within this game concerned mainly recognition of numerals and the order of the numerals in the counting sequence.



Figure 3. "The cup game", to find out under which cup an item is hidden by the clues given by other children.

The focus on numerals is also found in a play activity where children construct the shape of numerals of wooden sticks. The meaning of the numerals is not emphasised or discussed according to these documentations. The documentations rather show the children's competences in identifying numbers and numerals.

### *Differences in Children's Knowledge and Skills*

At the initial meetings of the programme, most documentations are directed towards activities where numbers and counting occur and which strongly emphasise the activities themselves or the children's existing skills that are discovered within these activities, rather than any learning or teaching acts involved. At the third meeting one documentation does however stand out from the other activity-focused ones. It is a written reflection saying:

We always count how many children there are at circle time. How many are not here? We count together and the children can answer, some know and some make guesses. The children who know have counted silently. Or they just know.

This shows a shift from merely observing when counting occurs in children's activities towards the differences in knowledge expressed by the children in a counting activity. Similar reflections can be found in the later documentations where this aspect is brought forward, for example when children are invited to sort caps with numerals written on them and discussions arise about plausible strategies, which offers the educator an opportunity to observe children's differences in number knowledge and counting skills.

### *Learning the Meaning of Numbers*

The fifth meeting provided fewer documentations but they had quite a different focus than earlier ones and brought to the fore an aspect that had not previously been discerned among the documentations and educators' reflections. Numbers were now made an object for learning with the children, for example arranging sets of objects in order of quantity and relating them to numerals. Another game was also documented: "the worm in the box" where ten pearls on a string was called "the worm" and the educator hides some of the pearls in a box, leaving the other pearls on the string outside the box (see Figure 4).



Figure 4. "The worm in the box", a game where some (of ten) pearls on a string are hidden in the box. The purpose is to find out how many are hidden based on the visible ones outside the box. The text on the documentation says "If you see 7 hearts... how many does the worm in the box have?" (in Swedish).

These documentations have a clear goal for learning and the documentations are directed towards the process of making meaning of numbers, orchestrated by the educator in different ways to challenge the children's conceptions of numbers and use of counting strategies. These documentations are in line with the documentation of "the cup game" at the previous meeting, since they emphasise a goal for learning about numbers, but differ from the previous one in the attention towards the meaning of numbers' cardinality and how children are about to learn this meaning.

One explicit example of the educator differentiating children's processes of learning the meaning of numbers and counting is an activity around a computer tablet, where a child's hand is visible showing four raised fingers, some counters are found on the table grouped as four together and one on the side and the game on the tablet showing a task of subtracting  $5 - \_ = 4$  (see Figure 5). This documentation reveals the new aspect differentiated by the educators: the relationship between representations and how to work with that with the preschool children.



Figure 5. Documentation of a game on the tablet computer that is complemented with concrete manipulatives and children's use of fingers.

As previously mentioned, one educator made use of milk bottle caps with numerals written on one side and letters on the other side. An explorative activity was taking place in the documentation where children were allowed to sort the caps in accordance with their symbolic nature (numerals or letters), symbolic value (ordering 1-9), symbolic similarity (categorising ones, twos, threes...) and use of strategy to determine the sets of ones, twos, threes, and so on. The process of learning to understand the meaning of numerals and numbers is focal in this documentation as well, even though the educator allowed the children to explore freely the different aspects of numbers in more spontaneous manners.

### *Summary of the Educators' Differentiating the Complexity of Learning Numbers and Counting in Preschool*

The last documentations are at first glance quite different in terms of the activities and in content, but they are similar in that they express learning processes where children explore numbers and counting primarily. The educators also add to the visual documentations that they have an idea of what is possible to learn in the documented activities, as in the game including the computer tablet added with other props and representations.

There is a great variety of documented activities, where similar ones as in the earlier meetings reappear later during the project, such as finding numerals shaped in nature (tree branches), but are presented in more nuanced ways where the process of learning is prominent. In other words, the educators are in the beginning of the programme differentiating aspects characterised by activities and occurrences, whereas they later in the programme differentiate the process of learning. The latter discerned aspects are closely related to the concept of mathematical knowledge for teaching in that the educators learn to reflect upon what it means to learn about numbers and counting in preschool and what teaching methods are used with success or are less successful.

## Conclusions

Our analysis of educators' differentiation of aspects of numbers and counting as content for learning in preschool reveals two main findings: The first is that the educators' shared experiences enable them to differentiate new aspects of the learning object. In other words, they are inspired by each other, which clearly shows over time where one documentation that stands out in one meeting usually is followed up by other educators, and are *extended*, in the following meeting and new aspects are thereby discerned. The second main finding is the progression in the aspects that are discerned by the educators through the process, from recognising the existence of counting as part of children's activities, through discovering children's differences in knowledge and skills, towards documenting the process of exploring numbers and counting as learning objects. In the former cases the educator is quite invisible but in the latter there is a conscious use of manipulative material and instruction, even though they are utilised in different ways.

The aspects found to be prominent in the educators' documentations change over time, and in chronological order. This indicates that there are some aspects that are foundational for other aspects to be discerned, such as the occurrences of numbers and counting in children's activities that are necessary for seeing and reflecting on differences in children's knowledge and skills, which in turn are necessary for reflections and actions regarding the learning process and the educators' own teaching acts. Venkat and Askew's (2012) point made from empirical intervention programmes is also relevant to address here: what is made possible to learn for children is related to the educators' mathematical knowledge for teaching, since this determines *what* content is chosen for learning (for example: numbers for enumeration only, or number meaning explored as part-whole-relationship) and *how* this content is explored (utilising available resources in sporadic or in goal-oriented manners). Increasing the differentiated aspects of the learning object entailed a direct focus on the learning process rather than only the occurrences of children counting or using numbers. It is related to educators' mathematical knowledge for teaching (Hill, et al., 2008), since they enhance their awareness of the complexity of learning and teaching numbers and counting in preschool. The results of our analysis are to be seen as a coherent picture of what it means to work with numbers and counting in preschool, and what knowledge is necessary for teaching this content.

According to the national curriculum, making the learning process the object for evaluation, is central in sustaining high quality education in preschool. But as our study shows, it is not self-evident how this way of documenting practice and processes for pedagogical purposes is to be done or is even possible without reflective actions. Our study may, however, shed light on some necessary features of professional development in the context of preschool education, in line with Timperley's (2008) conclusions: professional development should centre around educators' awareness of what is necessary to know in order to facilitate learning for their children and that both focusing on a knowledge area and time are necessary to take into account. Furthermore,

Timperley argues that successful professional development needs a demarcated area of knowledge, on which the teachers make thorough reflections. Our study supports Timperley's argument, in that the narrow content, numbers and counting, was investigated deeply and during a prolonged time and changes in the educators' perceptions were appearing gradually.

The study design allows us to discern patterns in the educators' perceptions as the content of the documentations change over time. One pattern stands out, as there is a reoccurring line in progression from a very general expression of numbers and counting as tools for other mathematical exploration, towards a sharpened focus on numbers and counting as the object of learning itself. Our conclusion is that a common content such as numbers and counting cause challenges to implement as learning objects, even though educators themselves are confident in them offering children mathematics to learn in preschool practice (Björklund & Barendregt, 2016). The play-based child-centred practice causes these challenges, since learning is supposed to occur in play and daily routines rather than as planned content (Oppermann et al., 2016). The general conclusion from our study is thereby that children in these Swedish preschools are offered a broad spectrum of mathematics in their daily activities in preschool, but are rarely offered the depth and complexity of any specific learning object. This conclusion is drawn based on the general view this study provides, that orientation towards a learning object is not a common way to teach in preschool practice and it is a challenging task for educators to differentiate the necessary aspects of numbers and counting that are important to conduct any deeper inquiry with the children. Nevertheless, this change in approach is important for improving the learning outcome in any school form (Kennedy, 2005). In the process of differentiating aspects of the learning object, the educators open up several new dimensions of numbers and counting as a content for learning: first through recognising the content as part of the children's activities (counting to find out the quantity of sets), then the ways numbers and counting are made use of, (measuring) towards exploring numbers as learning objects where there are many aspects for the children to discern and make sense of the learning object. Initially, the window of opportunities that appear on a daily basis, as the documentations show, provides the children with experiences of the kind most of them are familiar with, generalising their previous knowledge in different contexts rather than extending their knowledge through challenging questions or inquiry. In the later documentations, there are many more opportunities for learning since focus is directed towards a critical aspect of education and professional development – the children's process of making meaning of the target learning content: numbers and counting.

### *Significance of the Study*

Our study will contribute to a deeper understanding of the challenges but also opportunities that lie within the process of sharpening the focus on numbers and counting as a content for learning. This is essential for professional development among early childhood educators, since their basic education is general and teaching demands certain content knowledge, pedagogical knowledge and not least mathematical knowledge for teaching even with the youngest learners. The evaluation report done by the Swedish Schools Inspectorate (2016) shows the need for professional development, which our study has shown is made possible through collective reflective actions.

This study directs attention to a crucial feature of education, and in particular early childhood education, namely what is supposed to be learnt and how is this learning facilitated in the pedagogical activities offered to the children. Teaching, in the sense of making someone else aware of their surrounding world in a new or more nuanced way (Marton, 2015), has in preschool tradition settled with activities where opportunities to explore mathematical phenomena are made possible, in games, play or a milieu that offers stimulating materials (Oppermann et al., 2016). As recent studies (Sheridan, Williams, Sandberg, & Vuorinen, 2011) indicate, there is an

uncertainty about learning goals since the national curriculum emphasise goals to strive for (not to achieve at a certain age or level) and the importance of children's own initiatives in pedagogical practice. Preschool children are therefore rarely offered opportunities to challenge their conceptions of numbers and counting, content that is complex and basic understanding of which is not easily developed (also shown in Björklund & Barendregt, 2016). This study provides some idea of the reasons for this lack of teaching children the basics in number concepts and arithmetic. Focusing on children's activities alone and what learning content there may emerge will not support children's development to their full potential (as stated to be the preschool's mission by the curriculum), nor will it provide the practice evidence for the quality of their ways to stimulate children's learning (as meant to be done through the mandatory documentations). When educators differentiate more aspects of the learning content, including children's various perceptions of the same, the educators perceive the content for learning in ways that were not previously possible and their mathematical knowledge for teaching enhances. Now their focus on children learning about numbers and counting as shown in their documentations, contribute to their pedagogical mission in that they differentiate what children have to understand to make sense of and use numbers and counting in different situations.

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