

Promoting Creativity and Self-efficacy of Elementary Students through a Collaborative Research Task in Mathematics: A Case Study

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Abstract

There is a consensus among mathematicians and mathematics educators that creativity plays an essential role in doing mathematics. Creative students are self-regulated students who take control over processes and experience high self-efficacy beliefs. The aim of this case study was to promote mathematical creativity and self-efficacy of elementary students through a collaborative research task, and qualitatively elicit their efficacy-beliefs about performing creative mathematics tasks. The research questions asked about the students' ability to show creativity in the classroom and about the extent to which the task enriched their mathematics knowledge, and made them realize the beauty of mathematical creativity. Participants were 24 sixth graders who attended a college of education once a week, to learn mathematics. This study revealed the potential of student engagement in creative mathematical work that demanded meta-awareness, self-regulation, and self-efficacy. As the confidence to work mathematically was communicated by their peers, many students expressed a change in their attitudes to their own mathematical competence and were more willing to engage with unknown or challenging mathematical tasks. The contribution of this study is twofold: finding ways of developing the students' creativity and enhancing their performance, and engaging them in an activity that made them realize the beauty, wealth, and elegance of mathematics, thus enhancing their self-efficacy to learn mathematics. We recommend providing students with opportunities to enhance their mathematical creativity by using creative tasks.

Keywords: *creativity; self-efficacy; self-regulation; collaboration; performance-task; elementary students*

1. Introduction

In a changing technological society, innovations are recognized as the vehicle of economic and social growth and welfare. Promoting these innovations necessitates creativity (Shriki, 2013). Although the education system's central role is developing students' creativity, it is not often nurtured in schools (Sriraman, 2009). Several conditions may justify this situation, such as external pressures to cover the curriculum and to succeed in standardized tests; teachers' tendency to teach in a similar way to how they were taught as school students, which did not place an emphasis on creativity; relating creativity to giftedness, and therefore avoiding nurturing all students' creativity; teachers' difficulties in assessing students' creativity and its development due to lack of available tools (Shriki, 2013). Mathematicians and mathematics educators agree that creativity plays an essential role in doing mathematics. Creativity in mathematics may be characterized in several ways, such as divergent and flexible thinking, or "unusual and insightful solutions to a given problem" (Sriraman, 2009, p. 15), connections between domains, or the ability to produce work that is both novel (e.g., original or unexpected) and appropriate (e.g., useful or adaptive to task constraints) (Sternberg, 2004 p. 3). Of all the aspects of the various existing definitions of creativity, novelty or originality is widely acknowledged as the most appropriate because creativity is generally viewed as a process related to the generation of original ideas, approaches, or actions (Leikin, 2009). Working on mathematical tasks may influence not only the mathematical content that is learned, but also how students experience mathematics (Pepin, 2009). Thus, working on appropriate creative tasks may impact students' perceptions of mathematics as a creative domain. More recently, Sriraman (2009) claimed that "mathematical creativity ensures the growth of the field

of mathematics as a whole” (Srirman, 2009, p. 13). As such, promoting mathematical creativity is one of the aims of mathematics education.

1.1 Creativity - Theoretical Perspectives

There appears to be a consensus that the two defining characteristics of creativity are originality and usefulness. However, several questions reflecting the diversity of the field still have different answers: First, is creativity a property of people, products, or process? Those who view creativity as a property of people tend to focus on individual differences in people’s creativity or on the distinctive characteristics of creative people. Those who view creativity as a property of products tend to focus on case studies of creative production, and those who view creativity as a property of cognitive processing tend to focus on analyzing the steps involved in creative thinking or in teaching creative cognitive processing. The overarching definition of creativity seems to favor the idea that creativity involves the creation of new and useful products, including ideas as well as concrete objects; however, from this definition, it follows that creative people are those who create new and useful products, and creative cognitive processes occur whenever a new and useful product is created. Second, is creativity a personal or a social phenomenon? According to the personal view, creativity involves producing something new and useful with respect to the person doing the creating. According to the social view, creativity involves producing something new and useful with respect to the cultural environment (Sternberg, 2004).

Creative people generate ideas. Creative work requires applying and balancing three abilities that can all be developed; the synthetic, the analytic, and the practical abilities: Synthetic ability is the ability to generate novel, interesting ideas. Often, the person we call creative is a particularly good synthetic thinker who makes connections between things that other people do not recognize spontaneously. Analytic ability is typically considered to be critical thinking ability. Individuals with this skill analyze and evaluate ideas. They use analytic ability to work out the implications of a creative idea and to test it. Practical ability is the ability to translate theory into practice and abstract ideas into practical accomplishments.

Students frequently experience creative insights as they learn a new concept. This type of creativity focuses on the novel, original, personal, meaningful interpretation of experiences, actions, or events (Kaufman & Beghetto, 2009). The terms novelty and originality seem synonymous and indeed, some researchers use them interchangeably. Although we also use these terms interchangeably, each of them stresses different elements: Novel may refer to “new” while original may refer to “one of a kind” or “different from the norm.” While it seems likely that an original idea will also be new and vice versa, sometimes an idea raised in the classroom may be new to a student, but if other students have the same idea, it may not be original. This study adopts the view that the product of mathematical creativity in the classroom may be original ideas that are personally meaningful to the students and appropriate for the mathematical activity being considered. Originality may also manifest itself when a student examines many solutions to a problem, and then generates another that is different. Fluency and flexibility, the two aspects of divergent thinking, are also associated with creativity (Leikin, 2009). Fluency may be measured as the total number of unduplicated ideas generated (Jung, 2001), and flexibility may be evaluated by establishing if different solutions employ strategies based on different representations (Leikin, 2009).

The view of personal creativity, as a quality that can be developed in school students, requires a distinction between absolute and relative creativity (Leikin, 2009). Whereas absolute creativity is associated with remarkable historical works of prominent mathematicians, relative creativity refers to discoveries made by a specific person within a specific reference group (Leikin 2009). Creative students are successful self-regulated students who control and monitor their learning environment (Sternberg, 2004).

1.2 Self-regulation, High Order Thinking, and Creative Collaboration

Creative thinking is conceived as a meta-cognitive process supported by the awareness of a person with the ability to regulate a creative sequence. The combination of knowledge of one’s own cognition and action control as well as its evaluation and personal effort is assumed to result in creation. Meta-cognition is a substantial ingredient of creative thinking, as meta-cognition involves the knowledge of cognition and the regulation of cognition and action (L. Sanz & T. Sanz, 2013). Creative actions might benefit from meta-cognitive skills and vice versa, regarding the knowledge of one’s own cognition and the regulation of the creative process. Today, successful functioning demands an adaptable, thinking, autonomous self-regulated learner (Schunk & Zimmerman, 2007). Self-regulation is defined as “the degree to which students are meta-cognitively, motivationally, and behaviorally active participants in their own learning process” (Zimmerman, Bonner, & Kovach, 1996, p.167). Academic self-regulation refers to self-generated thoughts, feelings, and actions intended to attain specific educational goals. The most important competencies required of such a person include: a) cognitive competencies, such as problem solving, critical thinking, formulating

questions, searching for relevant information, making informed judgments, making efficient use of information, inventing and creating new things, and b) meta-cognitive competencies, such as self-reflection, or self-evaluation, all of which are needed for creative mathematics learning (Zimmerman et al., 1996). Mathematics research shows that self-regulation has an effect on mathematical performance (Fast, Lewis, Bryant, Bocian, & Cardullo, 2010). This challenging enterprise causes changes in the Instruction Learning Assessment culture (ILA) that may enhance creativity such as: various flexible open-spaced learning environments, teacher–student cooperation in learning, teacher considered a coach rather than a sole source of knowledge, development of meta-cognitive competencies, and high-order thinking performance tasks. This new culture fits the constructivists' approach to ILA in that instruction promotes the development of reflective active learners who make use of knowledge, investigate, construct meaning, and evaluate their own achievements. Task types are usually performance tasks that require integrative thinking processes, finding out connections and relationships, elaborations, generalizations, production of knowledge (Dembo & Eaton, 2000; Schunk & Zimmerman, 2007).

Self-regulated learners believe that opportunities to take on challenging tasks, practice their learning, develop a deep understanding of subject matter, and exert effort will give rise to academic success (Perry, Phillips, & Hutchinson, 2006). In part, these characteristics may help to explain why self-regulated learners usually exhibit a high sense of self-efficacy (Pintrich & Schunk, 2002). These learners hold incremental beliefs about intelligence (as opposed to fixed views of intelligence) and attribute their successes or failures to factors within their control, e.g., effort expended on a task, effective use of strategies (Dweck, & Master, 2008). The relationship between self-regulation and self-efficacy is twofold: First, both self-regulation and self-efficacy involve meta-cognition, and second, self-efficacy plays a crucial goal in every phase of self-regulation (Bandura, 1997).

1.3 Self-efficacy Beliefs

Self-efficacy is defined as people's beliefs about their capabilities to produce designated levels of performance that affect their lives. Self-efficacy beliefs determine how people feel, think, motivate themselves, and behave through four major processes: the cognitive, motivational, affective, and selection processes (Bandura, 1997). Scholars have reported that, regardless of previous achievement of ability, high-efficacious students work harder, persist longer, persevere in the face of adversity, have greater optimism and lower anxiety, and achieve more than low-efficacious students. They approach difficult tasks as challenges to be mastered rather than as threats to be avoided. Such students set themselves challenging goals and maintain strong commitment to them. They use more cognitive and meta-cognitive strategies, and achieve more than those who do not have strong beliefs (Pintrich, & DeGroot, 1990). An efficacious outlook lowers vulnerability to depression. In contrast, students who doubt their capabilities shy away from difficult tasks, which they view as personal threats. They have low aspirations and weak commitment to the goals they choose to pursue. When faced with difficult tasks, they dwell on their personal deficiencies and on the obstacles they will encounter, rather than concentrating on how to perform successfully. They give up quickly in the face of difficulties, and are slow to recover their sense of efficacy following failure. They lose faith in their capabilities very quickly and fall easy victim to stress and depression (Maier, & Curtin, 2005). Therefore, very important in mathematics education is to nurture young people who will approach threatening situations in learning mathematics with the confidence that they can exercise control over them. As people interpret the results of their achievements and judge the quality of their knowledge and skills through these personal beliefs (Pajares, 2005), self-efficacy, thus, operates as a key factor in the generative system of human competence (Bandura, 1997).

Evidence from many meta-analyses of more than two decades of study shows that efficacy beliefs contribute significantly to level of motivation and learning, socio-cognitive functioning, emotional well-being, and performance accomplishments. Therefore, they are crucial for educating young people (Bandura, 2005; Zimmerman, 2000). Self-efficacy has an effect on cognitive and meta-cognitive functioning, such as problem solving, decision making, analytical strategy use, self-evaluation, time management, and self-regulation strategies, all of which affect academic achievement (Bandura, 1997; Bandura, 2005). Efficacy beliefs play an essential role in all phases of self-regulation and achievement (Schunk & Zimmerman, 2006). When self-regulatory processes play an integral role in the development and use of study skills, students become more acutely aware of improvements in their academic achievement and experience a heightened sense of personal efficacy (Zimmerman et al., 1996). Efficacy beliefs provide students with a sense of agency to motivate their learning through use of self-regulatory processes. The more capable students judge themselves to be and the more challenging the goals they embrace, the better they are at monitoring their working time, at persisting, at solving problems, and at self-evaluating standards they use to judge the outcomes of their self-monitoring. The greater academic self-regulation of self-efficacious students produces higher academic achievements (Zimmerman, 2000).

Sternberg (2004) argued that all students have the capacity to be creators, and to experience the joy associated with making something new. Without efficacy beliefs, the students would not have invested time and effort (Sternberg, 2004). All that is left for educators is to occasion creativity.

With the aim of promoting mathematical creativity in the classroom, several studies (Leikin, 2009) have pointed out the importance of having students engage in tasks that may encourage some of the different aspects of mathematical creativity. One task of this kind is presented in this study. Our purpose was, first, to find out the extent to which the collaborative research task that we used promotes mathematical creativity of students, and second, to qualitatively elicit the students' efficacy-beliefs about performing creative tasks.

2. Questions

1. Based on knowledge already acquired, how capable are the students of showing the following aspects of creativity in the classroom: novelty, flexible divergent thinking and fluency, connection between different domains, collaboration, self-regulation, self-efficacy, and relaxation?
2. To what extent does the creative performance task enrich the students' mathematics knowledge?
3. To what extent do the students realize the beauty of mathematical creativity?

3. Methods

3.1 Participants

Sixth graders from elementary schools in a district in northern Israel (n=24) participated in the study. The students attended a college of education once a week in the afternoons, for three hours, where they had extra lessons in various subjects including mathematics.

3.2 Design

This case study was designed to: first, enhance creativity of mathematics students through the use of a collaborative research task; second, to receive the students' feedback on their experience through a five-question structured Likert-type questionnaire, where students were asked to rate their answers on a 5-point scale, and third, to qualitatively elicit tacit knowledge on the students' efficacy-beliefs about their capabilities to perform a creative mathematical task, and to describe and analyze their creativity. To that end, the students were asked to write openly about their efficacy beliefs to perform a creative mathematical task. The qualitative design consisted of systematic, yet flexible, guidelines for collecting and analyzing data to construct abstractions (Stake, 2010). Listening, observing, communicating, and remaining in the field of the study for a prolonged period allowed the researchers to first understand and hence create an authentic picture of the participants' creative thinking in regard to their capability to perform the task.

3.3 Data Analysis

To answer the research questions, the task process and products were first observed and analyzed for the various aspects of creativity that emerged. Second, the five-question feedback results were summed up and explained (See Table 1, in the Result's Section), and third, a qualitative analysis was performed to analyze the sentences the students wrote openly about their thoughts, feelings, and beliefs regarding their capability to perform a creative mathematical task. The unit of analysis was a sentence. The data were analyzed using constant comparative analysis (Stake, 2010). Recurring themes were examined and gathered under criteria, which were then categorized. The units were thematically coded into categories through three-phase coding: initial, axial, and selective coding (Charmaz, 2006). Each unit was compared with other units or with properties of a category.

3.4 Implementing the Task and Procedure of the Study

Implementing the task was the first step that endured two hours, followed by the feedback part. The students' open writing finalized the procedure. The research task had eight stages and combined the cognitive act of thinking with the physical act of using their hands to play with matches. The students were divided into pairs. Each pair was given 12 matches of identical length. The students' goal was to arrange the matches on a desk, and by moving them, to create as many polygons as they could, under two mandatory conditions: 1) The polygon had a given, unchangeable perimeter of 12 matches in length; 2) The polygons must have integer area units. This task was based on students' previous knowledge of basic characteristics of: rectangles, squares, right angles, other angles, isosceles triangles, and equilateral triangles. As students came from different schools and classes, they sometimes needed supplementary

knowledge of material not covered in class. This missing knowledge was provided by the teacher during the process.

3.4.1 Stage 1: Creating Rectangles and Squares

The students had to use all 12 identical matches to form as many different polygons as they could, with each polygon having an area expressed in integer units. One integer unit is a square created by four identical matches. The students were not allowed to use partial matches, and the polygons had to be continuous.

The first 15 minutes were devoted to coping with the demands of the given perimeter and the integer area units. The students were guided to start by creating the large polygons, which contained the most integer area units. As expected, the students started to create polygons containing right angles that enabled calculating the area, and therefore created three quadrilateral shapes: a 3x3 square, a 2x4 rectangle, and a 1x5 rectangle. The size of the square was nine integer area units, the size of the second was eight integer area units, and the size of the third was five integer area units, as seen in Figures 1–3:

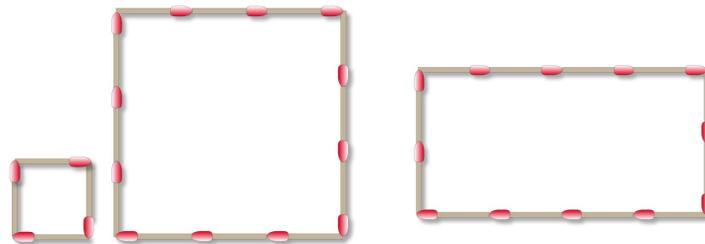


Figure 1.

A One-Unit Square and a
3x3-Unit Square

Figure 2.

A 2x4 Rectangle

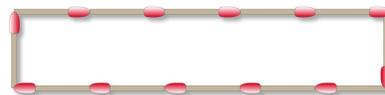


Figure 3.

A 1x5 Rectangle

The students were asked if the number of matches was 16, 20, or 24 (multiples of four), which rectangle would have the largest area? The students were supplied with more matches and created more rectangles. They came to the conclusion that for a 24-match length rectangle, a 6x6 size square is the quadrilateral shape with the largest area. Furthermore, they came up with the idea that of all the rectangle shapes with the same given perimeter, the square will have the maximum number of integer area units.

3.4.2 Stage 2: Moving Matches into Corners

The students were asked to try to create polygons of eight, seven, six, or five integer area units, which had right angles, using continuous forms other than rectangles. They had to make some changes in the rectangles already created. They moved two matches in one corner inside, thus reducing one integer area unit. They did the same in every corner of the square and reduced the integer area units from nine to five (Figure 4). Then they reduced the integer area units of the other two polygons from eight to six integer area units, as seen in Figure 5, and from eight to five as seen in Figure 6.

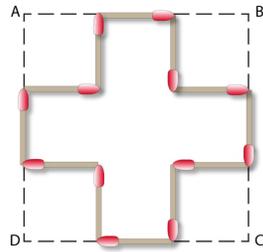


Figure 4.

Reduced Number of Integer Area Units,
from Nine to Five

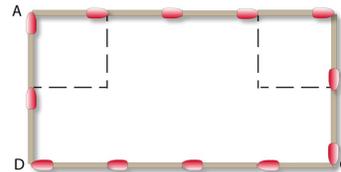


Figure 5.

Reduced Number of Integer Area Units,
from Eight to Six

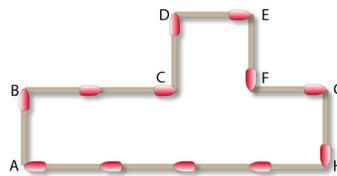


Figure 6.

Reduced Number of integer Area Units, from Eight to Five

3.4.3 Stage 3: Creating Different Polygon Shapes by Combining Different Rectangles

At this point of the research task, the students discovered that a five integer area unit polygon can be created in various shapes by combining rectangles:

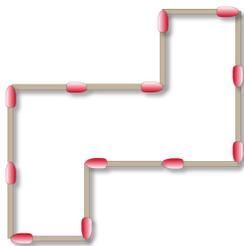


Figure 7.

A Five Integer Area Unit
Polygon

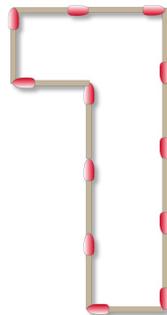


Figure 8.

Another Shape Created by a
Five Integer Area Unit Polygon

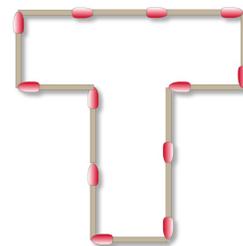


Figure 9.

A Five Integer Area Unit
Polygon, in Shape of the Letter
“T”

3.4.4 Stage 4: An Unsuccessful Attempt to Create a Four Integer Area Unit Polygon

At this point, the students tried to beat the teacher by forming smaller rectangles, but they broke one rule: the polygon perimeter was not continuous, as they put two matches on the same side, as seen in the following examples:



Figure 10.
Incorrect, Noncontinuous Polygon

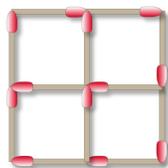


Figure 11.
Incorrect, Noncontinuous Polygon

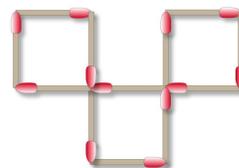


Figure 12.
Combining Polygons without Continuity

That idea was novel and original but did not adhere to the given rules.

3.4.5 Stage 5: Creating Three or Four Integer Area Unit Polygons with the Help of an Equilateral Triangle

When the students realized that they could not create polygons with less than five integer area units, the teacher hinted that the addition of equilateral triangles might be helpful, but he stressed the point that the triangular area could not be expressed in integer units. After a few minutes, two pairs of students succeeded in creating two similar four integer area unit polygons. Their principle was inversion while maintaining perimeter and integer area units. The students shifted equilateral triangles in and out of a four integer area unit polygon, as seen in Figures 13 and 14. Inspired by these possibilities, another pair of students created a shape that was based on a polygon with three integer area units, by shifting two equilateral triangles in and out, as seen in Figure 15.

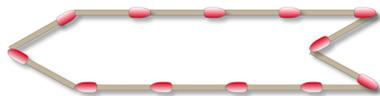


Figure 13.
Polygon Inversion While Maintaining Perimeter, and four Integer Area Units



Figure 14.
Polygon Inversion While Maintaining Perimeter, and Four Integer Area Units

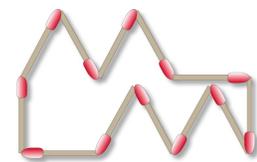


Figure 15.
Polygon Inversion While Maintaining Perimeter, and Three Integer Area Units

3.4.6 Stage 6: A Two Integer Area Unit Polygon: Combining Isosceles Triangles

The success in creating three and four integer area unit polygons inspired the students to try and create a two integer area unit polygon. Based on a two integer area unit polygon, they took out an isosceles triangle and added it to the top of the polygon. Each of the two equal sides of the isosceles triangle was two matches long. The polygon is displayed in Figure 16:

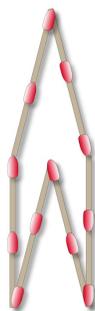


Figure 16.
A Two Integer Area Unit Polygon Created by Combining a Rectangle and an Isosceles Triangle

3.4.7 Stage 7: A Three, Four, Five, and Six Integer Area Unit Polygon Based on Pythagorean Theorem

Most of the students had previous knowledge of Pythagorean Triples and the relations between the length of the sides in the right-angled triangle, and most of them had knowledge of Pythagorean Theorem and its implications.

It was suggested to the students that they look for Pythagorean triples that had a perimeter length of 12, and create a triangle accordingly. Pythagorean Theorem was taught to several students who had not yet learned it. Of course, they immediately found that the triple was of 3, 4, 5 matches length, and according to this, they created a right-angled triangle of six integer area units, as illustrated in Figure 17. In other words, a new shape was created for a six integer area unit, in addition to the shape already created (illustrated in Figure 5), which, according to the second rule, also had a given perimeter length of 12 matches.

Moreover, by moving matches inward at vertex C of the triangle, we enriched our repertoire of forms with additional three, four, and five integer area unit polygons, as illustrated in Figure 18:

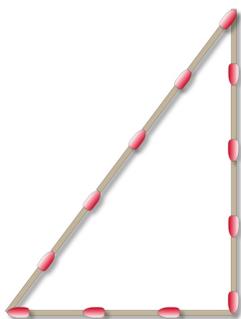


Figure 17.
A Triangle 12 Matches Long

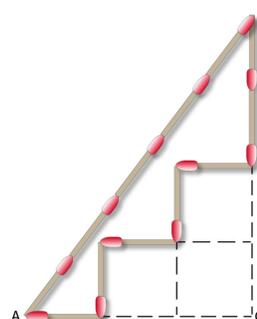


Figure 18.
Three to Five Integer Area Unit Polygons,
Created Using a Triangle

3.4.8 Stage 8: A One Integer Area Unit Polygon

The attempt to create a one integer area unit polygon by shifting triangles in and out did not work, because if we started with two matches, one match distance between them (1 1), then 10 matches remained. These 10 matches should be partitioned into 2 five matches. It was impossible to build two isosceles triangles (one in and one out) with them, as we could not break them and have 2.5 matches for each equal side of the isosceles triangle.

Two students suggested putting two pairs of matches in a parallel position, with half a match length distance between them, thus forming a one integer area unit polygon, and then shifting in and out isosceles triangles with a side two matches long, as illustrated in Figure 19.



Figure 19.
A One Integer Area Unit Polygon,
with Triangles Shifted In and Out

This innovative shape was evidence that the students used their previous knowledge of multiple fractions (2×0.5), and applied it to geometry. As for the question of finding the length of half a match, the students suggested calculating it using a ruler. The teacher suggested an alternative way: drawing a line the same length as a match and finding its mid-point using a ruler (without scale marks) and a compass.

The students created a variety of forms. The following figures illustrate some of them:

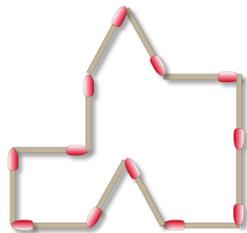


Figure 20.

A Five Integer Area Unit Polygon, with Triangles Shifted In and Out

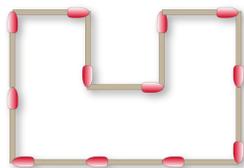


Figure 21.

A Five Integer Area Unit Polygon, with Matches Shifted Inward

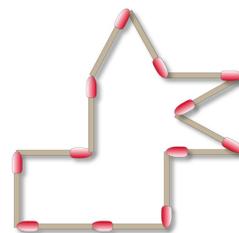


Figure 22.

A Four Integer Area Unit Polygon, with Triangles Shifted In and Out

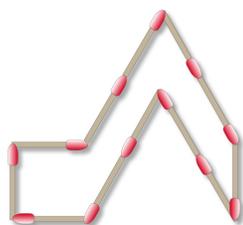


Figure 23.

A Three Integer Area Unit Polygon with Triangles Shifted In and Out



Figure 24.

A Three Integer Area Unit Polygon with Triangles Shifted In and Out

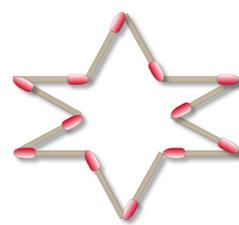


Figure 25.

A Three Integer Area Unit Polygon



Figure 26.

A Three Integer Area Unit Polygon

To sum up the activity, the teacher explained that using an advanced mathematical calculation (by Trigonometry), the maximum possible area of a regular polygon of 12 sides (matches) is 11.196 area units. This will be a regular polygon. Later, the students learned about regular pentagons, regular hexagons, regular heptagons, regular octagons etc.

4. Results

4.1 Creating a Variety of Polygons

The goal of creating as many and as different polygons as possible while maintaining perimeter and integer area units was a real challenge for the students: 18 students (75%) reduced the polygon from nine down to eight, seven, six, or five integer area unit polygons, 12 students (50%) produced three and four integer area unit polygons. Eight

students (33%) produced three to four integer area unit polygons with the help of equilateral triangles, eight students produced between three and six integer area unit polygons with the help of Pythagorean Theorem, four students produced two integer area unit polygons combining isosceles triangles, and two students suggested the one integer area unit polygon. This collaborative research task promoted the following aspects of mathematical creativity: novelty, flexible divergent thinking and fluency, connection between different domains, collaboration, self-regulation and self-efficacy, and relaxation and enjoyable environment, and play. These aspects will be discussed in the discussion section.

4.2 Results of the Feedback Part

At the end of the research activity, the students were asked to circle a score on a 5-point scale, for each of the five questions (5=fully agree; 1=disagree) of the questionnaire. 20 students answered the questionnaire. The results of the second part are shown in Table 1;

Table 1. Students' Answers to the Five Questions and Their Means (n=20)

Question	1	2	3	4	5
	How did you like the task?	To what extent was the task based on your previous knowledge?	Did you acquire new ideas while performing the task?	Did you acquire new mathematical knowledge while performing the task?	How ready are you to go on coping with creative research tasks in the future?
Mean	4.2	3.1	3.4	3.9	4.3

The mean scores for answers 1 and 5 were the highest. This result shows that the students enjoyed and were interested in the challenging task, and would like to be set such tasks in the future. The mean scores for answers 3 and 4 show that students did acquire new ideas, new mathematical knowledge, and new insights during the activity in general, either from the teacher and from other students or by working with their own hands and minds. The mean score for answer 2 shows that the task was based on previous knowledge except for the new ideas or insights gained during the task process.

4.3 Results of Students' Efficacy Beliefs Analysis

The students' openly written reflections were analyzed and the following categories/themes emerged:

*The ability to form a variety of new polygons was much unexpected, as remarked by one student:

"After having achieved five integer area unit polygons, we had the feeling that it was impossible to build polygons that contained smaller numbers of integer area units. Creating those polygons was a real surprise for all of us, and gave us the feeling that we can make it!"

* Divergent and flexible thinking, and fluency, allowed the fascinating creation of multiple shapes:

"It's because I think in many directions. Now I know, I can be an engineer one day."

* Learning new things and acquiring insights on the basis of previous knowledge enhances mathematics.

"If you don't learn from the past how can you excel in anything?"

* Working with friends was pleasant and helpful:

"I learned from him and did the same, we worked like mathematicians. If he can do it, I will do it too."

* This experience contributed to the students' self-confidence.

"Succeeding to create polygons empowered my beliefs in myself."

* This interesting task should be done with other students as well.

5. Discussion and Conclusions

The goal of creating as many and as different polygons as possible while maintaining perimeter and integer area units was fascinating. The students created a variety of shapes, such as shapes containing right angles only, shapes

that combined right angles and other angles, and shapes without right angles. With the aim of promoting mathematical creativity in the classroom, several studies have pointed out the importance of having students engaged in appropriate tasks that may encourage different aspects of mathematical creativity, as was done in this study. This collaborative research task successfully promoted the following aspects of mathematical creativity: novelty, originality and unexpected thinking, flexible divergent thinking, and fluency, connection between different domains of mathematics, collaboration, self-regulation, self-efficacy, and relaxation and enjoyable environment, and play.

5.1 Novelty, Originality, and Unexpected Thinking

In Stage 1, the students came up with a new idea about the largest area of the square in relation to other polygon shapes when the perimeter was given.

From Stage 2 onward, the students created new forms of polygons that they had not previously dealt with in their classrooms. Figures 13–26 show novelty and originality, and some were even unexpected. Analytic and synthetic abilities were demonstrated when they could imagine different areas that they had created by shifting squares and triangles in and out of rectangles. These polygons did not appear in their regular curriculum. Through this task, they enriched their polygon repertoire. Novelty is the crown of mathematics, as unexpected thinking and originality become the basis of useful adaptive and appropriate scientific and technological inventions in our lives (Sternberg, 2004). This is how mathematics contributes to society.

5.2 Flexible, Divergent Thinking and Fluency Allow Multiple Solutions

This collaborative research task promoted flexible, divergent thinking, which allowed the creation of multiple shapes. The notion of divergent thinking is attributed to Guilford (1968), who theoretically and empirically associated divergent production with creative potential (Sanz & Sanz, 2013). Divergent thinking, which facilitated production of multiple solutions, is one characteristic of creativity. The scientific community recognizes that divergent thinking tests reliably assess creative potential and therefore assumes that these tests are valid, useful, and predict the future performance of creative people (Runco & Acar, 2012). This flexible thinking was typical of all the stages of the task, allowing the students to pursue many different perspectives and creating a variety of polygon shapes while preserving the given feature of the perimeter size and the integer area units, as demonstrated in Stage 3. The multiple shapes of five integer area units (Figures: 4, 6, 7, 8, 9, 18, 29, 21) created by the students, shows the creative potential in geometry. The multiple possibilities demonstrated the aesthetics, wealth, and elegance of geometry. Divergent thinking is very important for the development of mathematics in that it enhances students' mathematical understanding when material is approached from different points of view. Mathematicians tackle a mission from different points of view. By encouraging our students to do the same as mathematicians, they too, learned to appreciate the value in tackling a mission from different viewpoints (Stupel & Ben-Chaim, 2013). Leikin (2009) claimed that multiple-solution tasks offer students the opportunity to come up with different solutions, in turn encouraging three hallmarks of mathematical creativity in school: fluency, flexibility, and novelty. Tasks which may produce creative divergent thinking should invite the students to search for many different solutions. Multiple ways of thinking is a basis for mathematical creativity in that it leads to unexpected novelties, which have the potential to enhance human society.

5.3 Connection between Different Domains of Mathematics

In this task, students used previous knowledge, e.g., perimeter and area calculation and Pythagorean Theorem to produce novel, original, unexpected geometric shapes. They learned new ideas that they did not know before, such as Pythagorean Theorem and used them for their own creations. They created polygons of one to nine integer area units while having an invariance feature of perimeter. The multiple productions for one geometric task demonstrate the connection between different areas of mathematics, such as algebra and geometry (Stage 8), which, according to Stupel and Ben-Chaim (2013), gives the students the sense that mathematics is an interconnected science and not a collection of isolated topics. Hence, they claim that mathematics textbooks worldwide are organized by specific problems that give the students the feeling that certain problems are connected to specific topics and, therefore, assume that each problem has one method or one solution. They emphasize that the NCTM standards (2000) also recommend that teachers present tasks that exhibit the connection between different mathematical domains. Although this is not easy, there is certainly a need to encourage such activities in class (Stupel & Ben-Chaim, 2013). Teachers would do well to apply knowledge from different mathematical domains, as much as possible, to promote high order thinking, which contributes to creativity.

5.4 Collaboration

Of course, it is not the task alone that may promote mathematical creativity. Several studies have investigated other

factors, which may have a creativity-promoting influence, such as collaboration. The student working with a peer, as in the present study, must evaluate the peer's idea, then his/her own idea, select appropriate ideas, and build on them. On the one hand, the peer's different background and knowledge base may contribute different perspectives for consideration; on the other hand, diversity may be very wide, causing difficulty in finding an agreed-upon solution (Kurtzberg & Amabile, 2001). We gave our students the chance to work collaboratively to spur creativity, as we all learn by examples, by watching techniques, strategies, and approaches that others use in the creative process. The literature review indicates that cooperative interaction has considerable impact on the stimulation of creativity (Sharma, 2014). Working collaboratively with the students afforded us a genuine feeling of working as mathematicians who are looking for creativity. In addition, students absorbed the enthusiasm and joy exhibited by many creative people as they make something new (Sternberg, 2003).

5.5 Self-regulation and Self-Efficacy – One of the Affective Aspects of Mathematical Creativity

Acquiring new insights on the basis of previous knowledge is typical of self-regulation. It occurred in the first stage, when the students said that the areas of the polygons were identical. The students initially created shapes that were familiar to them. Very soon, they realized that if the perimeter of two polygons is the same, their areas do not have to be identical. Self-regulation was perceived when students monitored their thinking processes, thought critically about the task to find answers, asked their friends questions, and tried new ways. They failed in Stage 4 and took another path. They demonstrated meta-cognitive ability when they reflected on their capabilities while performing mathematical tasks, and when evaluating their efforts and their experience. The teacher encouraged them to take control, to know how to defend their choice and take responsibility, to develop plans for completing the assignment, to assess their own and their peer's work. The teacher helped them believe in their own ability to be creative, which supports Sternberg's claim (2004) that all students have the capacity to be creators and to experience the joy associated with making something new. The teacher encouraged them to define and redefine problems, and to ask questions. He let them choose their own ways of solving problems, and choose again when they discovered that their selection was a mistake. When a student attempted to surmount an obstacle, he praised the effort, whether or not the student was entirely successful, which enhanced the students' self-efficacy. Creativity among children is more than cognition. Intrinsic motivation, openness, curiosity, and autonomy often play a role in children's creative efforts (Runco, 2006). Silver, Mesa, Morris, Star, and Benken (2009) found that experienced teachers have many aims when choosing tasks, including the aim of building students' self-confidence, which is negatively correlated with anxiety (Silver et al., 2009).

5.6 Relaxation, No Time Limit, and an Open Enjoyable Performance Task to Facilitate Creativity and Self-Efficacy

In a group environment such as a classroom, the teacher has several roles in the promotion of creativity. Besides choosing the task, the teacher has to foster a safe environment. Students in this study had low mathematics anxiety, were relaxed, tension free, and calm. Generating different solutions requires an open and stress-free mind, which contributes to the students' endeavor to succeed. Observations showed that they enjoyed the activities and were highly motivated. According to Jung (2001), providing a safe environment in a group situation, where individuals can try out novel ideas and question their own, may promote higher levels of creativity (Jung, 2001). Levenson (2011) found that when students feel free and relaxed and do not have time limits, they may come up with original ways of solving problems (Levenson, 2011). Part of being creative means investment of time, hard work, and delayed gratification. Hard work often does not bring immediate reward and the greatest rewards are often those that are delayed. The people who make the most of their abilities are those who wait for a reward (Sternberg, 2003).

5.7 Using Matches to Foster Creativity

The use of the matches was an attempt to make the activity exciting and enjoyable. It is almost universally accepted within the world of early education that children learn through play. The students worked with their hands and minds concurrently. Play, which involves moving the hands or body, contributes to learning by supporting children's development of meta-cognitive and self-regulatory skills, which are in turn crucial in the development of creativity. During play, children develop a sense of control and self-regulation of their own learning (Whitebread, Coltman, Jameson, & Lander, 2009). The teacher tried to be a role model for creativity by moving matches and thinking together with his students. The most powerful way to develop creativity in students is to be a role model (Sternberg & Williams, 2003). The teacher allowed time for creative thinking and encouraged creative ideas. During the play with the matches, the teacher asked the students to cross-fertilize ideas. This probably spurred creative ideas as the students enjoyed the activity, and did not show any anxiety. The teacher allowed the students to make mistakes. They took risks—after all, nothing was life-threatening (Sternberg, 2003). The act of moving matches around uses fine motor skills, and is an important activity in an era when people make daily use of touch screens. The following link

takes us to a GeoGebra applet that enables the dynamic construction of polygons on the computer screen. (We need to have GeoGebra downloaded to our computer; it can be done for free.)

Link: Construction of various polygons with 12 matches

<http://geogebraTube.org/student/m473921>

To summarize, creativity, as manifest in the mathematics classroom, is multifaceted. The teacher who aims to promote mathematical creativity may take several factors into consideration. One factor is choosing the task that may promote creativity, and another is facilitating the environment, including enhancing affective aspects, like self-efficacy, as done in this study.

6. Conclusions and Implications for the Future

Creative thinking is rapidly becoming a common purpose throughout the world (Storm & Storm, 2002). This need becomes fundamental in the case of mathematics, and an increasing number of educators, researchers, and agencies of education are asserting the need to foster mathematical creativity among students (Johnson, 2012; NCTM, 2000; Mann, 2006). Furthermore, mathematical creativity ensures the growth of the field of mathematics as a whole (Sriraman, 2009). A first step towards achieving this goal is to assist teachers to view creativity as inherent in learning (Beghetto & Kaufman, 2009), and to inspire teachers to believe that all students can become creative, as creativity is not an exclusive trait of the gifted (Rowlands, 2011).

This study has revealed the potential of student engagement in creative mathematical work by engaging students effectively in a creative collaborative performance task, which demanded meta-awareness, self-regulation, and self-efficacy. The result of engaging the students with a creative activity was twofold: first, it resulted in students' improvement of their creative abilities, and second, by communicating their confidence to work mathematically, many students expressed a change in their attitudes to their own mathematical competence and were more willing to engage with unknown or challenging mathematical tasks (Maier & Curtin, 2005; Pajares, 2005). Working on mathematical tasks may influence not only the mathematical content that is learned, but also how students experience mathematics (Pepin, 2009). Thus, working on this task had an impact on students' view of mathematics as a creative domain.

Johnson (2012) argues that the most recent revision of Benjamin Bloom's popular taxonomy of education objectives places creativity at the top of the cognitive domain, and that every 21st century skill list that he has seen includes creativity as one of the necessary abilities required by tomorrow's most productive workers. Nevertheless, not enough is being done in schools for developing creativity (Johnson, 2012). As professionals, we need to learn more about creativity and the creative process, and as teachers and educators, it is our duty to enhance and support creativity in education. Both have been done in this study.

Albert Einstein famously said that the world we have made as a result of the level of thinking we have done thus far creates problems that we cannot solve at the same level of thinking at which we created them. Let us think creatively about creativity (Johnson, 2012).

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