Pirouettes and protractors:

Dancing through mathematics

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An explanation of how the kinaesthetic context of dance may be used to investigate geometry is provided. Using a context such as dance makes mathematics learning accessible to all and engages students who may not otherwise be engaged.

The scene: Jennifer's pre-service numeracy class

"Class, today we are going to be doing ballet."

"In numeracy class?!"

Jennifer's students were very surprised to hear that they were going to be exploring ballet in a pre-service education class focused on numeracy. Many people see mathematics as a boring, rigid school subject and dance as a fun, creative extra-curricular activity. However, mathematics and dance have many similarities, including manipulating and exploring patterns, defining and solving problems, and appreciating beauty (Asaro, 2016; Parsley & Soriano, 2009). Mathematics concepts inherent in dance include patterns, transformations, counting, and geometry. While mathematical content may be the focus of a dance activity, mathematical proficiencies (e.g., problem-solving) are equally prevalent.

In line with suggestions in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA] n.d.-b), mathematics has moved away from being a subject where work is completed in an individual, silent, seated situation to a more active and interactive subject, which reflects an understanding that "learning, thought, creativity and intelligence are not processes of the brain alone, but of the whole body" (Hannaford, 2005, p. 15). Furthermore, the *Australian Curriculum: Mathematics* focuses on making connections between mathematics and other subject areas (ACARA, n.d.-c). As a result of such suggestions in the *Australian Curriculum*, as well as other curricula worldwide (e.g., Department for Education, 2013; Gouvernement du Québec Ministère de l'Éducation, 2001; National Council of Teachers of Mathematics, 2014; Ontario Ministry of Education, 2005), more examples of dance and movement in mathematics classes have been reported in academic and popular literature. Some dance/mathematics activities have been driven by researchers (e.g., Smith, King, & Hoyte, 2014), while others are based on the work of non-school-based arts educators (e.g., Rosenfeld, 2011, 2013). Several benefits have been reported from incorporating dance in mathematics, such as increased test scores/understanding (e.g., Parsley & Soriano, 2009; Smith et al., 2014) and positive affective changes (e.g., Werner, 2001; Wood, 2008).

Here, we present two activities in which students explore angles and related geometric properties in the context of ballet. These examples add to the growing body of literature featuring activities that integrate mathematics and dance. For instance, Larkin, Perez, and Webb (2004) provided 12 primary-level activities that addressed mathematical topics such as symmetry and common multiples. Schaffer and Stern (2014) described a "dance" involving loops of rope to create various two-dimensional shapes and three-dimensional figures. Finally, Hawes, Moss, Finch, and Katz (2012 /2013) presented a sequence of activities to explore geometric patterns and functions. Having provided an overview of a few different activities that incorporate mathematics and dance, we now turn to a discussion of two activities, focusing on angles, that have been successfully implemented in pre-service numeracy classes.

Setting the stage

In the two-year, post-degree teacher education program (MTeach) at Monash University in Melbourne, students take a mandatory numeracy unit, *Numeracy for Learners and Teachers.* The introduction of the unit was driven by the graduate expectations for teachers related to numeracy from the Australian Institute for Teaching and School Leadership (2014) and the related curriculum expectations in the *Australian Curriculum: Mathematics* (ACARA, n.d.-b), where numeracy is one of seven general capabilities. Thus, all teachers, regardless of year level or subject area specialty, are expected to have a high level of numeracy competencies and to incorporate numeracy in their teaching. The focus of this unit is to develop pre-service teachers' competence in numeracy and develop strategies to incorporate numeracy in their teaching. Each week, a different topic/curriculum area (e.g., sustainability, history) is the focus of in-class activities. The focus of one of the weeks was the arts, during which the activities discussed below were completed.

Act(ivity) 1: Angles with turnout

In this activity, students explored how angles are related to turnout. In some styles of dance (e.g., ballet, highland), dancers move with their legs turned out, as opposed to being in parallel. Turnout occurs by rotating the leg from the hip, not by simply turning out the foot. Dancers with 'perfect' turnout are able to fully rotate their legs outwards to form a 180° angle. After a brief demonstration by the instructor (Jennifer) of proper turnout technique, the students (in pairs) completed the activity, in which they had to measure the angle of turnout of their legs.

First, partners took turns tracing around each other's feet while standing on a large piece of paper in first position (i.e., heels together) with legs turned out. Students were reminded that turnout must come from the hip, after some students were observed initially attempting to turn out their legs by only 'cranking' their feet outwards in an attempt to exaggerate their degree of turnout. To further demonstrate and reinforce correct turnout technique, the instructor demonstrated that one must be able to do a plié (i.e., bend one's knees) and have one's knees over one's toes without having to bend forward at the waist, using the analogy of the up and down movement of an elevator. This discussion provided students with additional knowledge about ballet, which further exemplified the cross-curricular nature of this activity, in line with the *Australian Curriculum: Mathematics* (ACARA, n.d.-c) suggestions about making connections between subject areas.

Once both partners had tracings of their feet, students were tasked with measuring the degree of their turnout and comparing their measured turnout with perfect turnout (180°, a benchmark angle). This led to questions regarding the measurement process, including where they should draw lines that they could measure. Such queries hinted at students' past experiences and their understanding that intersecting straight lines were necessary for angle measurement. Students also questioned where such lines should be drawn (e.g., whether the location of the lines, inside/middle/outside the foot, would make a difference in the measurement). From these discussions, the class decided that measuring turnout by drawing lines on the insides of the feet would approximate turnout. A completed tracing is shown in Figure 1.

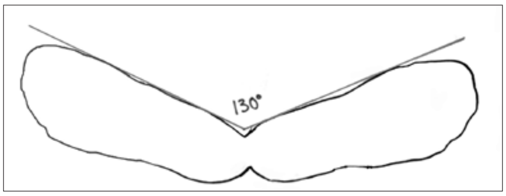

Figure 1. Sample tracing of feet in first position.

Students' fragile mathematical knowledge was evident when some students drew lines beginning on the outside of the heel and finishing within the foot. In a whole-class discussion, the instructor questioned whether the lines that these students had drawn were an accurate representation of the students' turnouts. This discussion prompted students to consider how drawing lines differently altered the final measurement of turnout.

Act(ivity) 2: Geometry with photos and movement

After completing Activity 1, students analysed four photos of dancers, in an activity inspired by Wasilewska (2012). In her paper, Wasilewska discusses various examples of shapes, patterns, angles, and symmetry present in dance. In Activity 2, students had the chance to explore mathematical concepts—particularly types of angles, parallel/perpendicular lines, and transformations—through photographs and movement.

This activity followed from Activity 1 and further explored the use of angles in dance. Students were required to annotate four photos to illustrate various mathematical properties, using mathematical tools (e.g., protractors, rulers). A sample annotated photo is shown in Figure 2.

Once the students had annotated the photos, they were encouraged to create movements and/or shapes with their bodies, if they had time, and their classmates would need to identify the mathematics present. Typically, two pairs would form a group, and one pair would create a movement/shape for the other pair to discuss. This activity is similar to the body-wise activity by Larkin et al. (2004), in which students act out mathematics vocabulary words.

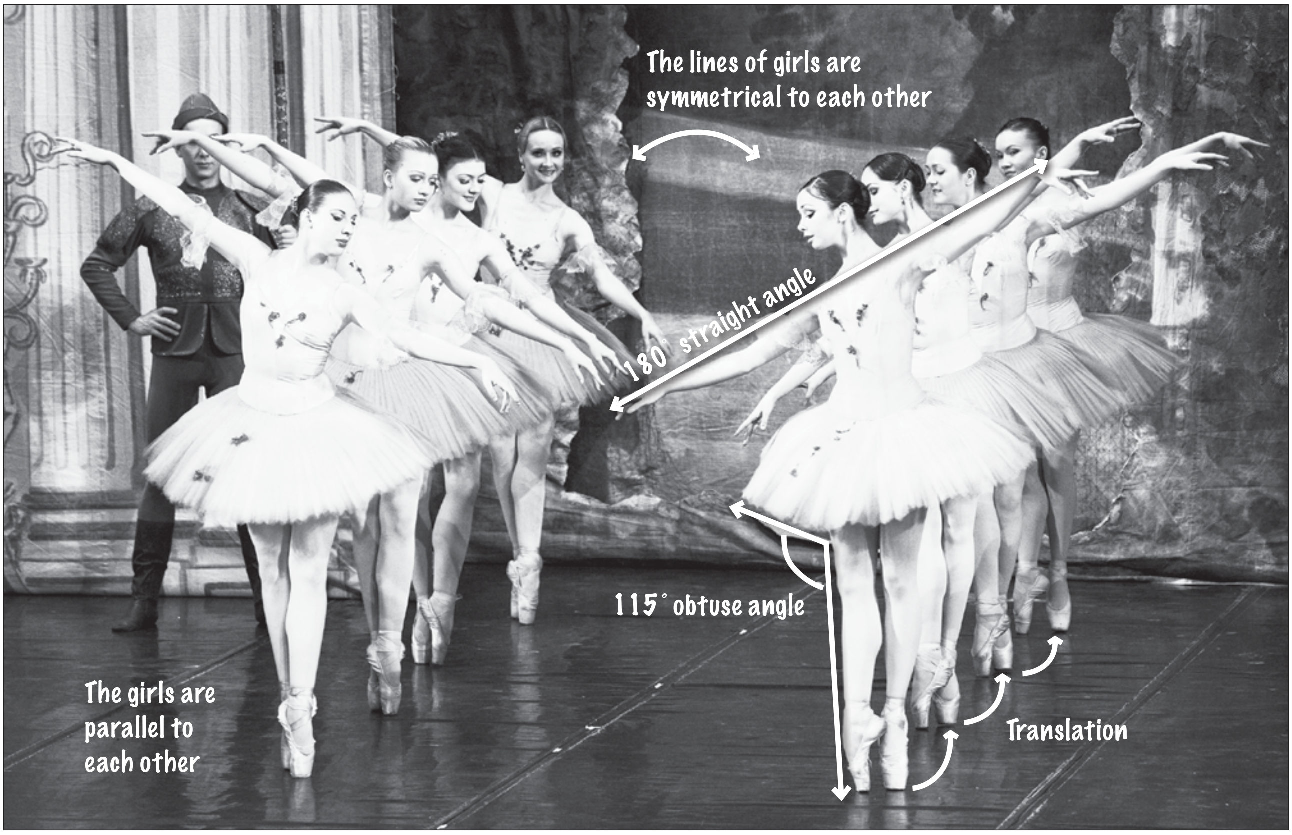

Figure 2. Annotated image from dance activity worksheet.

Our activity extended the body-wise activity as the shapes and movements that the students created with their bodies inevitably incorporated multiple mathematical concepts. For instance, a pair of students might create a symmetrical shape with their bodies that also incorporates various angle types. The students' creations were initially similar to the examples shown in the photographs in the first part of Activity 2. However, students soon began expressing themselves in novel, creative ways, such as building three-dimensional shapes with their bodies, which sometimes required additional students to be involved. While this activity was limited by the time available, students arguably would have come up with even more creative and diverse examples of mathematics with their bodies, given more time.

Coda

The two activities discussed in this article have many benefits, including—but not limited to—collaborative learning, accessibility, kinaesthetic learning, countering stereotypes, and cross-curricular learning. The students were highly engaged, both with the activities and with each other. The nature of the activities necessitated collaboration, and partners supported each other by tracing each other's feet, peer-tutoring to refresh and consolidate content knowledge, and co-creating shapes for classmates to guess.

The activities were accessible as neither the students nor the teacher needed dance expertise. This is in contrast to other examples of mathematics and dance integration that require a dance expert (e.g., Rosenfeld, 2011, 2013). While the instructor had a background in dance, the basic dance principles (e.g., turnout) that she shared in her demonstrations and discussions could easily be illustrated through pre-existing resources such as YouTube videos. Furthermore, our activities were not resource-laden, as they only required limited, inexpensive materials (large pieces of paper, markers, rulers, and protractors) and minimal classroom space (with the possible exception of the last part of Activity 2).

With their kinaesthetic approach, our activities illustrated to students that angles measure rotation and are not simply static 'V' shapes (i.e., a geometric figure comprised of connected rays), thus challenging common limited conceptual understandings (Smith et al., 2014). This kinaesthetic approach further promotes the accessible nature of these activities by allowing students to demonstrate their understanding in a different format, which may particularly benefit students with special needs (Skoning, 2008). Additionally, moving from an enacted/physical representation (creating angles with feet) to an iconic/visual representation (tracing) to a symbolic representation (written angle measurement) aligns with Bruner's (1966) theory of stages of representation for conceptual learning.

Furthermore, these activities dispelled stereotypes by showing that everyone can be successful in (and enjoy) mathematics and dance. Rosenfeld (2013) asserts that integrating the arts into the mathematics classroom allows participants "to challenge and reconfigure our conceptions of what math is and can be" (p. 208). Such activities may also counter stereotypes such as the common misconception that boys have better spatial sense than girls. Yet, girls tend to excel in, and have more experience with, dance and related activities (e.g., gymnastics) that require well-developed spatial skills (Asaro, 2016). Thus, girls (and other students with dance backgrounds) may feel more confident in participating in mathematics activities in a dance context. Indeed, several of the MTeach students who had self-identified as "not maths people" and/or "bad at maths" were far more engaged and demonstrated more confidence in these activities, due to their dance experience. These students acted in leadership roles, supporting their more mathematically confident/experienced peers who lacked experience with dance.

Finally, while ballet was used as the context in which mathematical exploration occurred, the activities also provided an opportunity for students to learn about ballet as a dance form, rather than merely using dance as a novel context for mathematics learning. Therefore, our activities provided an opportunity for true interdisciplinary learning (Brand & Triplett, 2012; Hayes, 2010).

Curtain call

In this paper, we presented two activities that allowed students to explore angles (and related concepts) and recognise their presence in dance (namely, ballet). Specifically, our activities explored the fundamental notion of angles as a measurement of rotation, scaffolding students' learning through an illustration of benchmark angles (e.g., 180°). Although the students were pre-service teachers, we envision that these interactive activities would be relevant and engaging in a primary school context. These activities highlight the connection between mathematics and dance, and allow students to collaboratively learn mathematics concepts kinaesthetically in a real-world context. As the activities are neither resource-laden nor require a dance expert, they are easy to implement, allowing for equity and access for all students, in line with recommendations from both the *Australian Curriculum: Mathematics* (ACARA, n.d.-d) and the *Melbourne Declaration on Educational Goals for Young Australians* (Ministerial Council on Education, Employment, Training and Youth Affairs, 2008).

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