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Life on the Number Line: Routes to Understanding Fraction Magnitude for Students With Difficulties Learning Mathematics

Russell Gersten, PhD[1], Robin F. Schumacher, PhD[2], and Nancy C. Jordan, PhD[3]

Abstract

Magnitude understanding is critical for students to develop a deep understanding of fractions and more advanced mathematics curriculum. The research reports in this special issue underscore magnitude understanding for fractions and emphasize number lines as both an assessment and an instructional tool. In this commentary, we discuss how number lines broaden the concept of fractions for students who are tied to the more general part–whole representations of area models. We also discuss how number lines, compared to other representations, are a superior and more mathematically correct way to explain fraction concepts.

Keywords

mathematics, fractions, number line

Jordan, Resnick, Rodrigues, Hansen, and Dyson (2017) noted that children with learning deficits in number and arithmetic operations, often referred to as dyscalculia, are characterized by a weak understanding of magnitude. In Piazza et al.'s (2010) work, 10-year-old children with dyscalculia performed similarly to typically achieving 5-year-olds for accuracy on numerical estimation tasks, which require magnitude understanding. In this special series, research reported by both Tian and Siegler (2017) and Jordan et al. indicate that the ability of a student in the upper elementary grades to represent the magnitude of a number (either a fraction or a whole number) on a number line with increasing accuracy is a solid indicator of how well a student will perform in upper level mathematics courses. These data align with the theoretical work of Case et. al. (1996) and the more recent integrated theory of number development by Siegler and Lortie-Forgues (2014), which emphasized the importance of magnitude understanding.

This research also suggests that number line estimation tasks, which assess fraction magnitude understanding, should seriously be considered as an efficient screening measure to discern students likely to require intervention. Furthermore, interventions focused on magnitude understanding through number line activities for struggling students in fourth grade (and beyond if necessary) are a logical next step. Fuchs, Malone, Schumacher, Namkung, and Wang (2017) demonstrated in 5 consecutive years that intervention intentionally designed to enhance magnitude and number line understanding is both feasible and effective.

We therefore focus this commentary on magnitude understanding by way of the number line representation. We discuss why the number line is a critical—if often underused—tool for developing students' knowledge of magnitude understanding for fractions. Additionally, we consider the potential for number line instruction to provide students with conceptual foundations for the sometimes arcane procedures involved in adding, subtracting, multiplying, and dividing fractions.

Living on the Number Line

The senior author received an email one night about 8 years ago from Richard Askey, a prominent mathematician who has worked for 15 years to make U.S. mathematics instruction more mathematically accurate. The humorous, but odd, email began, "Mathematicians spend most of their lives

[1]Instructional Research Group, Los Alamitos, CA, USA
[2]Vanderbilt University, Nashville, TN, USA
[3]University of Delaware, Newark, USA

Corresponding Author:
Russell Gersten, Instructional Research Group, 4281 Katella Avenue, Suite 205, Los Alamitos, CA 90720, USA.
Email: rgersten@inresg.org

living on a number line." Though certainly not true for all mathematicians, this premise has intrigued us over the years. So we propose, based on the findings from the studies in this issue, that students struggling with mathematics should start spending *more* of their life on the number line.

Why is the number line critical for magnitude understanding? There are many reasons. The first reason is that the number line *broadens the concept of fractions* in a way that part–whole representations cannot. Many educators—especially in special education, we suspect—rely on part–whole models rather than number lines to teach most fraction concepts. Although introducing fractions through part–whole models makes sense in that it grounds itself in building upon children's prior knowledge and makes crucial links between fractions and geometry/measurement, overreliance on this interpretation of a fraction can lead to overly narrow thinking about what fractions represent. In the research of Jordan et al., one struggling sixth grader, when asked what a fraction is, remarked, "It's a pie!" Similarly, several low-performing fourth-grade students who participated in the Fuchs et al. interventions proclaimed that a fraction is "the shaded part." These two similar anecdotal stories allude to how the overreliance on part–whole models for teaching fractions can generate false understandings.

As mentioned, area models do provide a useful context when introducing the idea of fractional amounts by building on students' prior knowledge and helping students understand that a fraction can be part(s) of a set as well as part(s) of a whole. Identifying a fraction that represents a part of a set but with a denominator that does not match the set is especially challenging for students (e.g., mark 3/5 of 10 circles). For these reasons, we are not suggesting leaving part–whole instruction behind; instead, we advocate shifting toward an integrated approach that includes and emphasizes the number line as fraction instruction begins in third grade. Furthermore, Fuchs et al. (2017) found that before intervention began, the low-performing students in their studies (i.e., fourth-grade students scoring in the lower third on a nationally normed test) had minimal problems with tasks involving area models but struggled with magnitude comparisons and correctly placing fractions on a number line. This is yet more evidence that our instructional efforts should be spent on the more difficult concept of magnitude, rather than the more accessible part–whole understanding of fractions as represented by area models.

A second reason the number line is critical to student learning is that it is the *most mathematically correct way to represent numbers, and especially fractions.* A leading mathematician, H.-H. Wu (2010), notes that one reason fractions are:

> the bane of elementary school students is, without a doubt, the fact that *the concept of a fraction, unlike that of whole numbers, is an abstraction*. Whereas the learning of whole numbers can be grounded on the counting of fingers, there is no obvious replacement of one's fingers for the learning of fractions. (p. 174)

By this, he means that all operations involving whole numbers (even multiplying 907 by 312) are simply extrapolations of operations that can be performed with fingers or concrete objects. Fractions, however, do not function this way. Instead, they force students to confront very abstract, complex mathematical ideas. For instance, by fourth grade most students know that there are an infinite number of whole numbers. With fractions, however, students must expand their understanding of infinity to also learn there are an infinite number of fractions between two whole numbers (e.g., 1 and 2) and even between two fractions (e.g., between ½ and ¾). They also must grapple with the idea that a given fraction can be written an infinite number of ways, all of which are accurate (e.g., three-fourths can be written as ¾ or .75, 12/16, 300/400). With these difficult and abstract concepts, the number line offers a visual and mathematically correct way to help facilitate these understandings.

These are two critical reasons the number line should be emphasized when teaching students about magnitude, which has been neglected in elementary education classrooms in the United States until recently. Encouraging the use of number lines (and multiple fraction representations) to teach fractions is now stressed in virtually all contemporary state standards (Council of Chief State School Officers [CCSSO] & National Governors Association Center for Best Practices [NGA Center], 2016; Florida Department of Education, 2016). However, it remains unclear to what extent it appears in current intervention programs for students with mathematics difficulties (MD) (aside from the Fraction Face-Off! program described by Fuchs and colleagues).

As mentioned, the importance of the number line to fraction magnitude understanding, both in assessment and for instruction, emerged as a major theme in all three articles in this issue. We should look beyond fraction magnitude, however, and think about the potential that number lines have for fraction instruction in a broader sense. As Wu (2010) pointed out, fractions are abstract and cannot be connected directly to concrete objects such as fingers. The number line, therefore, can be a representational tool for fraction understandings beyond that of fraction magnitude. For example, the number line is useful for and a mathematically correct way to teach fraction calculations (e.g., addition, subtraction, multiplication, division). However, using the number line for fraction calculation instruction is only possible if students have a fluent understanding of how fraction magnitudes are represented on a number line. Hence, the research in these articles is of clear importance when considering not only magnitude understanding but also the *potential* for a wider range of instructional approaches for integrating the number line into teaching fractions. Wu (2010) proposed this broader use of number lines throughout instruction on all four operations involving fractions because it is the only representation that mathematically reflects addition, subtraction, multiplication, and division. It also sets the stage for students to become familiar with the conventions used to plot linear functions in Pre-Algebra and Algebra 1.

Increasingly, the community of research mathematicians, mathematics educators, special education researchers, and cognitive and developmental psychologists has concluded that the number line is central to effective fractions instruction. It is a significant tool for building fraction understandings, a point that is reinforced in all three of the research articles in this special series, which specifically look at how students with MD learn fraction concepts. This special series offers a rich research base for the utility of number lines to both teach and assess fraction magnitudes for struggling students and lays the groundwork for using number lines to teach fraction operations for students with MD. So, in moving forward, we have more questions. Can instructional settings across the United States that teach mathematics to students with learning disabilities and students with difficulties in mathematics truly move away from such a large emphasis on area models (e.g., pizza, regional diagrams) in favor of emphasizing the number line? Or are teachers too tied to what's familiar? What sort of professional learning activities will help teachers of students with learning disabilities to build fraction proficiency in this important domain? All are important areas for additional research and attention.

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