

# Coding in senior school mathematics with live editing

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## Introduction

This article is based on a problem-solving task for senior secondary school students. The skills and concepts involved in the task are related to the topic of linear programming. In the task, linear programming is applied to optimise the value of an investment portfolio. The context of the task centres around a scenario in which a large sum of money has been bequeathed in a will. As the tale unfolds, the instructions in the will become more and more complicated, and, in turn, this means that progressively more sophisticated techniques in linear programming need to be applied.

In the task, the students use technology to assist them in finding optimal solutions. The technology that they use is MATLAB (MathWorks, 2016), which is matrix-based software commonly used in industry and in university courses related to science, technology, engineering and mathematics. The task is presented as an interactive computer program. Using the live editor option in MATLAB, however, the script of the program is made to resemble a user-friendly worksheet.

## The story unfolds

The tale begins with an introduction of the scenario:

Your dear uncle Albert has passed away and left you with \$5 000 000 in his will. There are some conditions, however. You are not allowed to spend the money until you are 21. In the meantime, you are to invest it wisely in property and shares. Uncle Albert always said, “Don’t put all your eggs in one basket,” and he has stipulated that you must invest no more than two thirds of the money in shares. You have been advised that the rate of return on property is 8% p.a. whilst shares are returning 10% p.a. Also, you have been made aware that, on a scale from 1–10, shares have a risk rating of 7 whilst property has a risk rating

of 4. You are to aim to maximise your return on the investments but keep the average risk down to a level of 5.

At this stage, the problem can be solved using a two-dimensional graphical method. It is the sort of problem that students would have solved previously in class using a pencil and a ruler. In this case, they are assisted by technology in the production of the graph of the feasible region, but they have to interpret the problem themselves and thereby construct the mathematical model of the objective and the constraints.

Once the students have constructed the mathematical model of the objective function and the constraints, they enter this into some MATLAB code. The essential elements of the code are given to the students and they adapt it accordingly. When this section of the code is run it produces a graph of the feasible region from which the students can determine the optimal solution. The graph of the feasible region is shown in Figure 1 below.

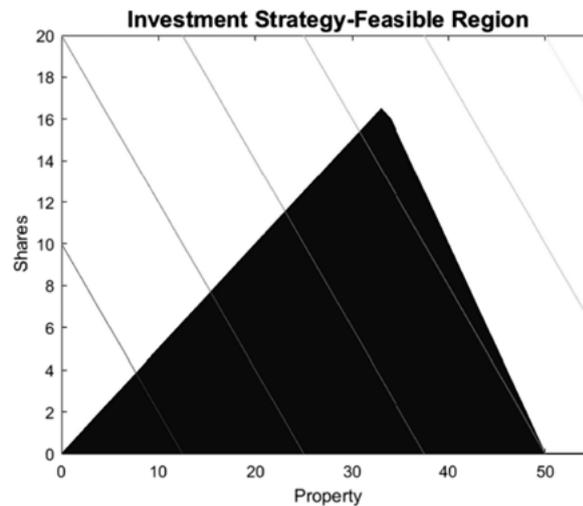


Figure 1. Graph of the feasible region.

The story continues, however, and the plot becomes a little more complex:

Just when you thought you had your investment strategy all worked out, the solicitor rang and explained that Uncle Albert had updated his will. The conditions have been changed slightly. In summary the conditions are now that you have to invest in:

- property (return 8%; risk rating 4);
- shares (no more than two thirds of the total investment; return 10%; risk rating 7);
- government bonds (at least 10% of the total investment; return of 4%; risk rating 2).

As before, you are to maximise your return but keep the risk rating of your investment below 5.

The problem can no longer be solved using a two-dimensional approach. The students are now asked to use the *simplex method* (Johnson, 2016). This involves constructing a mathematical model of the objective function and the constraints including slack variables. Having set up their model in matrix form, the students need to find the solution using the simplex method. Once again, the students are given technological assistance. The row operations that need to be carried out on the matrix can be performed using the MATLAB function **pivot.m**. This function takes the form **pivot(M,r,c)** where  $M$  is the matrix, and  $r$  and  $c$  represent the row and column position of the pivot element. The students decide on the pivot element and then the pivot function carries out the row operations.

There is a final twist in the story, however:

Somewhat embarrassed, the solicitor has rung again. Yet another updated version of the will has been discovered! In this final version, the conditions are that you are to invest in:

- property in outer suburbs (return 6%; risk rating 2);
- property in city/riverside (return 12%; risk rating 9);
- shares (return 10%; risk rating 5);
- government bonds (return 3%; risk rating 1);
- commercial enterprises (return 15%; risk rating 10).

You are also to ensure that:

- Any money that you do not invest will be kept in a holding account and will earn 1.9% p.a. interest. This money would not be considered part of the investment.
- You are to maximise your return but keep the average risk below a rating of 4.5.
- At least 10% of the investment should be in government bonds.
- The amount invested in shares is to be no greater than the total amount invested in property.

The problem has now gone past the stage of being solved in two dimensions, and using the simplex method manually would be laborious even with the aid of the pivot function (Trick, 2016). The students are now given the MATLAB function **linprog.m**. This function will find the solution for the students and only requires them to construct the mathematical model which includes the objective function and the constraints.

## Discussion

### The use of technology

The use of technology in this task raises some questions for discussion. First, which aspects of the task are performed by the students without the aid of technology? Second, which aspects of the task are performed for the students by the technology? Third, does the use of technology enhance the task?

In the first section of the task, the students construct the mathematical model without the aid of technology. The technology produces the graph of the feasible region which the students interpret to find the optimal solution. The students, therefore, need to demonstrate conceptual understanding and skill in linear programming, whilst the technology performs the routine work in drawing the graph.

In the second section of the task, the students have to construct the mathematical model, and decide on the pivot elements to use in the simplex method. The technology performs the row operations on the matrix based on the choice of pivot elements. Once again, the students need to demonstrate an understanding of the model and the method. The technology performs the routine aspects of the method that the students would previously done in class exercises using pencil paper.

In the third section of the task, the students have to construct the mathematical model which by this stage has become more complex. The technology then performs the linear programming required to produce the solution. The students' main focus is on the translation of the problem into a mathematical model, leaving all of the routine work to be performed by the technology.

It is important to note that the third section is preceded by two other sections in which the students have been required to demonstrate their understanding of the underlying processes. Also, the routine work involved in drawing the graphs of feasible regions and carrying out the row operations in the simplex method would have been practised in class using pencil and paper. It would be unfair in this context to consider the **linprog.m** function as a 'black box'.

In each of the first three sections of the task, the complexity of the problem increases. Technology is used at progressively more sophisticated levels to cater for this increase in complexity. Also, as noted above, the use of the technology is focused on the procedural aspects of the task, thereby allowing the students to concentrate on the concepts and the mathematical modelling. It could be argued, therefore, that the use of technology enhances the task by allowing the problem to become more complex, and by allowing the students to focus on the higher order thinking components of the task.

In the final section of the task, the students are encouraged to exercise critical thinking, reflecting on the strengths and limitations of the techniques used throughout the task.

The types of knowledge that the students draw upon when working through the task can be aligned with pedagogical frameworks associated with secondary school learning. In the *dimensions of learning framework* (Marzano, 1992), there are two types of knowledge: the first of these is referred to as *declarative knowledge*, and the second is referred to as *procedural knowledge*. Declarative knowledge is composed of concepts and facts, whereas procedural knowledge is associated with skills. In the task, the students use mainly declarative knowledge, whereas the technology performs much of the procedural work.

In the *21st century learning design framework* (ITL, 2012), *knowledge construction* is one of six dimensions that make up the framework. In the dimension of knowledge construction, it is recommended that the emphasis be on thinking skills including critical thinking, analysis, and synthesis. Activities that favour the application of knowledge in a different context, especially where two academic disciplines are connected, are further recommended. In the task, the students are applying their knowledge of linear programming to a new context and one which connects mathematics with business.

### **The teaching and learning context and relationship to Australian curricula**

The task was given as an assessment piece to senior school students at Ormiston College in Queensland. The students were studying the subject Mathematics C which is based on the syllabus approved by the Queensland Curriculum and Assessment Authority. The task was categorised as an extended modelling and problem-solving task. Since the students were in Year 11, the task was scaffolded in line with the recommendations of the syllabus. The task is also relevant to the learning outcomes of Unit 2 of the subject Specialist Mathematics from the Australian Curriculum for Senior Secondary.

### **The presentation of the task**

In this task the students are using a computer program. They are adapting lines of code given to them and adding lines of code of their own. The task is not presented, however, as a decontextualised exercise in coding for mathematics. The task is presented as a story which links to mathematical modelling which in turn is enhanced by the use of technology. Coding is integrated with the mathematical problem solving.

The balance of narrative, application of mathematical ideas and technology in the task is enhanced through the use of the live editing facility in MATLAB. Using this facility, the comments that are usually added to a computer script to make it easier for another user to follow and to assist in pinpointing errors, can be replaced by a text section in the program. Before live editing was available, comments in a MATLAB script would be made by preceding them with a % sign. These comments would be automatically coloured in green to show that they were not active lines of code but only text for documentation purposes. Using live editing in MATLAB these comments can now be replaced with text that includes headings and images. Mathematical symbols can also be included through the use of a built-in equation editor.

Through the use of live editing, the task can be presented in the style of a worksheet (as shown in the appendix). This belies the fact that it is in fact a computer program which is fully documented and which the students can use in an interactive manner. As well as being user-friendly in appearance, it also convenient that the task and the code are combined in one instrument. Moreover, the code is in sections which can be run independently. This means

that students can test a section at a time and, if necessary, correct any errors. The output can be included in the body of the program or, as illustrated in Figure 2 below, in a sidebar.

Use the linprog function to find the optimal solution to your investment strategy.

The linprog function finds the minimum of  $f^T x \leq b$  such that 
$$\begin{cases} A \cdot x \leq b \\ A = eq = b \\ lb \leq x \leq ub \end{cases}$$

This allows for inequalities, equalities and lower and upper bounds. Note that:

- The command `[x, fval]=linprog(f,A,b,[],[],[0 0 0 0 0])` is the appropriate format for this problem since no equalities are involved and it is only necessary to set the lower bound to zero.
- f represents the objective function, A and b represent the constraints.
- The function finds a minimum and so for problems that involve finding a maximum, it is necessary to multiply all the elements of the objective function by negative one.

```
f=-1*[0.06;0.12;0.18;0.03;0.15;0.019]
A=[1,1,1,1,1,1;
  -2.5,4.5,0.5,-3.5,5.5,0;
  0.1,0.1,0.1,-0.9,0.1,0]
b=[5000000,0,0]
[x,fval]=linprog(f,A,b,[],[],[0 0 0 0 0 0])
```

Output from the sidebar:

```
f = 6x1 double
-0.0600
-0.1200
-0.1000
-0.0300
-0.1500
-0.0190

A = 3x6 double
 1.0000  1.0000  1.0000  1.0000  1.0000  1.0000
-2.5000  4.5000  0.5000 -3.5000  5.5000  0
 0.1000  0.1000  0.1000 -0.9000  0.1000  0

b = 3x1 double
5000000  0  0

Optimization terminated.
x = 6x1 double
1.0e+06
 0.1667
 0.0000
-4.3333
 0.5000
 0.0000
 0.0000

fval = -4.5833e+05
```

Figure 2. Output of the program appears in sidebar.

## Conclusion

In this paper, an example is offered of a problem-solving task for senior secondary school students which was given in the context of a story. As the story unfolds, the task requires progressively more complex forms of linear programming to be applied. Coding in MATLAB is used throughout the task in such a way that it supports the increasing complexity of the task, and performs routine procedural work for the students, thereby allowing them to focus on the mathematical concepts. The task is presented to the students in MATLAB code. Using the live editor facility in MATLAB, the code is made to resemble a worksheet. This approach makes technology widely used in universities and industry accessible and relevant to secondary school mathematics students.

## References

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## Appendix

The task is presented in a form that resembles a worksheet.

### Uncle Albert's legacy: A tale of growth and diversity

#### Background story

Your dear uncle Albert has passed away and left you with \$5 000 000 in his will. There are some conditions, however. You are not allowed to spend the money until you are 21. In the meantime you are to invest it wisely in property and shares. Uncle Albert always said, "Don't put all your eggs in one basket," and he has stipulated that you must invest no more than two thirds of the money in shares. You have been advised that the rate of return on property is 7% p.a. whilst shares are returning 9% p.a. Also, you have been made aware that, on a scale from 1–10, shares have a risk rating of 7 whilst property has a risk rating of 4. You are to aim to maximise your return on the investments but keep the average risk down to a level of 5.

#### Part A: Two-dimensional graphing

Construct a mathematical model of your investment showing the objective function and the constraints. Enter your model into the text space provided below.

Adapt the code below so that it will produce a graph of the feasible region.

To achieve this you should:

- edit lines 5, 6, and 7 of the code below;
- add suitable lines of code to title the graph and label the axes.

```
x = 0:d; % domain for graph
a1= 1;b1= 1;c1= 0; % Edit the values of a1, b1 and c1 to suit your problem
a2= -2;b2= 1;c2= 0; % Edit the values of a2, b2 and c2 to suit your problem
a3= -1;b3= 2;c3= 0; % Edit the values of a3, b3 and c3 to suit your problem
y1 = max((c1-a1*x)/b1,0); % a1*x+b1*x<=c
y2 = max((c2-a2*x)/b2,0); % a2*x+b2*x<=c
y3 = max((c3-a3*x)/b3,0); % a3*x+b3*x<=c3
ytop = min([y1; y2; y3]); % array of minima
area(x,ytop); % filled area plot
hold on;
[u v] = meshgrid(0:6000, 0:6000);
contour(u,v,0.07*u + 0.09*v,50
```

According to your model, how much should you invest in property and how much should you invest in shares? Enter your solution with justification in the space below.

### Part B: Simplex method

Just when you thought you had your investment strategy all worked out, the solicitor rang and explained that Uncle Albert had updated his will. The conditions have been changed slightly. In summary the conditions are now that you have to invest in:

- property (return 8%; risk rating 4);
- shares (no more than two thirds of the total investment; return 10%; risk rating 7);
- government bonds (at least 9% of the total investment; return of 4%; risk rating 2).

As before, you are to maximise your return but keep the risk rating of your investment below 5.

Construct a new mathematical model of your investment showing the objective function and the constraints. Enter your model in the text space provided below.

Use the simplex method to find an optimum solution to your investment problem. Make use of the MATLAB function **pivot.m** to assist you with the row operations procedures. In the code section provided below you should:

1. Enter a matrix A with 5 rows and 8 columns to represent the initial tableau.
2. Use the pivot function repeatedly to produce the solution matrix.

Note:

- The matrix A represents the initial tableau in which the first four rows contain the constraints and the last row contains the objective function.
- Slack variables need to be represented in the matrix A.
- The pivot function takes the form  $\text{pivot}(A,r,c)$  where the inputs are the matrix A, and r and c are the row and column positions of the pivot.

### Part C: Linear programming in MATLAB

Use the simplex method to find an optimum solution to your investment problem. Make use of the MATLAB function **pivot.m** to assist you with the row operations procedures. Type your code in the space provided below.

Somewhat embarrassed, the solicitor has rung again. Yet another updated version of the will has been discovered! In this final version, the conditions are that you are to invest in:

- property in outer suburbs (return 6%; risk rating 2);
- property in city/riverside (return 12%; risk rating 9);
- shares (return 10%; risk rating 5);
- government bonds (return 3%; risk rating 1);
- commercial enterprises (return 15%; risk rating 10).

You are also to ensure that:

- Any money that you do not invest will be kept in a holding account and will earn 1.9% p.a. interest. This money would not be considered part of the investment.
- You are to maximise your return but keep the average risk below a rating of 4.5.
- At least 20% of the investment should be in government bonds.
- The amount invested in shares is to be no greater than the total amount invested in property.

Construct a mathematical model of your investment showing the objective function and the constraints. Enter your model in the text space provided below.

Use the linprog function to find the optimal solution to your investment strategy.

The linprog function finds the minimum of  $f^T * x \leq b$  such that

$$\begin{cases} A * x \leq b \\ A * eq = b \\ lb \leq x \leq ub \end{cases}$$

This allows for equalities, inequalities, and lower and upper bounds. Note that:

- The command `[x,fval]=linprog(f,A,b,[],[],[0 0 0 0 0])` is the appropriate format for this problem since no equalities are involved and it is only necessary to set the lower bound to zero.
- `f` represents the objective function, `A` and `b` represent the constraints.
- The function finds a minimum and so for problems that involve finding a maximum, it is necessary to multiply all the elements of the objective function by negative one.

According to your model

1. What is the optimal solution to your investment strategy?
2. How much should you allocate to each type of investment?
3. How much money will your investment earn in one year?

Enter your responses to this section in the text space below.

#### Part D: Strengths and limitations

1. Describe one strength and one limitation of each of the mathematical models you used in Parts A, B and C above.
2. Identify one significant assumption that was made in Part C and describe using examples calculations the associated mathematical effect of this assumption.
3. Comment succinctly on the reasonableness of your results.

Enter your responses to this section in the text space below.