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Attitudinal and Cognitive Beliefs of Two Preservice Secondary Mathematics Teachers

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Abstract

The objective of this paper is to present attitudinal and cognitive beliefs of two preservice secondary mathematics teachers about teaching geometric transformations (GTs) using Geometer's Sketchpad (GSP). The study comprised ten task-based interviews, a series of five each with two participants, senior undergraduate preservice mathematics teachers, at a medium-sized public university in the Rocky Mountain Region of the United States of America. Radical constructivist grounded theory (RCGT) was the theoretical framework used both to guide the study process as well as to integrate the research participants, the researcher, research process, and the data. The results of the study include in vivo categories of the participants' beliefs associated with attitude and cognition of teaching GTs with GSP. These include developing confidence with GSP, efficiency of using GSP for teaching GTs, exploring GTs with GSP, conjecturing GTs using GSP, supporting and engaging students in learning GTs, and understanding GTs with GSP. Some pedagogical and policy implications of these categories are also addressed.

Introduction

Technology integration is an essential part of mathematics education for meaningful and effective teaching and learning in many countries. The National Council of Teachers of Mathematics (NCTM, 2000) highlights the importance of technology in teaching and learning mathematics by including technology as one of the overarching themes of its six principles. In its 2015 position statement, "Strategic Use of Technology in Mathematics Education," NCTM states that "the capabilities of technology enhance how students and educators learn, experience, communicate, and do mathematics" (p. 1).

The Common Core State Standards in Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) also highlights the use of technological tools in teaching and learning mathematics to make sense of mathematical problems, to reason about mathematical problems abstractly, to model mathematical problems, and to enhance algebraic thinking. Indeed, how teachers use technological tools in a math class and why they use the tools the way they do, may depend both on their motivation to use the tool as well as their core beliefs about the technology application in mathematics education (Ertmer, 2006). For example, some mathematics teachers use technological tools in teaching math-specific mathematics content, including algebra, statistics, and geometry. Other teachers use them to support mathematical practices in the classroom, such as engaging English language learners in mathematics classrooms, teaching mathematical modeling, integrating standards in mathematics teaching, assessing students' learning of mathematics, or creating interactive online spaces for students to learn and share their mathematical understanding (Polly, 2015). A teacher can, therefore, use technological tools to make math contents interesting to learners (Ertmer et al., 2012) Abstract mathematical concepts comprising equations or formulas can be made visible through graphic displays (Foley & Ojeda, 2007) and collaborative or individual online platforms can be created for problem-solving (Garry, 1997).

Teachers' existing beliefs about technological tools may play a vital role in their effectiveness in the mathematics classroom (Chai, Wong & Teo, 2011; Chen, 2008; Leatham, 2002). Although some studies report inconsistencies in beliefs and teacher practices, teachers' practices do tend to reflect their beliefs (Ertmer et al., 2012). These inconsistencies are due to classroom and institutional factors, available resources, limited understanding, and inconsistent or conflicting beliefs (Chen, 2008). Zhao and Cziko (2001) maintain four key elements in teachers' beliefs about how they may use the technological tools--achieve higher-level goals, sustain the higher-level goals, ability to use the tools effectively, and motivation to acquire the resources to use technology in teaching mathematics. Their beliefs, therefore, function as "a filter through which they determine the priorities of different factors" (p. 67).

Many researchers and scholars characterize teacher beliefs about technology integration as no use of technology, pre-mastery use of technology, post-mastery use of technology, and exploratory use of technology for teaching mathematics (Misfeldt et al., 2016). Erens and Eichler (2015) categorize mathematics teachers as initiators, explorers, rein forcers, and symbiotic collaborators, based upon their expressed beliefs about technology integration in teaching mathematics. Chen (2011) classifies teachers' beliefs as instrumental and empowerment, based on how they use the tools in the classroom. In addition to the aforementioned categories, Cullen and Greene (2011) discuss preservice teacher beliefs and attitudes toward technology integration. They found that teachers who have positive attitudes toward technology integration also have positive beliefs about the tools and its application in teaching mathematics, and those who have negative attitudes have negative beliefs about the use of the tools. A positive belief about technology integration helps teachers and students in the effective use of tools in mathematics teaching and learning. This suggests that for a successful implementation of technology integration in the mathematics classroom, teachers should have positive beliefs so that they can focus on technological, pedagogical and content knowledge (TPACK) (Mishra & Koehler, 2006; Niess, 2005). Many mathematics education programs currently integrate TPACK into mathematics teacher education programs to enhance the knowledge of preservice and inservice teachers in terms of content, pedagogy, and technology in a holistic fashion (Hunter, 2015), in order to have a positive influence on their beliefs about technology. With a successful TPACK in teacher education programs, practicing and future teachers may use technological tools in mathematics classrooms as demonstrating, disseminating, exploring, connecting, and thinking tools (Cuevas, 2010). These applications of TPACK in programs may play a significant role in developing positive attitudes and beliefs about technology integration among mathematics teachers (Misfeldt et al., 2016).

Most of the studies on mathematics teacher beliefs about technology integration are centered around the general views on the use of technological tools in day-to-day mathematics classes. Many studies of beliefs of preservice or inservice mathematics teachers categorize their beliefs between the dichotomies of positive or negative beliefs and constructivist or instrumentalist beliefs. Some studies discuss these views in terms of the function of technology in teaching and learning mathematics, such as technology for algebraic thinking and reasoning, technology for the meaning of mathematical relations, technology for interpretation, and contextualization of mathematical content and pedagogy (Polly, 2015).

The studies mentioned above draw conclusions either from quantitative surveys or descriptive qualitative data. The data in qualitative studies on the beliefs of mathematics teachers tends to be presented with such inadequate details that the reader is unable to grasp the context and questions that might have a connection to the expressed beliefs. Both quantitative and qualitative research traditions on teacher beliefs about technology integration appear, then, to revolve around the categories of beliefs and descriptions of those categories. The study in this paper, however, considers research into teachers' beliefs about technology integration as a symbiosis between researcher and participants, a tool to give voice to the research participants with the investigator, construction of meaning through cognitive and adaptive function, and balancing the theory and practice of analysis and interpretation through fit and viability of categories within the context (Belbase, 2015 & 2016). This theoretical framework helps readers to understand the perspective and viewpoint of the researcher through which he performed the data collection, analysis, and interpretation in this study. The research question for the study was: What beliefs do preservice secondary mathematics teachers hold about teaching geometric transformations using Geometer's Sketchpad? There were six key categorical findings of preservice secondary mathematics teacher beliefs about teaching geometric transformations (GTs) using Geometer's Sketchpad (GSP) in the study. In this paper, I present two of the six key categories, attitudinal and cognitive beliefs of teaching GTs with GSP. These categories are subdivided into six subcategories which are discussed in detail in the findings and discussion sections.

For purposes of this study, attitudinal beliefs are those beliefs that are influenced by or the result of one's attitude toward the use of GSP in teaching GTs. Preservice or inservice mathematics teachers may develop their attitudinal beliefs about the use of the tool in teaching GTs or other content areas of mathematics based on their general attitude (as positive or negative) toward the use of any technological tool. These beliefs are therefore associated with the personal attitude which affects use of the tools in teaching and learning mathematics (Johnson, 2013; Mahmod, Dahlan, Ramayah, & Karia, 2005). Cognitive beliefs are those beliefs associated with one's cognitive ability to use the tool by making a connection to knowledge and skills in teaching appropriate content, with an increase in perception of both usefulness and ease of use (Bere & Rambe, 2013).

RCGT is introduced as the theoretical framing of the study in the next section. Then, I highlight the major steps of the methodological trail in the study, present and discuss the results, and finally summarize the findings with implications in this paper.

Theoretical Framework

My epistemology throughout this study has been guided by radical constructivism and the methodology has been influenced by interpretive ground theory. I synthesized five assumptions of radical constructivist grounded theory (RCGT) by integrating radical constructivism (e.g., Von Glasersfeld, 1978, 1995) and grounded theory (e.g., Charmaz, 2006; Strauss & Corbin, 1998) to guide the study process, including data construction, description, analysis, and interpretation. I discuss each assumption in the following subsections (Belbase, 2016, 2015, 2014).

Symbiotic Relationships

My anticipation was that the research participants and researchers would develop symbiotic relationships through the process of co-constructing knowledge from research. Both in radical constructivism and grounded theory methodology, this symbiosis between investigators and participants is a natural phenomenon (Charmaz, 2006; Corbin & Strauss, 2008; Steffe & Thompson, 2000; Von Glasersfeld, 1995). Data was constructed through task-based interviews (Goldin, 2000; Maher, 1998) during which the participants contributed to the study through their participation, construction of narratives, and reflections on their experiences of using GSP (Steffe, 2002; Steffe & Thompson, 2000).

Voice of the Researcher and the Participants

I assumed that the research would carry participants' voices in different forms, such as vignettes, protocols, narratives, and life stories. These voices are direct expressions of the participants in different genres (both verbal and nonverbal). The research process and outcomes carry the voices of the participants in the first-person perspective (direct voice) through the narrative protocols, and the researcher's voice in the second- and the third-person perspectives (indirect voice) in the form of my interpretive and reflexive accounts (Bergkamp, 2010; Hertz, 1997; Warfield, 2013).

Research as a Cognitive Function

Next, I considered the research process as an active cognitive process. While conducting task-based interviews with participants, I decided in the task situation itself, what questions to ask, what activities to observe, and how to record and perform in situ data construction (Friedhoff et al., 2013). I performed coding, categorizing, and theoretical memoing of data. The coding, categorizing, and theoretical memoing continued as developmental process. From this process, codes were not fixed from the beginning but modified according to new concepts and meanings coming from the latter analyses by addition of new codes. Through these processes, I learned about new codes coming from the latter parts of interviews that supported the major themes. I maintained theoretical sampling and constant comparison to the central concepts or categories throughout additional interviews (Charmaz, 2006). At the same time, I engaged the participants in interactions during the task-based interviews. The series of interactions helped us in the "organization of our experiential world" (Von Glasersfeld, 1990, p. 19), forming and shaping our knowledge and beliefs. The entire research process was itself an active cognitive function (Bailyn, 1977).

Research as an Adaptive Function

The next assumption is the construction of new knowledge from research as an adaptive function. In the present study, the development of codes and categories was not a one-time activity, but was an ongoing process throughout the research (Lichtenstein, 2000). This means that the construction of grounded categories was established through reorganization of concepts or themes with changing data, context, and interpretation during the task-based interviews. In this sense, the study is associated with the development of qualitative, constructive, and adaptive grounded categories (Layder, 1998; Welsh, 2009). The research participants and I shared experiences, meanings, and contexts while working through problems related to reflection, rotation, translation, and composite transformation using GSP. Therefore, we adapted to the new experiences in terms of categories and meanings through the research activity.

Fit and Viability of Theory (Praxis)

The final assumption is that the major categories or themes from the data should be examined with two praxis criteria, criteria of fit and criteria of viability (Von Glasersfeld, 1995). I examined each category from the data to see if they resonate with the participant's expressed beliefs (Charmaz, 2006; Glaser & Strauss, 1967; Strauss & Corbin, 1998). Also, I observed those categories in terms of viable explanation of their beliefs in the context of teaching GTs with GSP. While doing this, I also foresaw the "transformative possibilities of the research process and product" (Rodwell, 1998, p. 79). To maintain the fit and viability of the categories to their expressed beliefs, I requested a fellow graduate student to serve in peer debriefing, reviewing, and auditing the codes, categories, and concepts that came up from the data. The peer-reviewer and I worked together to make sure that the inquiry process through the coding and categorizing accomplished the goal to construct meaningful categories from the interview data. The peer reviewer was a critical other to provide me an "affective and intellectual dimensions of the inquiry…" (Rodwell, 1998, p. 194) by aligning the categories to the significant findings from the data.

I employed these five assumptions of RCGT, symbiotic relation, voice, research as a cognitive function, adaptive function, and praxis, for the Study of Preservice Secondary Mathematics Teachers' Beliefs about Teaching Geometric Transformations Using Geometer's Sketchpad, by outlining the role of the participants and researcher (this author) and identifying our position in the research process.

Method

In this section, I discuss the research process in terms of selection of participants, administration of five task-based interviews with each participant, post-interview analysis of the data, writing theoretical memos, and construction of codes and categories. I then discuss each of these steps in the subsequent subsections.

Selection of the Participants

I consulted a class of twenty students in methods of teaching mathematics for preservice secondary mathematics teachers in the fall of 2013 at a medium-sized public university in the Rocky Mountain Region of the United States. I selected two participants (one male and one female) depending on availability of their time for interviews and their interest to volunteer in the study. The male participant was a returning student to college whereas the female participant was a continuing college student and both aimed to be mathematics teachers after graduation.

Administration of Interviews

The first two interviews took place at the end of the fall 2013 term during the Methods of Teaching Mathematics course, and the remaining three interviews were conducted in the spring 2014 during student teaching internship. Each interview was focused on a different area of geometric transformations: reflection, rotation, translation, and composite transformation. The interviews lasted between 37-86 minutes. All the interview episodes were recorded using a digital video recorder. I conducted the first two interviews myself and recorded them for transcribing and analysing. A graduate student of mathematics education assisted me to conduct the final three interviews. I observed the interviews and took note of important points. I transcribed the interview data verbatim for analysis and interpretation.

Writing Theoretical Memos

I wrote a reflective memo after each interview to support the analysis and interpretation of the data. These notes included major points that were discussed during the interview sessions, the observations of the interview process, and nonverbal expressions of the participants. These memos helped me in constructing themes and categories during the coding process. It helped me in constant comparison of the codes, categories, and themes. The first (initial) codes from the data were connected to broader concepts or categories, leading to the construction of final, more focused, categories of participants' beliefs. The writing of memos also helped me in forming second- and third-order interpretive accounts during interpretation of the data.

Analysis and Interpretation

I carried out the analyses and interpretations of the interview data in two phases. In the first step, I performed a classificatory analysis and interpretation using the principles of grounded theory (Charmaz, 2005; Strauss & Corbin, 1998). This kind of analysis was based on the grounded theory approach to finding concepts and categories from the pieces of data. In the second phase, I analyzed and interpreted the data using a holistic approach (Hall, 2008). This analysis was based on the constructivist approach to finding meanings of data. The whole analysis and interpretive approach was guided by the five assumptions of RCGT. I transcribed the interview data verbatim for each interview episode, and the transcribed texts were used for the analysis and interpretation. I carried out analyses of data with open coding, axial coding, and selective coding to construct codes, categories, and themes (Corbin & Strauss, 2008; Strauss & Corbin, 1998). I supported theoretical sensitivity by remaining open and reflective to the elements of theoretical importance in the data (Charmaz, 2006; Corbin & Strauss, 2008).

Results and Discussion

The subcategories related to preservice secondary mathematics teachers' attitudinal and cognitive beliefs about teaching GTs with GSP are developing confidence with GSP, the efficiency of using GSP for teaching GTs, exploring GTs with GSP, conjecturing about GTs using GSP, supporting and engaging students in learning GTs, and understanding GTs with GSP. I discuss each of these subcategories below under a different subsection.

Developing confidence with GSP: Temporal, visual, and relational aspects

The participants articulated their beliefs about their self-confidence in using GSP in the context of teaching and learning GTs. The following sample of an interview transcript in Table 1 is an example in which they expressed their confidence in using GSP for teaching GTs. The higher/lower confidence level in using GSP may come through their ability to use the tool, and hence it represents their cognitive belief.

Table 1. Interview transcript on 'developing confidence with GSP'

Researcher: Do you think you have/have not developed confidence in using GSP?
Cathy: Yes, I do have confidence in using GSP.
Jack: Yea. I am getting there. I would be more confident when I have time to do it. I just haven't spent enough time on it. I would like to spend more time on it.
Researcher: What has brought up that confidence?
Cathy: Foundations of Geometry class, that wonderful class.
Researcher: In your experience how does GSP help in developing confidence in learning transformations?
Jack: Let's say kids are struggling, even with they say graphing, but they are really good in computer. Think how confident they are gonna be if they can do this, and they can see it. They are actually gonna be able to construct it. You know, maybe a kid is struggling with it. All of a sudden that kid sees it in the computer what's happening. Maybe that just turns him around and builds his huge confidence of what he is seeing.
Cathy: Yes, but only if the teacher is able to make that because I could see it as an older teacher does not build their confidence to be saying 'yes' or 'let me check'. So, if you as a teacher are confident enough then you can look at that and be able to prompt them if it is wrong. Why did you do that? ...bla bla bla... So, if you are confident enough, you can definitely build your students' confidence. If you are not confident, then they just gonna be the same.

Cathy expresses her confidence in using GSP for classroom activities while teaching GTs. She reiterates that she learned to use GSP in the Foundations of Geometry course she took a year ago in "…that wonderful class" meaning that for her, the Foundations of Geometry was a milestone in learning about GSP. The use of GSP in problem-solving and writing proofs in geometry developed a high-level of confidence. She expresses her confidence in doing different explorations and constructions about GTs with GSP. Seeing the process of GTs in the dynamic environment of GSP developed her confidence and she seems to think that it can also help students develop their confidence with using the tool when they can see what is going on with the reflection, rotation, translation, and their composites. She says, "If you as a teacher are confident enough then you can look at that and be able to prompt them if it is wrong." She considers that students' confidence in using the tool and solving various problems related to GTs with GSP depends on the teachers' confidence in doing them. She believes that the teacher confidence increases the student confidence about the tool and processes while working on GTs. On the other hand, Jack shows a limited confidence with using GSP for teaching GTs. He says, "I would be more confident when I have time to do it." He considers that doing something by oneself in GSP develops his or her confidence in using the tool. The level of confidence is influenced by what one can see. He claims that, "Maybe that just turns him around and builds his enormous confidence with what he is seeing." Even a student who is struggling in learning GTs may have excellent computer skills, and he or she can use GSP to find out more about GTs.

When a student sees the process of reflection, rotation, or translation in the computer, it just changes his or her ability to grasp the ideas in GTs. For Jack, when a student can see it (GT process) on the computer (with GSP), that ultimately adds to his or her confidence. Therefore, Jack appears to believe that one's confidence level depends on the opportunity to use the tool, ability to see what is going on in the processes with a GT, and how long he or she has gone through struggles during the learning. Therefore, Cathy and Jack's beliefs about developing confidence with GSP in teaching GTs are associated with temporal, visual and relational factors that provide a theoretical basis to associate these beliefs to the literature of teacher beliefs in the third-order interpretation.

One can see three important points from this transcript in relation to Cathy and Jack's sense of confidence in using GSP to teach GTs. First, it is about students' ability to construct. Jack reveals his thinking that his confidence with using GSP may be reflected in his students' ability in doing construction about GTs with the tool. The second point is about what his students can see when they construct GTs with the use of GSP. If the students can see what is going on within a GT that may change their whole understanding of GTs by using GSP. The third point comes from Cathy's view about prompting on what students are doing and developing their sense of confidence. That means a teacher's confidence in using GSP for teaching GTs can be viewed indirectly from the students' ability to use the tool and follow the prompts and construct something about GTs with the tool. These beliefs are analogous to literature and theory of teacher beliefs about confidence in using technological tools in teaching and learning. Many researchers and authors (e.g., Eu, 2013; Keller, 2010) have highlighted the issue of student learning associated with their confidence in using technology. Lack of confidence in the use of GSP creates a dilemma, and it leads to avoidance of using the tool for teaching and learning of GTs in the classroom (Tsamir, 2015). Cathy seems confident in using the tool for teaching and learning of GTs, but she does not see it as important at the beginning. She prefers prompting over use of the tool to enhance students' confidence in the concepts of GTs, whereas Jack, despite his limited experience with GSP, feels that students' creation and visualization of GT processes may improve their confidence.

Efficiency of Using GSP for Teaching GTs: Enhancing, Exploring, Understanding, and being able to Access

The participants expressed their beliefs about the efficiency of using GSP in teaching and learning GTs. In this subsection, efficiency is associated with the comfort and competence of using the tool and doing things in a shorter time with the ease of teaching and learning. The following interview transcript in Table 2 shows an example of participants' sense of efficiency in using GSP for teaching GTs. The sense of efficacy shows their attitudinal beliefs about the use of the tool in teaching GTs.

Cathy does not consider that GSP is a teaching tool. However, she accepts that it enhances teaching. She considers GSP neither as a teaching nor as a learning tool. She could use it for teaching, but she does not like doing that at first in her lesson: "I would want them to explore and deepen their understanding." She wants her students to do constructions and other procedures with paper and pencil. She expresses her thought that students can be engaged in exploring other properties without going into details of constructions by using shortcuts that GSP offers. For her, GSP does not do the procedure, but it can show the concepts in a clearer way. She reveals her view that it is an easy tool for a poor teacher who is not well prepared to deal with the content. For such teachers, GSP does everything very quickly and easily. He or she does not have to go through the details of constructions and procedures. For Cathy, it lends a straightforward approach to enrich teaching GTs. Cathy expresses her thoughts that GSP makes the learning of GTs interesting to those students who either hate math or find it tedious. Therefore, she appears to believe that GSP is not a teaching and learning tool, but it is an exploring and practicing tool for students. She says, “I have to, honestly, get more comfortable with the idea of teaching with it.”

Table 2. Interview transcript on ‘efficiency of using the GSP for teaching GTs’

Researcher: Do you think that GSP is an appropriate tool for teaching transformations?
Cathy: Not to teach, but to enhance teaching.
Researcher: GSP is a tool for teaching GTs or learning GTs?
Cathy: Kinda neither. I would say it is for exploring a lot like. I would teach with it, and I would not expect kids to learn from it. But, I would want them to explore and deepen their understanding.
Researcher: Do you think/don't think that GSP is useful in teaching GTs?
Cathy: I think it could be. I have to honestly get more comfortable with the idea of teaching with it, because I have to figure out the procedural side. But, like again you can only rotate. Ok, you have to say rotate. Maybe if we can do this- ‘How you find the center of rotation?’ This is how I am gonna do it. I don't know. If I have figured out the procedural side, I would feel more comfortable in teaching.
Researcher: Does the use of GSP make teaching transformations easy?
Cathy: Yea. I don't think you have to, like if you were a poor teacher, in my opinion, teach it. You can go into rotations now. Go to GSP, click and rotate. Figure out what you need to do. So, I think, yes it would make really easy because you don't have to do anything. With that even if you use it use it correctly. You make sure that they know what a rotation is, not that just you go and click that button.
Jack: I would like to think so. It depends on your access (of GSP) in the classroom. If you have access, the students actually see what's happening. If you have access, that's nice.
Researcher: Does it make it faster?
Cathy: Yes, so much faster cause you don't have to hand draw.
Jack: I don't think it's faster. If they already have the knowledge base, and you are just going and saying we are doing this, then yes. If you have to build on to their knowledge and how to use the program first, I think it's gonna go slower. That takes a lot of time.
Researcher: Does GSP make it more interesting?
Cathy: To an average student that hates math and is not really into it, I think GSP makes it a lot interesting. Personally, I like math, so I am gonna find interest on pencil and paper cause I am constructing it still while students don't. Early they find it tedious. They don't want to do that. They are not getting to a point fast enough. So, they are getting lost. For them, it is a lot more interesting.
Jack: Oh, it definitely makes lessons more interesting. Like as I said I think in Sketchpad you can import pictures. You can lay stuff over on top of them and show them real-life applications. They can build the Ferris wheel. They can build their own stuff and see it. It's not like a picture and trying to spin it. That's just a picture. With this (GSP) they can actually build it.

For Jack, teaching GTs with GSP is easy if it is accessible to all the students in the class. He thinks that GTs can be taught very well by using GSP. He accepts that a greater accessibility of GSP in the classroom makes the teaching of GTs easier, faster, and more meaningful. He says, “If you have access, the students actually see what's happening.” However, he considers students’ foundational knowledge of geometry and GSP as the factors of efficiency. If students are not ready with such foundational knowledge of geometry or the technology (i.e., GSP), then they may take a longer time while working with GSP. It can be interesting to some students who know the basics, but it may be frustrating for others due to a lack of foundations. Jack further expresses that, “You can lay stuff over on top of them and show them real-life applications.” Nevertheless, a teacher can make lessons on GTs interesting by using GSP and importing pictures in it, for example. The teacher can use the images for different GT functions, not just the geometric patterns or polygons. He seems to believe that the students can build on the things they already know by using GSP, provided they have access to this tool.

The sense of efficiency is different for Cathy and Jack in using GSP for teaching GTs. Cathy’s focus is on developing conceptual understanding of GTs before the exploration of different properties with GSP. The shortcuts of GSP may not help students to understand the details of GT processes. For her, other hands-on activities with paper and pencil are more important than using GSP for the conceptual teaching of GTs, whereas for Jack, foundational knowledge of the content and technology and accessibility of the tools in the classroom make teaching and learning of GTs more efficient. These views resonate with the theory of teacher beliefs concerning the use of technology for efficient teaching of mathematics. Some researchers and authors (e.g., Tajuddin, 2009) discussed the instructional efficiency of technological tools in teaching and learning mathematics. They focused on availability (or accessibility), portability, manipulability, applicability, performability, and precision as part of the efficiency of a technological tool for teaching mathematics. Cathy is interested in using GSP for exploring after teaching procedures and concepts of GTs. Jack is interested in using GSP for teaching concepts once procedures are done with paper and pencil. It seems that both agree on the view that students can learn geometric transformations with a greater efficiency by constructing, conjecturing, and exploring properties of GTs with GSP (Almeqdadi, 2000; Kor & Lim, 2009; Niess, 2005). The difference is only about purposeful uses and timing of using the tool for teaching and learning GTs. Therefore, both Cathy and Jack believe that GSP provides a greater efficiency in either teaching or learning and practice of exploring GTs.

Exploring GTs with GSP: Surface Stuff versus Depth and Learning Curve

The participants stated their beliefs about exploring different properties of GTs using GSP in the context of teaching and learning. Here, the meaning of exploring is associated with the investigation of properties using conjectures and proofs (both formal and informal). I have identified some key points of the participants' beliefs about exploring GTs with the use of GSP. I have discussed these points in the second and the third-order interpretations of their beliefs. The following interview transcript in Table 3 is an example to show their beliefs about the use of GSP for exploring properties of GTs. The features of GTs discussed are associated with the geometric properties. These beliefs are associated with their perceived usefulness of the tool, and hence they are cognitive beliefs.

Table 3. Interview transcript on 'exploring GTs with GSP'

Researcher: Do you think you can/cannot explore properties of GTs with GSP? Cathy: Yes, I can, and we should explore that. Researcher: Why should you? Cathy: Because I want to explore, it is not just what's subject, you are not gonna want surface stuff but want to dig into the topic. Researcher: Does this GSP help in exploring properties of any transformations or even composite transformations? Cathy: Yes, it does. Um, you can do, there is on reflections, it's like a Mira, stand up like this, you don't have to draw it every time. You have to set it next to it and look through it. That's the one. There you can. You, kind have to look through and draw it if you gonna explore. So, that makes that part have them faster, I guess, and get the result of their conjectures a lot faster. And, if they are exploring the area they don't have to go and find the height and the base to find the area (of a triangle). They can just directly go to the area because that's the conjecture, and that's what they need. Jack: Um, you could talk about how the shapes are congruent. You can talk about angle of rotation. You can talk about distance between them (points), but I don't know if that's what we gonna do with rotation. I know my high school classes were of direct instructions. Kids can be bored. In a group-work, they can explore some real-life stuff on their own too. It basically keeps them not just listening to me rather see them each other as groups that are more effective sometimes. You can explore like generalized formulas with them which allow a greater algebraic expression. GSP has a lot of learning curve.

Cathy believes that she can explore the properties of GTs by using GSP. She also iterates that a teacher should do it to dig deeper into the topic: "…you are not gonna want surface stuff but want to dig into the topic." She accepts that GSP is helpful to explore different properties of not just a single transformation, but it is helpful to explore the properties of composite transformations. For her, use of GSP makes the process of GTs faster, and this supports in proving conjectures. Also, GSP being a shortcut tool, Cathy seems to believe that students do not have to go through details of steps while measuring areas, for example. She claims that, "They can just directly go to the area because that's the conjecture, and that's what they need." If the conjecture is about showing that a GT preserves area, then they can directly go to measure the areas of the object and the image and compare them. Therefore, Cathy appears to believe that exploring GTs with GSP is helpful to develop conceptual understanding. However, this view contradicts her earlier view that she would not use GSP in the classroom to teach the concepts. In the same line, Jack reveals his thinking that GSP can help him explore the shapes (geometric structures) if they are congruent. For him, GSP helps in exploring angles of rotations, the distance between points (from the object to the image), and create proofs. He sees limitations of direct instruction in terms of student motivation, and he prefers group work for exploring real-life experience related to GTs by using GSP. He expresses, "I know my high school classes were of direct instruction. Kids can be bored." He even thinks that students can explore the algebraic properties of reflection (or other transformations) relating to the coordinates of the object and the image points. Further, he accepts that students can generalize the algebraic expressions of a GT process. In this way, he believes that GSP has 'a lot of learning curve'.

Here, Cathy and Jack seem to agree that GSP is helpful in exploring the properties of GTs. Cathy accepts that it leads into a deeper understanding of the topic (properties). It also provides a shortcut to explore the properties. Jack focuses on different properties, such as the angle of rotation and distance. The process of exploring GTs with GSP is also related to ownership. When students study the properties themselves using the tool, they may own it (both the process and the content they learned). These beliefs have a connection to the literature and theory of teacher beliefs and hence can be further reinterpreted in the next level.

The major points from the above discussion include a sense of necessity to promote exploration with GSP, a deeper understanding of GTs with GSP, direct exploration without details of constructions, the scope of explorations with GSP, and learning curve with GSP. These beliefs have significance in the literature of teacher beliefs. Many researchers and authors (e.g., De Villiers, 1998; Dywer & Pfiefer, 1999; Keyton, 1997; Manouchehri et al., 1998; Olive, 1992, 1993, 1998; Shaffer, 1995; Tyler, 1992) have highlighted the use of GSP for exploring various geometric properties during teaching and learning of mathematics. The two preservice teachers in this study seem to have a sense of necessity to explore different features of GTs with GSP. The only point of difference is about when to begin exploring the features of GTs. Cathy considers that it could be after students know procedures and concepts. Jack thinks that use of GSP could be at any time during teaching and learning of GTs. The purpose of using GSP in teaching GTs is not just to solve problems, but also "to explore the potential of GSP" (Nordin et al., 2010, p. 114). Both Cathy and Jack seem to believe that GSP has a great potential to explore different properties of GTs using its dynamic features. Cathy considers that it helps students to dig deeper into a topic within GTs, and it could be linked with Van Hiele's idea of geometrical thinking (Abdullah & Zakaria, 2013). Van Hiele's five levels of visualization, analysis, informal deduction, formal deduction, and rigor could be well integrated with teaching and learning GTs with GSP (Usiskin, 1982; Van Hiele, 1986, 1985).

One of the important points to note in this discussion is 'developing a learning curve.' Jack expresses his thought that GSP offers a learning curve while teaching GTs. The notion of the 'learning curve' is very powerful in terms of using GSP for teaching and learning of GTs in a flexible way. The learning curve may follow Van Hiele's (1985) levels-- visualization, analysis, informal deduction, formal deduction, and rigor--or it may go along another path of learning GTs with GSP in a non-linear fashion. The cluster of geometrical thinking may not follow Van Hiele's (1985) model (Bennett, 1994; Burger & Shaughnessy, 1986; Clements et al., 1999; Mayberry, 1983). However, an exploration of GTs may turn into a foundation for constructing proof about congruence or similarity (Giamati, 1995). Therefore, Cathy and Jack's beliefs about exploring GTs with GSP have potential to extend the pedagogical affordance of the tool. For Cathy, GSP provides shortcut processes for studying properties of GTs, and for Jack, it has a lot of learning curves.

Conjecturing about GTs Using GSP: Conservation and Articulation of Properties

The participants' beliefs about making inferences for teaching and learning GTs with GSP include conjecturing as a part of teaching and learning mathematics in which students and the teacher may guess a relationship, property, or a solution to a problem. It is like a hypothesis in research. The difference is the scope. A conjecture has a narrow scope within a concept or a problem, whereas a hypothesis has a broader scope and stronger impact. The following interview transcript in Table 4 is an example that shows what the participants believe about conjecturing. The conjecturing in this section is related to their view of making conjectures from the constructions they did with reflection, translation, rotation, and composite transformations during the interview sessions. Conjecturing and reasoning with those conjectures are related to cognitive functions, and hence the beliefs associated are cognitive beliefs.

Cathy considers that GSP can serve as a tool for making and proving conjectures related to different properties associated with GTs. She thinks that it is possible to make and test the conjectures about side lengths, angles, orientations, areas, and perimeters. She accepts that these conjectures are helpful to understand properties of different GTs. However, she does not think that students can prove these conjectures in a formal way: "…they can say based on their construction when they reflected on this line, they may translate it, and they rotate it." That means students can verbally explain their task, and they can explore by measurement of area under any GTs (within reflection, rotation, and translation) and observe if it is preserved. For this process, they do not have to go through the measurement of bases and heights of triangles to find their areas. They can use the built-in tool for area measurement. She explains that, "Instead of measuring the bases and creating heights they decide to just measure the areas because they know that what the area is meant." She accepts that they can give verbal explanations of the conjectures and discuss them with measurements, but she contends that that is not a formal proof. Therefore, she believes that making and proving conjectures about GTs is possible by using GSP; however, the students (high-schoolers) are not able to do it in a formal way: "I think them explaining, easily explain and come up with, I guess not a proof, but a kind of proof about side lengths (informally)."

Table 4. Interview transcript on 'conjecturing about GTs using GSP'

Researcher: What conjecture students or we can make from this (construction of object and image under a rotation)?
Cathy: As in like angles stay preserved, side lengths are preserved. Orientation is not preserved but, perimeters preserved, areas preserved. I think that would be more difficult for them, I guess. I think them explaining, easily explain and come up with, I guess not a proof, but a kind of proof about side lengths. They can come up with that and make it very, almost exact, they can see it, know it why is that, but then you get to the area, I think even the orientation changing, the area would look different for them. So, they would have difficulty in that in perimeter even with explaining that all side lengths are the same. They might have more difficulty than saying getting from all side lengths are the same, what does that mean about the perimeter.
Jack: It gonna be pretty much the exploration or that kind of thing. I would choose angles, how they are related to each point and each object. Even you could talk about with angles and sides and how they are the same.
Researcher: So, what conjecture could we or the students make about this composite transformation?
Cathy: Like the area is congruent throughout these shapes, or perimeter or side lengths, like you can reflect down into. They can explain to me like what they are doing. So, like they decided that they are gonna explore the area. So, they can say based on their construction when they reflected on this line, they may translate it, and they rotate it. Um, that it is a construction. So, they don't move independently. And, like I guess explain what they did. They decided that they were gonna measure the areas. And areas depend on base and height cause, this is a triangle, and it depends on one half base times height. Instead of measuring the bases and creating heights they decided to just measure the areas because they knew that what the area is meant. The triangles are all the same.
Jack: Probably it just maintains all those properties. When you get into the translations students like to list the properties each one maintains different things, reverses orientations. So, that they could talk about that. I would choose angles, how they are related to each point and each object. It would be interesting to see if they can actually put it into words what they are seeing. It would be interesting to hear their own language what they say. That would be a kind cool to hear. But, I don't know I mean you would see how they understand congruence and similarity, equal angles and linearity. You can see if they really understand those kinds of things that making them. Write something or vocalize what you are actually doing. One thing I think just kind of fun to make them do, like a presentation. If you have the opportunity to put that on your screen up there and have them try to tell in class to other. Sometimes they put into words what you can't like another student understands them from reflecting, like to do jigsaw such a fun.

Jack also thinks that it is possible to use GSP for making conjectures about different geometrical properties associated with GTs and proving them. He says that students can use GSP to explore different geometric properties about GTs: "Even you could talk about with angles and sides and how they are the same." He states that the geometric properties, for example, congruence, similarity, lengths, and angles, are preserved under a GT. However, he is not sure about how the students can prove these conjectures by using the tools in GSP, claiming that, "It would be interesting to see if they can actually put it into words what they are seeing. It would be interesting to hear their language what they say." He comments that it would be nice to listen to the students' explanations of proofs about conjectures. He even considers students making presentations as an approach to sharing their verbal explanations of proofs to others. Sometimes their words could be more interesting than the teacher's. Therefore, he appears to believe that making conjecture is an important part of teaching and learning of GTs with GSP within the limitations of students' ability to produce formal proofs, not just a verbal explanation.

Both Cathy and Jack accept that the use of GSP can be helpful in making and proving conjectures about properties of GTs. Cathy's emphasis is on properties of lengths, perimeters, areas, and orientations. However, she contemplates that it might be difficult for students to prove them. She then focuses on students' explanations as part of their informal proofs of conjectures. However, for Jack, students would relate points, angles, sides and each object on the image and pre-image. He is also concerned if students can put their proofs into words what they see in the processes of GTs by using GSP. These views provide some details for theoretical interpretation of their beliefs in a broader sense in the third-order interpretation. Some of the key ideas which have emerged from this discussion include conjectures about preserving different structures (sides, angles, areas, and perimeters), the process in conjecturing (construction, measurement, and articulation of the process), and sharing information (making the presentation in the class or doing a jigsaw). These beliefs are supported by Hoyles and Jones (1998) who state that, "In developing dynamic geometry contexts the following factors are critical--encouraging the learners to make conjectures focusing on the relationships between geometrical objects and providing the means for the students to explain their actions and their results" (p. 124). This view clearly supports Cathy and Jack's reflection about making conjectures related to different properties that are preserved under a GT. The interpretation of conjectures can be extended to Crane et al. (1996) who discuss different kinds of conjectures with GSP. Some of these conjectures are related to angles (vertical and exterior), lines (lines in a pair, parallel lines, and tangents to circles), and polygons (triangles, quadrilaterals, trapezoids, rhombus, parallelograms, and rectangles). These conjectures could be a part of the discussion in any GT process with GSP. They (students) may discuss different kinds of conjectures in terms of construction of geometrical structures, and measurement of angles, lengths, and areas. Therefore, Cathy and Jack's views of making conjectures, explaining them, and sharing them in the classroom have been supported and supplemented by the literature as important aspects of teaching GTs with the use of GSP.

Engaging and Supporting Students in Learning GTs: Describing, Holding Attention, and Laying Out Steps

The participants articulated their beliefs about engaging and supporting students while teaching GTs with GSP. The interview transcript in the Table 5 shows their expressed beliefs within this subcategory. Here, holding attention and drawing students' interest to the tool and content is associated with developing a sense of connection to the tool. Such belief is related to attitudinal beliefs because these activities help teachers to develop a positive relationship between the students and the contents with the help of the technological tool.

Table 5. Interview transcript on 'engaging and supporting students in learning GTs'

Researcher: What do you think would engage them (students) while learning GTs with GSP?
Cathy: Um, honestly I think the assessment at the end, like the exploration, is good enough. Like for exploring, I don't think if we can take it out too long, but I think the assessment would be right after it. I think them being able to draw a picture and describing what that picture is. More like being able to make their own picture instead of saying draw a triangle and rotate the triangle. But, like doing that and then maybe using only rotation or using the transformations we have already used. Make your picture cool. I don't know how that instruction would be, but then can be- reflect it and rotate it.
Jack: I can just think about what's gonna interest them, what's gonna hold their attention. If you are just talking about rotation (or any GT), you are just using formal terms, I think you gonna lose their attention in ten minutes. One of the cool things about GSP is that's where video games come from. Kids love video games. You can talk about, pull them, and see what kinds of games are like. Even you can find a YouTube clip to show them and how they pick out a rotation. You can show them that's where it is coming from.
Researcher: How will you help your students to test and explore that conjecture (about the area of object and image polygons under a transformation)?
Cathy: Um, so, start them out with a polygon or a line. Maybe a triangle, cause, a triangle is easier to see and find the area of. So, you can take the measure of each side, find the length of each and compare them. They can measure all of them. I don't think that they understand the proof. Or I think they would think that construction is the proof. I think they could verbalize may be a proof. I don't know if they can make any formal but may be verbalized yea.
Jack: You can design an activity in which they are doing these kinds of stuff. I would give them step-by-step instructions so they get it. It saves time. Because one thing about Sketchpad is it's time-consuming if you are having them reteach the program. If you put the step-by-step instruction in front of them and you can walk around and help them with it if they need. I think that would be better and it saves time. We have a little time in the classroom for that stuff.

Cathy reflects formal assessment as a tool to engage students in learning: "Like for exploring I don't think if we can take it out too long, but I think the assessment would be right after it." She appears to think that the assessment should be done right after the teaching and learning activities so that students are involved in the process of learning. Most often a teacher can move around and see what students are doing. A teacher can use students' construction of any GT as means to assess their learning and engage them with it. For Cathy, a teacher's prompts like 'rotate it and reflect it' and students' responses to the prompts can be a helpful tool for continuous assessment of students' learning in the class, motivating and engaging them in the learning. She says that, "…you can take the measure of each side, find the length of each and compare them." She focuses on engaging and helping students in measuring and comparing side lengths of object and image polygons.

Cathy thinks that helping students in making conjectures and proving them are important aspects of teaching and learning GTs with GSP. She expresses doubt that students can prove a conjecture in a formal way, although they may talk and verbalize the proof: "I don't think that they understand proof. Or I think they would think that construction is the proof." In a similar vein, Jack considers that a formal assessment of learning can be done by engaging students in hands-on constructions with GSP. He considers that such constructions related to GTs with GSP allow students to see what is going on, and they can explain it to the class. While doing this, in his view, student interest and attention should be the central aspect of teaching and learning. Otherwise, the teacher may lose their attention within a few minutes. He says that, "I can just think about what's gonna interest them, what's gonna hold their attention." Therefore, he accepts that the assessment is a part of engaging students in making connections with relevant things (in life). He explains that, "Even you can find a YouTube clip to show them and how they pick out a rotation. You can show them that's where it is coming from." That means a teacher can pull out relevant examples from other media files to demonstrate animation features and can link them with GSP and GT process. He thinks that step-by-step instruction could help students to follow thoroughly and can reach a level where they can begin their work independently. He states that, "If you put the step-by-step instruction in front of them and you can walk around and help them with it if they need."

Cathy and Jack portray their beliefs about student assessment as a part of engaging and supporting them in learning. They accept that construction should not be left without description and students' explanation of their construction will help them make sense of what they have done. Also, student interest and attention are key factors in engaging them in a meaningful activity. Finally, visualization of the GT process motivates them in making connections between mathematics and the real world. These interpretations of their beliefs are further reinforced by the literature on teacher beliefs. Some key points related to what engages students while teaching and learning of GTs with GSP include assessment, construction followed by a description, interest and attention, and visualization. These belief constructs are in parity with Tamblyn's (2009) hands-on activities for engaging students instead of just presenting information through constructions, graphs, and tables. Cathy and Jack's beliefs in this category are sustained by Hollebrands' (2007) views that students can be engaged in different mathematical activities where they not only construct image and pre-image (object) under a GT, but they also interpret the results and compare (assess) the results when their construction changes by dragging.

Understanding GTs with GSP: Skipping or Quickening Steps, Visualizing, and Solidifying

The participants voiced their beliefs about teaching GTs with GSP and reflected their views in terms of confidence, a sense of efficiency, and the process of exploring different properties of GTs. The following interview transcript in Table 6 demonstrates their expressed beliefs within this subcategory. The function of understanding is related to one's cognition and hence the related beliefs are cognitive beliefs.

Table 6. Interview transcript on 'understanding GTs with GSP'

Researcher: Do you think that GSP is helpful in the procedural understanding of rotation? How?
Cathy: Yes, I think so. I do think that paper and pencil is very important. So, I would like to, after this, go and maybe give them paper and pencil type situation and say now rotate this shape over this and then make sure that they really, actually, know what they did, but I think they have to mark the point where they gonna rotate. They have to graph the figure and rotate the figure. I think that procedurally it (GSP) skips steps, but not really skipping steps. It is just quickening the steps.
Jack: That it gets cool. It's because I can see exactly what's happening. Especially, if you can show how to do an animation where they can see it's spinning. It is procedural. You rotated through the seventy- five degrees where's my point gone and I think that's kinda cool.
Researcher: So, what about the conceptual understanding of rotation (by using GSP)?
Cathy: I think that's why I would go back to the paper and pencil to see that they actually get conceptual. It won't take you that long to do that with paper and pencil. With paper and pencil, you have to absolutely know how to do it. So, while GSP would give you conceptual understanding with the assessment part. I would show them paper and pencil activity and solidify; yes, you did know conceptually what you were doing.
Jack: I think it helps with the concept of the angles, especially when you draw something like this, where it is seen, which angle which, and what's happening. I think that helps with the concept of a rotation. Most of them know what a rotation is just by the definition of the word. But, can they visualize it? Being able to draw the lines and measure these angles up here I think that really helps.

Cathy suggests that GSP skips steps in details of procedures. That's why she favors paper and pencil activity first to develop a procedural understanding of any GT process: "I do think that paper and pencil is very important." Although she values using GSP in the classroom practice of GTs, she wishes to use paper and pencil activities first to give the concepts. The dynamic feature in GSP can show what happens when an object is reflected, rotated or translated. However, the detail of the GT processes may not be observable in the shortcut tools. For this, one should use GSP to demonstrate the process, not just the product. In that sense, she considers that the animation in GSP can develop a better understanding of the GTs. The animated process in GSP can make the processes of GTs visible and dynamic. It also helps in the measurement and development of concepts of GTs once procedures are done. For Cathy, 'GSP skips steps' means it makes the steps faster rather than shorter: "I think that procedurally it (GSP) skips steps, but not really skipping steps. It is just quickening the steps." In contrast, Jack contemplates that the use of GSP may enhance both procedural as well as conceptual understanding of GTs. For him, what we see about GTs with GSP is procedural. The animation of GTs shows what is going on. At the same time, he thinks that GSP helps in understanding the concepts of change in angles with different GTs, especially in rotation. For him, an understanding of GTs with GSP is related to one's ability to see what's happening: "That it gets cool. It's because I can see exactly what's happening." He expresses that students' ability to construct lines and image of an object under a GT and measure the angles (and possibly sides) helps them in making sense of what is happening. He reiterates the act of seeing in relation to understanding. Therefore, he believes that GSP is helpful to develop both procedural and conceptual understanding of a GT.

Cathy and Jack articulate their beliefs about understanding GTs with the use of GSP at four levels, which are indirect process (quickening steps), direct process (animation), spatial understanding (visualization), and reasoning (measurement). These interpretations of their beliefs are embraced in the literature and theory of teacher beliefs. Many researchers and authors (e.g., De Villiers, 1999; Flores, 2010; Khairiree, 2006; Saunders, 1998) have highlighted the different aspects of understanding geometry with GSP. De Villiers (1999) talks about five aspects of understanding geometric proofs--explanation, discovery, verification, challenge, and systematization--that help in developing an understanding of geometrical relationships, in general. It seems that these aspects are equally valid for understanding different GTs with GSP. Hollebrands (2004) discusses the intuitive understanding of geometric transformations with GSP. She highlights key aspects of understanding GTs in terms of knowledge of transformations (both procedural and conceptual) -- reflection, translations, and rotation. Her notion of understanding GTs is related to students' ability to construct object and image with appropriate reference. These references are line of reflection, the point of rotation, and vector of translation. She feels that students' understanding of these transformations is related to their ability to construct, conjecture, measure, verify, and explain (Hollebrands, 2004).

Limitations

I conducted this study by interviewing two research participants on different task situations of reflection, rotation, translation, and composite transformations using Geometer's Sketchpad. Therefore, the scope of the findings and discussion of the study is limited to those two cases. The results of this study cannot be generalized to other preservice secondary mathematics teachers due to the very small sample size of interviews. Also, the results were drawn from the coding and categorizing of the interview data within the period of about six months. During this period, the participants' beliefs about use of the technological tool (GSP) for teaching GTs might have been influenced by several factors, such as time lag in the use of the tools, and latest experiences of teaching mathematics (with or without using the tool) in the schools while they were doing teaching practicum. Research findings in terms of the major categories of attitudinal and cognitive beliefs and their subcategories are the researcher's subjective constructs from coding and categorizing of the interview data. These constructs are not an objective ontological reality of participants' beliefs, but the second- or third-order interpretations of the researcher about their belief. Therefore, the portrayal of the participants' beliefs is 'subjective interpreted beliefs'.

Conclusion

Use of GSP Develops a Sense of Confidence in Teaching GTs

The interview transcripts in Tables 1, 2, and 3 and subsequent interpretive layers depict Cathy and Jack's beliefs associated with developing confidence with GSP, the efficiency of using GSP for teaching GTs, and exploring GTs with GSP. These beliefs seem to be similar to the beliefs associated with affect, but they are different in the sense that affective beliefs are more related to internal perceptions, feelings, experiences, anxieties, excitements, and values; however, attitudinal beliefs are consequences of self-confidence and sense of ownership and power to use the tool. Cathy has a sense of high confidence in using GSP. She seems to believe that teacher confidence directly influences student confidence in using the tool. She expresses that prompting students helps them to increase their sense of confidence. For her, the Foundations of Geometry course was critical to developing self-confidence in using the tool. She appears to believe that seeing what is going on in GSP while working on GTs helps students in developing confidence. She considers GSP as a practicing or exploring tool after mastering the procedures of GTs. For her, procedures precede the concepts in relation to teaching GTs with GSP. She has a greater ability to explore GTs with GSP in depth; however, she gets confused with some backward problem-solving (e.g., finding the center of rotation when the object and image under a rotation are given).

Jack expresses his belief that he has a sense of getting into learning and using the tools in GSP (i.e. being able to use GSP for teaching GTs); however, he still has a lack of confidence. He feels it is a prerequisite to spend more time on the tool (GSP) for teaching GTs. For him, both the doing (i.e. hands-on) and seeing (i.e. observation) of GT processes with GSP help his students in building confidence. His limited experience of using GSP seems to inhibit his sense of confidence. He appears to believe that efficiency of GSP as a tool depends on access, one's ability to see what's happening, and background knowledge. For him, GSP enables students to achieve on a better learning curve than without it. He has positive beliefs about the usefulness of GSP for teaching GTs; however, his beliefs about his own ability to apply the tool appear to be negative due to his limited experience with it.

Students' construction of an idea within any GTs by using GSP leads to two possibilities. The first is the construction as a mechanism to build on what he or she already knows. Students construct new ideas on their old ideas when they construct any polygon or geometrical figure and do a transformation (Ernest, 1986). The construction seems to be a cognitive function. The second is that construction as a transformation of students' thinking and reasoning is a metacognitive function. When students construct any geometrical object and then reflect, rotate, or translate it under a given condition, then they transfer their constructions through building up new ideas (Von Glasersfeld, 1989). At the same time, they gain a higher level of thinking and reasoning through meta-cognitive construction when they can see and experience what is happening with what they are constructing with GSP. That means what they can see and experience opens the door into the metacognitive world. A teacher's prompt takes them to a deeper level of understanding in this abstract world. Therefore, these points are interconnected in building the confidence of students' learning of a GT with GSP.

Use of GSP Supports Understanding GTs

The interview transcripts in Tables 4, 5, and 6 and subsequent interpretive layers portray Cathy and Jack's beliefs associated with conjecturing about GTs using GSP, supporting and engaging students in learning GTs, and understanding GTs with GSP. Cathy expresses her beliefs about conjecturing properties that are preserved (e.g., lengths, angles), an explanation as key to understanding, deriving meaning, interrelated structures in GTs, and use of direct measurement for manipulating the properties of GTs using built-in tools in GSP. She prefers engaging and supporting students in assessing their learning, exploring properties of GTs, helping them in constructing and describing what they have done by starting with easier things that students already know. She believes that students don't understand the formal proof, and for them, construction is a proof. She further thinks that they can verbalize the constructions as proof. She believes that students develop an understanding of GTs with paper and pencil before the use of GSP. That means she thinks that students' understanding of the procedure before concepts is necessary because, for her, the use of GSP keeps on skipping steps.

Jack believes that he can help students in exploring properties of GTs and finding interrelations of different geometric features (e.g., angles, sides, areas, and perimeters). He accepts that he, as a teacher, should be able to help students express their work on GTs in words that may develop their mathematical language, and it may enhance their understanding GTs processes with GSP. He appears to think that some key aspects of cognition of GT processes with GSP are related to their interest, attention, ability to see the application of GTs (e.g., in video games), and design of activities in class through hands-on constructions. Jack feels that students' ability to see what's happening (visualization) provides them a deeper understanding of the process.

These conclusions are well connected to the theory of teacher beliefs. According to Giamati (1995), students make conjectures, and then they vocalize them through peer or group sharing. He says that, "…students can use it to test a wide variety of conjectures once they have mastered this construction tool" (p. 456). Students can make a variety of conjectures, and all of them may not lead to a reasonable proof or verification. In such a case, students may learn why the conjecture was not reasonable (Giamati, 1995). Giamati (1995) further suggests that students gain "deeper understanding of the problem using their scripts to explore it and make conjectures than they would have if the results had merely been explained to them" (pp. 457-458). Tamblyn (2009) proposes important pedagogical strategies in relation to making conjectures and proving or verifying them. They suggest that students' conjecturing that leads to problem-solving and class discussion may help in examining their conjectures. In these activities, they find GSP useful as a tool for exploring and investigating different relationships to develop conceptual understanding.

Implications of the Study

Influence of Beliefs on Practice

The participants' beliefs associated with attitude have psychological and pedagogical implications in the areas of sense of confidence, efficiency, access, the order of use, ease of use, and knowledge to use GSP for teaching GTs. Preservice teachers' sense of confidence in the use of GSP for teaching/learning might influence their perceived usefulness and ease of application. Use of GSP may depend on preservice teachers' beliefs about the efficiency of the tool in terms of access, the order of use, ease of use, and knowledge to use for teaching GTs.

The findings within the category 'Beliefs Associated with Cognition' have psychological and pedagogical implications in the areas of conjecturing and developing mathematical language by students, informal proofs by verbalization and constructions, and procedural versus conceptual understanding of GTs with GSP. Teachers may help students in making conjectures and developing their mathematical language about interrelated structures and properties of GTs with GSP and testing them. Their beliefs about how they think of engaging students may influence their decisions to apply formal or informal proofs through construction, visualization, and verbalization of GT processes with GSP. Their beliefs about students' understanding (either as instrumental or relational) of GTs with GSP may influence their anticipated practice of using the tool.

Subtle Elements in Use of GSP

Some points discussed in the findings include a lack of understanding a proof, measurement for studying properties, step-by-step instruction, and walking around and helping students. Jiang and McClintock (1997) view suggests that university teacher educators need to create an environment for preservice teachers to "develop new learning styles, performing investigations and experimentation, using the results of studies, formulating thoughtful conjectures, verifying the conjectures, discussing possible extensions, generalizations, and applications" (p. 129). The use of GSP can help in achieving these objectives in mathematics teacher education. The use of GSP can help in developing students' "geometric intuitions" (p. 134). Therefore, teachers can help students in the process of "investigate-conjecture-verify" a conjecture or a problem (Jiang & McClintock, 1997, p. 135).

Cathy's view that 'students don't understand the formal proof' is supported by Bucher and Edwards (2011). Bucher and Edwards (2011) state that, "Although we discuss the importance of proof frequently in our geometry courses, curiously we ignore our own advice when teaching students about geometric transformations" (p. 716). According to these authors, textbooks as well do not focus much on formal justification or proof about geometric transformations. That means the proof has not been the focus of high school geometry (in the US). This issue is accompanied by the current practice of "algorithmic approach to transformational geometry--one that omits rigorous proof--misses the opportunity to connect the rigid motions to a host of other geometric topics" (Bucher & Edwards, 2011, pp. 716-717).

The measurement of sides, angles, areas, and perimeters may contribute an easy way for teachers to teach GTs with GSP. However, the students' understanding of the GT properties may still be incomplete due to a lack of logical/abstract proof. The step-by-step process may provide a clear pathway for the students to follow, do constructions, and observe properties. This kind of practice seems to be linear, and teacher directed. Nonetheless, this approach provides students clear direction and guidelines to begin from a certain state and end at a certain point (through measurement) without loss of time and without being confused. This view is supported by the literature as well. Cole (1999) says that, "As a teacher, you are always assessing your students. You walk around as they work on the task or projects, observing groups, conversing with students, spot-teaching concepts, and skills, and checking for understanding" (p. 224). Cole (1999) further says, "Teachers walk around, observing and talking with students, for many reasons, making sure that the students are doing what they are supposed to be just one of these reasons" (p. 225). Cole connects walking around activity with making observations, doing on-the-spot assessment, making notes, and balancing equity in the classroom. Therefore, the use of GSP or other tools in teaching GTs or other contents has subtle elements of conjecturing and proving various geometrical properties in mathematics that should be in focus of mathematics teacher education programs.

Use of Technology for Understanding

Skemp (1976) introduced two different forms of understanding–instrumental understanding and relational understanding. Usiskin (2012) thinks that instrumental understanding is related to procedural understanding, and relational understanding is related to conceptual understanding. For Usiskin (2012), mathematical understanding may include both instrumental understanding (i.e., procedural understanding) and relational understanding (i.e., conceptual understanding). Many other researchers and authors (e.g., De Villiers, 1999; Flores, 2010; Khairiree, 2006; Saunders, 1998) discuss the role of the dynamic geometry environment of GSP for developing a deep understanding of geometry in general. It is extremely important to conceptualize different understanding of geometry in terms of visual understanding (understanding from observation), real understanding (understanding at depth), shallow understanding (surface level understanding), differential understanding (understanding that helps in differentiation of properties), computational understanding (understanding from calculations), and spatial understanding (understanding of space in terms of distance, area, and volume). The participants in this study could not make these different understandings explicit in their narratives; however, their accounts of the future teaching of GTs with GSP have many of these elements of understanding.

Policy to Integrate Technology in Mathematics Education

The finding of this study has policy implications in terms of integrating technological tools in preservice and inservice mathematics teacher education to develop positive attitudinal and cognitive beliefs about the use of the tools, usefulness of the tools, and ability to use different tools. The mathematics teacher education programs at universities and other institutions should orient toward reforming mathematics education, modernizing the classroom practices with technological tools, and transforming the mathematics education by imparting positive beliefs of technology integration. Teacher beliefs about developing confidence with GSP from temporal, visual and relational aspects emphasize developing and implementing teacher education policy that supports technological and pedagogical knowledge. The view about efficiency of using GSP for enhancing, exploring, understanding GTs indicates need of upgrading state-of-the-art infrastructure of mathematics teacher education for the next generation. The ultra-modern technology in mathematics teacher education not only promotes confidence and efficient use of technology, but it also instills habits of exploring different pathways of learning mathematics on different learning curves.

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