

The Use of Manipulatives in Mathematics Education

Ernest Larbi* Okyere Mavis
Catholic University College of Ghana, Fiapre, P. O. Box 363, Sunyani

Abstract

The study was designed to investigate the efficacy of using algebra tile manipulatives in junior high school students' performance. The study sample comprised 56 students from two schools purposely selected from two towns within the Komenda Edina Eguafo Abirem municipality. The students were made up of two groups; the experimental and the control group. Each group was taught the same algebra units over a period of four weeks. However, the experimental group was taught using algebra tile manipulatives whilst the control group was taught using the conventional 'talk and chalk' method. The instruments used for data collection were mathematics achievement pretest and posttest. Students' achievements on the posttest were analysed using percentages, mean, standard deviation and the independent t-test. The findings of the study were that, those who were taught through extensive use of algebra tiles performed significantly better. Thus the use of the algebra tiles proved very effective and promising approach to teaching and learning algebra, and that the tiles also improved students' thinking process as they solved problems in algebra. On the basis of these findings, it is recommended that algebra tiles should be used as a tool to introducing students to algebraic concepts.

Keywords: Manipulatives, instruction, performance, learning

1. Introduction

The role played by mathematics in almost all areas of development in life cannot be underestimated. Mathematics serves as backbone to all technological advancement in the world. There can be no meaningful development in this modern world of technological era without adequate and sufficient knowledge of mathematics. The study of mathematics enhances one's understanding of the world through the language of symbols and abstract representation of phenomena. It is a subject that is very important for the academic excellence of people irrespective of programme of study. Knowledge in mathematics is applied in almost every school subject. In Ghana, mathematics features prominently as one of the core subjects in the curricula of basic schools, senior high schools and colleges of education. Mathematics features as one of the critical filters for entry into higher educational programmes. In addition, many universities having realised the importance of mathematical knowledge have embedded some mathematical courses in the various course to be studied in the non-mathematics programmes. For instance, students supposed to undertake a research work as part of their programmes can only be successful when they have acquired basic knowledge in statistics, a branch of mathematical knowledge.

Despite the importance of mathematics in human development, many investigations have shown that students in secondary schools are not very much interested in learning mathematics (Eshun, 2000; Awanta, 2000). Also available records indicate unsatisfactory mathematics performance of students in the West African School Certificate Examination (WASCE). The Chief Examiners' Report of West Africa Examinations Council for the past few years have highlighted students' weakness in solving problems in mathematics (WAEC Chief examiners' Report, 2005, 2006, 2007). Some of the persistent weaknesses which have been identified by the Chief Examiner are students' inability to use mathematical skills and concepts to

1. Remove brackets correctly from an equation. That is failure to use distributive property of multiplication over addition
2. Simplify algebraic expressions after expanding the product of two binomials.

A study by Yara (2009) showed that most students have perceived mathematics as a subject with many technical terms which are difficult to remember. However, a critical study of the subject shows its interrelated body of knowledge in which topics are arranged such that earlier topics are prerequisite to those that follows especially in Ghana and most part of the world where spiral curriculum is practiced to enable concepts to be taught in bits at all levels.

What apparently comes to mind on the issue of difficult nature of mathematics is perhaps the way mathematics is taught or presented in our schools. A major task of every mathematics teacher is how to make the subject meaningful to learners. According to Yackel, Cobb and Wood (as cited in al – Absi & Nofal, 2010), mathematics knowledge is not intrinsic and as a result develops from the learners interaction with the environment and people. This knowledge manifests itself based on the kind of interpretations given by the learner. Thus learning is a product of what goes on in a learners learning environment. This therefore calls for the tutors or instructors to create conducive and productive environment for learning as no or little learning can take place in a threatening environment. Mathematics should be presented in a way that meets learners learning styles and thought processes. According to Coombs (1970), education is made up of two components - inputs and outputs. Inputs are human and

material resources and outputs are outcomes of the educational process. Inputs and outputs form a whole and for one to assess the educational system in order to improve its performance, effects of one component on the other must be examined. Instructional resources which are educational inputs are of vital importance to the teaching of any subject in the school curriculum. Most especially is mathematics which seems a bit abstract to learners. Hence, psychologists and mathematics educators are of the view that, mathematics should not be taught in a teacher dominant lesson, but instead, lessons must focus on learners' knowledge construction and hence should be placed at the center of the teaching and learning process to enable them explore and interact with materials to aid knowledge acquisition.

1.2 Research Question/Hypothesis

The following research question and hypothesis have been formulated to guide the study.

1. How comparative are students in both the experimental and the control group in using the distributive property?
2. How comparable are the proficiency levels of the students in the control and the experimental groups, in factoring algebraic expressions?
3. There is no significant difference in the mean scores of students taught using algebra tiles and that of students taught without such materials.

2. Review of Related Literature

2.1 Methods of Teaching

According to Fletcher (2009), various teaching methods are used in teaching mathematical concepts to varying degrees of success. These methods are 'transmission' and 'interactive' approaches, and research has shown 'interactive' to be more effective than the 'transmission' approach.

In the transmission approach which is also known as traditional teaching method or teacher centered instruction, the teacher acts as a reserve of knowledge. The teacher who sees himself as the sole supplier of knowledge takes control over almost every activity in the teaching and learning process. His or her duty is to transmit or explain facts and procedures to learners. Learners are only asked to check if they are following the taught procedures. Such approach creates boredom in class, encourages passive attitude among learners and make them feel they have nothing to contribute (Fletcher, 2009). This method of teaching is also called non-participatory teaching method because students do not participate in the lesson. Lesson is however conducted through explicit teacher explanation through lectures and teacher-led demonstrations.

Thornton (1995) has observed that quite a number of schools still depend on such approaches which he termed "mechanistic" approach to teaching. A 1988 National Assessment for Educational Progress (NAEP) survey gathered the following results on how mathematics was being taught at the secondary school level.

Typical mathematics instruction apparently consists of listening to teacher explanations, watching him solve problems on the chalkboard using a mathematics textbook, and working alone to solve problems on worksheets. Over one half of the students reported never working in small groups to solve mathematical problems. Over eighty percent claimed that they had never worked on independent projects or investigation in mathematics class (Silver-Linquist, Carpenter, Brown, Kouba & Swafford as cited in Thornton, 1995, p. 3).

This situation contradicts the vision of mathematics instruction indicated by the standards during the educational reform in the United States of America (US) which has been 'knowing' mathematics is 'doing' mathematics (National Council of Teachers of Mathematics [NCTM], 1989).

On the other hand, the interactive approach is the situation where the learner is placed at the centre of the learning process and seeks knowledge or information to solve a problem. A teacher using this approach believes that knowledge is constructed by the learner. The teacher's duty therefore is to choose appropriate learning tasks for learners, make the purpose of activities clear and encourage them to explore and verbalise their mathematics thinking. This approach helps learners to gather, discover or create knowledge in the course of an activity having a purpose. This active process is different from simply mastering facts and procedures.

Regarding the general improvement in the teaching and learning of mathematics, Talmadge & Eash (as cited in Blosser, 1985) asserted that instructional techniques are important, but the use of instructional materials or manipulatives also influences learners' achievement, and helps them to both use process skills and transfer of learning to many situations. Instructional materials or manipulatives provide the physical media through which the intents of the curriculum are experienced. These physical media appeal to the senses of the learners which bring things that are far beyond their environment near. In other words, they make imaginations more vivid and accurate.

2.2 Definition of Manipulative Materials

There have been numerous definitions of manipulatives by several authors. Kennedy (1986) defines

manipulative as “objects that appeal to several senses and that can be touched, moved about, rearranged, and otherwise handled by children” (p. 6). Smith (2009) defines manipulatives as “physical objects that are used as teaching tool to engage students in hand-on learning of mathematics” (p.20). Thus manipulatives are materials from our own environment that learners can use to learn or form mathematical concepts. In other word, any material or object that helps learner to understand mathematics. Such materials help to reduce the abstract nature of mathematics as perceived by many students. Although the National Council of Teachers of Mathematics encourage the use of manipulatives at all levels, Heddens (1997) cautions it must be used with care, else students are made to believe that two mathematical worlds exist; manipulative and symbolic. Heddens asserts that one of the best ways of developing mathematical ideas is through activities with physical materials. Students learn best when they are active participants in the learning process. They assimilate knowledge when given the opportunity to explore, investigate question, record, share, and talk about discoveries. Fletcher (2009) adds that, “manipulating familiar objects that inspire confidence is the beginning of getting a sense of structure, and that the structure eventually emerges in the form of a generalisation or expression” (p. 32).

The uses of instructional materials enable learners to understand lesson easily. Their uses facilitate acquisition of knowledge by learners, help make discovered facts stick firmly to their minds and result in better performance. The uses of the manipulatives arouse learners’ interest and promote active involvement in the lesson (Munger, 2007). Learning basically occurs when learners interact with the environment and encounter some experiences through which discoveries and relationships are made among concepts. When learners are placed at the centre of instruction, they are able to discover new relationships between materials learnt and understanding grows from within. According to Resnick and Ford (1984), teaching methods should allow learners to participate in some of the creative processes that mathematicians have enjoyed through centuries from which they were able to discover certain generalisations and principles.

One good reason for using manipulatives is that they have positive effect on learners’ achievement when learners are allowed to use concrete objects to model, and internalise abstract concepts. Manipulatives not only allow students to construct their own cognitive models for abstract mathematical ideas and processes, they also provide a common language with which to communicate these models to the teacher and other students (Sowell, 1989; Ruzic & O’Connell, 2001). According to Heddens (1997), teachers will receive more insight into students’ mathematics understanding through the use of manipulatives by:

1. listening to students talk about their mathematics thinking
2. observing students working individually and in cooperative groups
3. asking why and how questions rather than asking:
 - i. yes or no questions
 - ii. for results of calculating activities
 - iii. for answers
4. having students write a solution to a problem rather than by only responding with correct or incorrect values (p. 49).

Thus manipulatives are considered useful to students in the learning of mathematics as well as a tool used by teachers to introduce mathematical concepts and to assess their understanding.

The motive behind the use of manipulatives is that individual students learn in different ways, when manipulatives are used, the senses are brought into learning and they also act as visual representation of mathematical concepts. In addition to meeting the needs of students who learn best in this way, manipulatives afford the teacher new ways of presenting a topic. A sound lesson on any mathematical topic should involve multiple instructional methods. Incorporating several different instructional techniques increases the possibility that all students will develop mathematical understanding through at least one method. When manipulatives are used and children placed at the centre of the learning process, the role of the teacher changes from transmitter of knowledge to being a facilitator of learners’ discovery (Fletcher, 2009).

2.3 Multiplying and Factoring Expressions

According to Russel (2011), distributive property makes it easier to work with numbers and has many applications to learning algebra and mathematics as a whole. She referred to distributive property as breaking or separating an expression into parts. Distributive property helps with mental mathematics and should be taught to students as a method to multiply much quicker. Studies indicate that inability of students to understand the uses and application of the distributive property act as a barrier to successful learning of mathematics especially in algebra (Sfard, 1994; Norton & Irvin, 2007). According to Thompson and Fleming (2003), many students come to the study of early algebra with poor understanding of arithmetic. Macgregor (as cited in Norton & Irvin, 2007) asserted that it is likely that failure to understand the structures of arithmetic (eg. commutative law, distributive law, fraction, integers and operations) will place an added cognitive load on students when it comes to the study of algebra. The implication from the above discussion is that without sufficient knowledge in arithmetic and its structures, students learning of algebra will be inhibited. For instance, a child who is well informed about the use

of distributive property will find it easier working through the following exercises.

1. $4 \times 53 = 4(50 + 3)$
 $(4 \times 50) + (4 \times 3)$
 $200 + 12$
 212
2. $3 \times 99 = 3(100 - 1)$
 $300 - 3$
 297

Students should have lots of opportunity to split numbers apart using the distributive property which greatly assists the mental processes (Russell, 2011). Stacey and MacGregor (1997) noted an important part of algebra learning as transformational processes. Clearly without transformational tools of arithmetic, students are likely to be burdened with added cognitive load and struggle to move from operational to the structural phase of algebraic thinking. Thus without the foundation of numeracy, its generalisation would seem difficult to students. Norton and Irvin (2007) in a study noted that students' main difficulty in learning algebraic concepts is their inability to understand the structures of arithmetic. They found that students' struggle was associated with lack of understanding of arithmetic concepts, including those associated with equivalence, operations with negative integers, and the distributive law (property). Blume and Mitchell (as cited in Jones, n.d) in a study to investigate 83 eight graders' use of distributive property found that fewer than 10% of them could state the distributive property of multiplication over addition. They noted that the primary emphasis in texts at the 6th – 8th grade level was on completing the pattern of sums and products, and considered it to be symptomatic of the common classroom malady known as symbol pushing. Seng (2010) in a study to investigate the error pattern in solving problems in algebra found that students tended to make distributive error in bracket expansion.

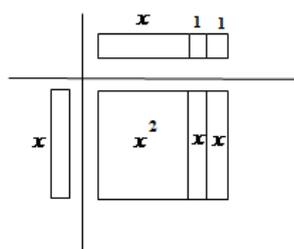


Figure 1. The distributive property.

A remedy to enhancing students' ability to the use of the distributive property is the use of algebra tile manipulative. By using these tiles, learners make greater connections with the ability to use the distributive property (Russell, 2011). According to Picciotto and Wah (1993), working with manipulatives gives students hand-on experience with variables and can help them avoid common mistakes. The use of algebra tiles can quickly eradicate beginners' confusion between $2x$, $2 + x$ and x^2 . Consider these expressions in figure 1. $x(x + 2) = x^2 + 2x$

The purpose of the above activity is to enable students develop a visual understanding of the distributive property (Picciotto & Wah, 1993). Figure 1 is similar to factoring an expression. Starting with this form of factoring, guides students to develop a feel for the distributive property, which is then easily introduced, initially in the form; multiply every block along the left by every block across the top. Marshall (n.d) asserted that it is in factoring polynomials that the tiles provide students with the greatest rewards. The tiles make factoring so easy that difficult problems turn into simple puzzle type exercises. With the use of manipulative, students begin to understand and accept mathematics without all fears that has been associated with it throughout the years (Marshall, n.d). Factoring and distributing with manipulatives can help students avoid another common mistake: "distributing the square" $(x+4)^2$ can easily be built with tiles and clearly indicates that $(x+4)^2$ is not equal to $x^2 + 16$.

Sharp (1995) conducted a study involving five high school algebra classes. Two rural high school algebra classes (100% white) and three suburban high school algebra classes (85% white, 10% African American, 5% Hispanic) were used. The students in the treatment group were instructed using algebra tiles to add, subtract, multiply and to factorise algebraic expressions. The control group was also taught the same units without such materials. Results indicated no significant difference between the groups. However, results of daily narrative data indicated that majority of the students instructed using the algebra tiles indicated that the tiles added a mental imagery that made learning easier. They indicated that they found it easy to think about algebraic manipulations when they visualised the tiles.

3. Methodology

This study was quasi-experiment, employing pre-test, post-test nonequivalent group design. Quasi experiment is a form of experimental research extensively used in the social sciences, psychology and education due to lack of complete random assignment of respondents (Nwadinigwe, 2002; Shuttleworth, 2008). It is a design often used in classroom experiments where experimental and control groups are such naturally assembled groups as intact classes. This was used to also enable the effect of the instructional materials in the teaching and learning of mathematics to be examined in natural settings.

In addition, the nonequivalent group, pretest-posttest design was used to partially eliminate a major limitation of the posttest only design. Therefore, if it is found that one group performs better than the other on the posttest, the initial differences (if the groups were in fact similar on the pretest) could be ruled out and the normal development (resulting from instruction) explains the differences.

The target population for the study was all JHS 2 students in the 61 schools within the Komenda-Edina-Eguafo-Abirem Municipality in the Central Region of Ghana. The sample for the study was made up of 56 students sampled from two schools purposively selected from two towns in this municipality. The choice of these schools from the two different towns was influenced by such factors as proximity and time constraints and also to avoid contamination of treatment.

Two kinds of Mathematics Achievement Test were developed for the study. One for pretest and the other for posttest. To ensure validity of the instruments, two teachers currently teaching mathematics at the JHS level were given copies of the achievement test to assess the quality of each item in the context of clarity, ambiguity and generality. This was done in addition to using the recommended textbooks and the syllabus for JHS in the item construction. Suggestions received from the colleagues and the teachers on the field were incorporated to refine the content of the MAT making it more relevant and suitable for the purpose of the study. The refined instruments were then pilot-tested in two different Junior High Schools in the Brong-Ahafo Region. The reliability coefficient of the various sections of the achievement test ranged from 0.71 to 0.91 and were found suitable to use since a classroom test can have a reliability coefficient of 0.7 or higher (Wells & Wollack, 2003). The results of the pilot test also helped to modify some of the items of the MAT.

Prior to the treatment, the students of the two junior high schools were pretested to check their entry characteristics. Results from the pretest showed no significant difference between the two schools. School A ($M=18.58$, $SD=6.15$) and school B ($M=16.70$, $SD=7.62$); $t(51) = 0.982$, $p=0.331$. Since $p=0.331 > 0.05$. The students were then taught the same units in algebra components of the curriculum.

One school was assigned the experimental group and the other the control group. The experimental group was taught using the algebra tiles to enhance instruction and explanation of the concepts whilst the control group was taught using the traditional teaching method. After the four weeks instructional period, the students were tested using the MAT developed for the study to determine the effectiveness of the manipulatives used. The data collected was analysed using SPSS version 20.

4. RESULTS AND DISCUSSION

Research Question 1: How comparative are students in both the experimental and the control group in using the distributive property?

Table 1: Distribution of Credits for the use of the Distributive Property

Score	Experimental Group		of Number of Students	Control Group	
	Number of Students	Percentage of students		Number of Students	Percentage of students
17 ---- 20	2	8%	0	0%	
13 ---- 16	3	12%	2	6.5%	
9 ---- 12	9	36%	7	22.6%	
5 ---- 8	7	28%	16	51.6%	
1 ---- 4	4	16%	6	19.4%	

Table 1 shows the distribution of credits for the use of the distributive property. Tasks or items in this section required students to explicitly use the distributive property to expand an algebraic expression. The use of the distributive property was in two folds; multiplication of a monomial by a binomial and multiplication of two binomials. Result from Table 1 shows that most students in the experimental group were able to use the distributive property as compared to their counterparts in the control group. A critical analysis of Table 1 indicates that whilst students in the experimental group seem to obtain high scores, the performance of students in the control group is relatively lower. The score interval from 17 to 20 registered 2 students in the experimental group with no student from the control group matching up to this performance. However, low score interval from 1 to 4 has more students in control group than those in the experimental group. Thus the performance of the experimental and control group seem to be inversely related in favour of the experimental group.

The overall students mean score for the items requiring the use of distributive property in the expansion of algebraic expression was 8.07. A score of 10 is considered the average performance on the required skills to perform the tasks in this section. The average performance 8.07 being less than 10 indicates that students had some difficulty in using the distributive property. The experimental group mean of 9.22 suggests a better performance as compared to the mean performance of the control group of 7.15

This finding is in agreement with the works of Norton and Irvin (2007) and Seng (2010) who found that students have difficulties in the use of the distributive property. However the results in the study shows that students in the experimental group performed better than their control group counterparts indicating that continual exposure of the students to the use of algebra tiles in learning could alleviate their difficulty in the use of the distributive property.

Research Question 2: How comparable are the proficiency levels of the students in the control and the experimental groups, in factoring algebraic expressions?

The comparative performance of students in the experimental and the control group in the factoring of algebraic expression is presented in Figure 2.

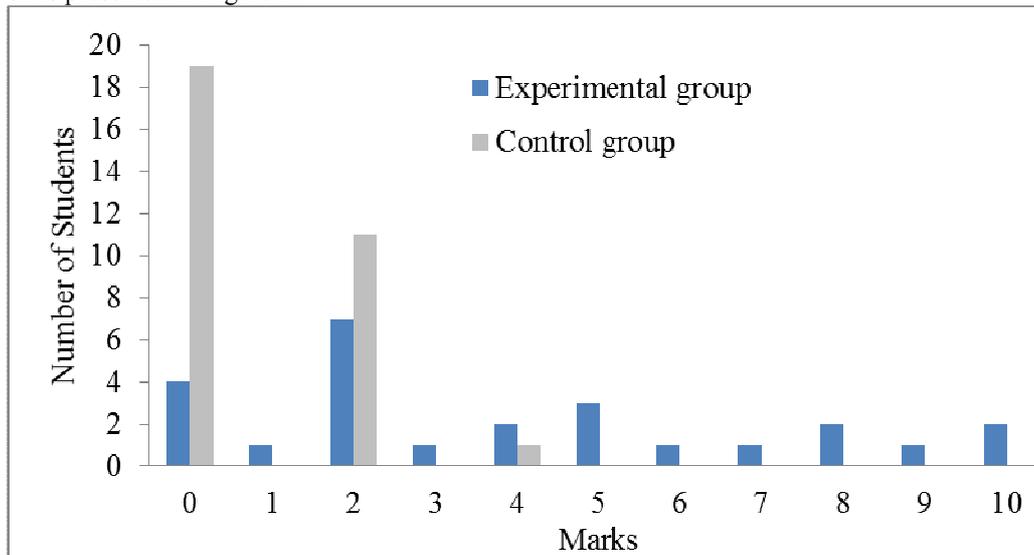


Figure 2. Distribution of scores of students on factorisation.

Figure 2 shows that as many as nineteen representing 61.3% of students in the control group had no score compared to four, representing 16% of the students in the experimental group. Eleven students, representing 35% from the control group compared to seven students, representing 28% from the experimental group scored two.

As the scores obtained in this section kept increasing, indicating better performance, the number of students in the control group obtaining such scores kept reducing, making way for the students in the experimental group. Whilst no students from the control group scored above five, seven students representing 28% in the experimental group scored above this mark. It can be inferred from this result that students in the study, who were instructed using the algebra tiles performed relatively better than their counterparts who were instructed without the use of such materials.

Table 3: An extract of t-test Comparison of the Posttest of Mean Scores of Students in the Experimental and the Control groups on Factorisation

Group	N	\bar{X}	SD	df	t	p	Decision
Experimental	25	3.96	3.21				
Control	31	0.84	1.13	54	4.639	0.00	Significant

Table 3 shows a significant difference in the mean score of the experimental group ($M = 3.96$, $SD = 3.21$) and the control group ($M = 0.84$, $SD = 1.13$); $t(54) = 4.64$, $p = 0.00$. The magnitude of the difference in the means was very large ($\eta^2 = 0.285$). Since $p < 0.05$, there is no evidence to retain the null hypothesis. Hence we

reject the null hypothesis of no difference and uphold the decision that there was a significant difference between the experimental group and the control group with regard to their performance on the factorisation section of the post test. The analysis showed that 28.5% of the variation in the scores on the post test of the two groups was explained by the instructional strategy.

This result shows that the use of algebra tiles has a statistically significant effect on students' performance in factorisation. Students taught using the tiles had a higher mean score (3.96), which meant a better performance than that of the control group (0.84) taught without such materials.

This finding supports that of Sharp (1995), who observed that students instructed by the use of algebra tiles benefited most since their interest was aroused and sustained throughout the activities associated with the use of such materials. Also this finding corroborates Thornton (1995), who in his study observed that students who were exposed to extensive use of algebra tiles improved in their ability to perform abstract operations on polynomials. The conclusion drawn was that algebra manipulatives enhanced greater understanding of the concepts covered.

Research Question 3: There is no significant difference in the mean scores of students taught using algebra tiles and that of students taught without such materials.

Table 4: An extract of t-test Comparison of the Posttest of Mean Scores of Students in the Experimental and the Control groups on the Posttest

Group	N	\bar{X}	SD	df	t	p	Decision
Experimental	25	32.16	10.48				
				54	4.797	0.000	Significant
Control	31	20.35	7.94				

N=56

The result in Table 4 shows that there was statistically significant difference between the mean scores of the experimental group (M=32.16, SD=10.48) and that of the control group (M=20.35, SD=7.94); $t(54) = 4.797$, $p = 0.000$. The magnitude of the difference in the means was very large (eta squared=0.298). Since $p < 0.05$, the decision that there is a significant difference between the experimental group and the control group with respect to the use of the manipulative 'Algebra Tiles' is upheld. An eta squared value of 0.298 was obtained from the analysis suggesting that 29.8% of the variance in the scores of the experimental and the control group could be explained by the instructional strategy.

The finding of a significant difference between the two groups in favour of those exposed to the use of the algebra-tiles suggests that students' performance might have improved through the use of the tiles which might have helped them in concept formation and as a result enhanced understanding of the relevant concepts. Findings by some researchers (e.g. Fennema, as cited in Thornton, 1995) which suggest that most students gain very little regarding to understanding of mathematical concepts through the use of manipulatives are not supported by findings of this study. Findings from this study rather uphold the assertion that manipulatives offer important opportunity for students to link hands-on experience to understanding of mathematical concepts (Kurumeh, Chiawa & Ibrahim, 2010; Suydam & Higgins, 1977).

Agusiobo (as cited in Onasanya & Omosewo, 2010) asserted that learning is an activity that takes place in a contact and not in a vacuum. He adds that when students are instructed using manipulatives, their active involvement and their ability to make discovery enable them to develop a consolidated library of knowledge. The results of this study is not surprising since through the use of manipulatives, students activeness in the class and their ability to make observation and discovery in an unhurried manner might have improved understanding. This is an indication that students were well informed of concepts underlying the manipulation of algebraic expression. That is the use of manipulatives must have enhanced learning by providing opportunities for exploration and concept representation which enabled them to view mathematical ideas as an integrated whole but not isolated facts to be learned or memorised. In other words, emphasis was laid on conceptual understanding rather than procedural understanding. This is supported by recent educational theories which are promoting conceptual understanding rather than teaching procedures and memorising facts and formulas (Sowell, 1989; Heddens, 1997).

In addition, the hands-on activities might as well have encouraged and enabled all learners with different characteristics and abilities to benefit greatly from the variety of the learning experience provided in the approach.

CONCLUSIONS

The following conclusions were drawn, based on the findings of the study:

1. The junior high school students exhibited low skills regarding the use of the distributive property, however those in the experimental group taught using the algebra tiles manipulative outperformed their counterparts in the control group.
2. The use of the algebra tiles enabled students in the experimental group to demonstrate proficient skills in factoring algebraic expression than their counterparts in the control group.
3. Students taught with the manipulatives-algebra tiles performed significantly better than those taught without such materials.

IMPLICATIONS OF THE FINDINGS FOR CLASSROOM PRACTICE

Algebra is representation of ideas using symbols. Expressing mathematical thought in symbolic language is the ability to read with understanding and reason logically.

Algebra tiles provide a meaningful way of teaching algebra to students. The use of algebra tiles enables students to model mathematical ideas which are essential to the learning of mathematics and related disciplines. If students are to learn the most basic concepts in algebra and mathematics as a whole, starting from basic school and beyond, it is important that teachers make use of methods that are most effective and materials that enhance concept formation. One such material that has met with considerable success through research is the use of algebra tiles, which is a major finding in this study. The study showed that the use of algebra tiles enabled students to build adequate knowledge in basic operations on algebra.

Lessons associated with the use of manipulative materials also enable students with different learning styles to benefit equally, as concepts are explained (auditory learners), demonstrated for learners to see (visual learners) and allowing learners to manipulative or model concepts themselves (kinesthetic learners). It also provides freedom students may require for learning. Time and experiences in the class enrich students learning. Students can learn from their experiences and connect the mathematics ideas to these experiences.

When students are actively involved in the manipulation of the algebra tiles, their interest in learning mathematics is aroused. Such foundations help students to understand and appreciate mathematics. Thus effective use of algebra tiles contributes to conceptualisation and understanding of algebra in the learning of mathematics.

RECOMMENDATIONS

This study was conducted using only two schools. This makes it difficult to generalise the results. However the results and findings can be inferred to students of the same characteristic and to a large extent to improve on the method of instruction as far as teaching and learning of mathematics is concerned. The following recommendations are made to guide the teaching of algebra and mathematics:

1. Algebra tiles should be used in teaching the distributive property to all students to enable them discover the discernable pattern in removing brackets from an expression. In doing this emphasis must be laid on "process" and not "product".
2. As much as possible, mathematics educators should use manipulatives such as algebra tiles for effective teaching of all the units in algebra in the school curriculum.

REFERENCES

- al – Absi, M. M., & Nofal, M. B. (2010). The effect of using manipulatives on the mathematical achievement of the first grade students. *Damascus University Journal*, 26(4), 37 – 54.
- Awanta, E. K. (2000). Helping students overcome mathematics anxiety. *Journal of the Mathematical Association of Ghana*, 12, 59-63.
- Blosser, P. E. (1985). Research related to instructional materials for science. Retrieved July 22, 2010 from <http://www.ericdigests.org/pre-923/science.htm>
- Coombs, P. H. (1970). *The World educational crisis: A system analysis*. New York: Oxford University Press.
- Eshun, B. A. (2000). Sex-differences in attitude of students towards mathematics in secondary school. *Journal of Mathematical Association of Ghana*, 12, 1-12.
- Fletcher, J. A. (2009). Learning algebraic concepts through group discussion. *Journal of Science and Mathematics Education*, 4, 31- 47.
- Heddens, J. W. (1997). Improving mathematics teaching by using manipulatives. Retrieved October 23, 2010, from <http://www.fed.cuhk.edu.hk/~flee/edumath/9706/13hedden.html>
- Jones, S. (n.d). The role of manipulatives in introducing and developing mathematical concepts in elementary and middle grades. Retrieved on October 23, 2010, from http://www.resourceroom.net/math/Jones_mathmanip.asp
- Kennedy, M. L. (1986). A rationale. *Arithmetic Teacher*, 33, 6-7.
- Kurumeh, M. S., Chiawa, M. A., & Ibrahim, M. O. (2010). Dienes multibase blocks approach: An effective

- strategy for improving interest in number bases among secondary school students in mathematics. *Research Journal of Mathematics and Statistics*, 2(3), 101-104.
- Marshall, R. (n.d.). Multiplying and factoring polynomials using algebra tiles. Retrieved April 4, 2011, from <http://mypages.iit.edu/~smile/ma8711.html>
- Ministry of Education, Science and Sports. (2007). Teaching syllabus for mathematics (junior high school). Accra: Curriculum Research and Development Division.
- Munger, D. (2007). Children learn and retain math better using manipulatives. Retrieved November 19, 2011, from http://scienceblogs.com/cognitivedaily/2007/10/children_learn_and_retain_math.php
- National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Retrieved September 9, 2010, from <http://www.nctm.org/>
- Norton, S., & Irvin, J. (2007). A concrete approach to teaching symbolic algebra. *Mathematics: Essential Practice*, 2, 551-560.
- Nwadinigwe, I. P. (2002). *Fundamentals of research methods and statistics*. Lagos: Vitaman Educational Books and Publishers.
- Onasanya, S. A., & Omosowo, E. O. (2010). Effect of improvised and standard instructional materials on secondary school students' academic performance in physics in Ilorin, Nigeria. *Singapore Journal of Scientific Research*, 1(1), 68-76.
- Picciotto, H., & Wah, A. (1993). A new algebra: Tool, themes, concepts. *Journal of Mathematical Behaviour*, 12(1), 19-42.
- Resnick, L. B., & Ford, W. W. (1984). *The psychology of mathematics for instruction*. Hillsdale: Lawrence Erlbaum Associates.
- Russel, D. (2011). The distributive property. Retrieved April 22, 2011, from <http://math.about.com/od/algebra/a/distributive.htm>
- Ruzic, R., & O'Connell, K. (2001). Manipulatives enhancement literature review. Retrieved February 11, 2011, from <http://www.cast.org/ncac/Manipulative1666.cfm>
- Seng, K. L. (2010). An error analysis of form 2 (grade 7) students in simplifying algebraic expressions: A descriptive study. *Electronic Journal of Research in Educational Psychology*, 8(1), 139-162.
- Sfard, A. (1994). The gains and pitfalls of reification-the case of algebra. *Educational Studies in Mathematics*, 26, 191-228.
- Shuttleworth, M. (2008). Quasi-experimental design. Retrieved March 6, 2011, from <http://www.experiment-resources.com/quasi-experimenatal-design.html>
- Smith, S. S. (2009). *Early childhood mathematics* (4th ed.). Boston: Pearson Education Using Manipulatives. Retrieved October 20, 2010, from <http://www.teachervision.fen.com/pro-dev/teachigmethods/48934.html>
- Sowell, E. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education*, 20, 498-505.
- Stacey, K., & MacGregor, M. (1997). Ideas about symbolism that students bring to algebra. *The Mathematics Teacher*, 19(2), 110-113.
- Suydam, M., & Higgins, J. (1977). Activity-based learning in elementary school mathematics: Recommendations from research. Retrieved February 28, 2011, from <http://www.eric.ed.gov/PDFS/ED144840.pdf>
- Thompson, S., & Fleming, N. (2003). Summing it up: Mathematics achievement in Australian schools, TIMSS 2002. Retrieved April 10, 2011, from http://www.timss.acer.edu.au/documents/TIMSS_02_Math_ES.pdf
- Thornton, G. K. (1995). Algebra tiles and learning styles. Retrieved October 23, 2010, from <http://ir.lib.sfu.ca/bitstream/1892/8013/1/b17449029.pdf>
- Wells, C. S., & Wollack, J. A. (2003). An instructor's guide to understanding test reliability. Retrieved January 10, 2011, from <http://testing.wisc.edu/Reliability.pdf>
- West Africa Examination Council. (2005). Chief Examiners' Report for the Basic Education Certificate Examination. Accra : Wisdom Press.
- West Africa Examination Council. (2006). Chief Examiners' Reports on Senior Secondary Certificate Examination. Accra: Wisdom Press.
- West Africa Examination Council. (2007). Chief Examiners' Reports on Senior Secondary Certificate Examination. Accra: Wisdom Press.
- Yara, P. O. (2009). Students attitude towards mathematics and academic achievement in some selected secondary schools in southeastern Nigeria. *European Journal of Scientific Research*, 36(3), 336-341.