

An Investigation into Challenges Faced by Secondary School Teachers and Pupils in Algebraic Linear Equations: A Case of Mufulira District, Zambia

Koji Samuel^{1*} Dr. H.M. Mulenga² Mukuka Angel³

1. Mufulira College of Education, P.O BOX 40400, Mufulira, Zambia, Central Africa

2. School of Mathematics and Natural Sciences, Copperbelt University, P.O.BOX 21692, Kitwe, Zambia

3. Mukuba University, School of Mathematics and Computer Science, P.O BOX 20382, Kitwe, Zambia

Abstract

This paper investigates the challenges faced by secondary school teachers and pupils in the teaching and learning of algebraic linear equations. The study involved 80 grade 11 pupils and 15 teachers of mathematics, drawn from 4 selected secondary schools in Mufulira district, Zambia in Central Africa. A descriptive survey method was employed to obtain in-depth and richer information about the central issues under study. To address the three research questions, an assessment test for the pupils was administered followed by separate interviews for teachers and pupils respectively. Some identified challenges include: (i) lack of pre-requisite knowledge by pupils, (ii) lack of conceptual, procedural and strategic knowledge and skills required for solving linear equations and (iii) inappropriate approaches and methods used in the teaching of algebraic linear equations. The study also compared responses on items from teachers' and pupil's interviews and found out that there was a strong relationship between teachers' and pupils' opinions on the challenges that pupils face in solving algebraic linear equations and what needs to be done to improve the situation. It is therefore, proposed that the teaching and learning of secondary school mathematics need to focus on transforming the quality land scape that has been identified especially the transition from arithmetic to algebra. Pupils need to be exposed to formulating equations from situations with which they are familiar and that teachers get regular refresher courses to keep them up-dated with new developments in the teaching and learning of mathematics.

Keywords: Challenges, Algebra, Linear equations, Secondary school teachers and pupils.

1. INTRODUCTION

Mathematics is an important tool for the development and improvement of a person's intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought. When learners have acquired enough knowledge in mathematics they develop numeracy, reasoning, thinking skill and problem solving skills. In the Zambian school curriculum, mathematics is featured as one of the core subjects in all the options for both the academic and the practical career pathways. As a result, there is always a greater pressure on learners to succeed in mathematics than there is in any other subject as is the case with England and Wales (Cockcroft; 1982). It is also argued that any person who is ignorant in mathematics would be at the mercy of others and would be easily cheated.

Despite the perceived usefulness of mathematics, there has been repeated failures in the subject by many learners in Zambia (MOE, 1996). Most performance reports by the Ministry of General Education in Zambia as well as the Examinations Council of Zambia are littered with concerns on poor performance especially in natural sciences of which mathematics is part and parcel. Such concerns are a clear indication that interventions in the teaching of mathematics must be designed to mitigate the learning problems faced by pupils especially in areas such as algebra and linear equations in particular. Therefore, the present study focused on identifying the challenges that secondary school teachers and pupils face in algebraic linear equations and charted the way forward on best practices in a quest to mitigate the identified problems. It is proposed that exposing these challenges will call for interventions that will eventually lead to improved quality of learning mathematics and its applications to real life.

1.1. Statement of the problem

The poor performance of pupils in mathematics and linear equations in particular has been a thing of concern to mathematics educators, parents and government. The chief examiner's annual reports in mathematics in the Joint Examinations for the School Certificate and General Certificate of Education Ordinary Level (JESCGCE) conducted by the Examinations Council of Zambia (ECZ) are good evidences of those facts. Mathematics educators have put in efforts aimed at identifying the major problems associated with secondary school mathematics. Despite all these noble efforts, the problem of poor achievement in mathematics has continued to rear its head. It is based on this fact that this research identified linear equations in algebra as a core difficult area where pupil's performance has always been low.

1.2. Objectives of the study

The study explored the following objectives:

- (i) To find out pupils' difficulties in solving algebraic linear equations
- (ii) To find out teachers' and pupils' opinions regarding the difficulties that pupils face in solving algebraic linear equations.
- (iii) To find out measures to undertake in mitigating the identified teaching and learning problems in algebraic linear equations.

1.3. Research questions

The above research objectives translate to the following research questions:

- (i) What are pupils' difficulties in solving algebraic linear equations?
- (ii) What opinions do teachers and pupils hold regarding the difficulties that pupils face in solving algebraic linear equations?
- (iii) What measures should be undertaken in mitigating the identified teaching and learning problems in algebraic linear equations?

2. LITERATURE REVIEW

Literature on the past research point out that most of the challenges that pupils often encounter when solving equations are attributed to their inadequate understanding of the pre requisite knowledge. As Skemp (1971) indicated, mathematics is hierarchical in nature, so learning of higher order concepts is successful only when related lower order concepts are fully grasped by the learners. Van de Walle (2004) posited that pupils need to make connections between old and new knowledge: They need to engage in reflective thinking, sifting through existing ideas to find those that seem most useful in giving meaning to new concepts being learnt. In his book, *the process of Education* (1960: 11-16), Bruner also indicated that; "if earlier learning is to render later learning easier, it must do so by providing a general picture in terms of which the relations between things encountered earlier and later are made as clear as possible" His argument was that schools should not waste time by postponing the teaching of important areas and he further indicated that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. This notion underpins Bruner's idea of the spiral curriculum which states: "A curriculum as it develops should revisit the basic ideas repeatedly, building upon them until the student has grasped the formal apparatus that goes with them (p. 13)".

Furthermore, literature indicate that students often face challenges in mathematics content, especially when trying to make sense of abstract concepts such as solving equations. Specifically, researchers have identified three common challenges that students often face when attempting to solve equations:

- (i) Lack of symbolic understanding of variables and coefficients within an equation (Kilpatrick & Izsak, 2008; Poon & Leung, 2010);
- (ii) Lack of understanding of the meaning of the equal sign (Knuth et al., 2006); and
- (iii) Reliance on procedural knowledge without conceptual understanding (Capraro & Joffrion, 2006; Siegler, 2003; Star, 2005).

These findings by previous researchers are not unique to previous studies but also apply to the present study. Therefore, it is clear that teachers need to help pupils acquire knowledge and skills in solving linear equations by undertaking a number of measures such as the ones proposed by the present study (Refer to discussion of the findings). Past researchers have also investigated the importance of external representations for solving equations. Caglayan and Olive (2010) conducted a qualitative study in which eighth grade students ($n = 24$) solved equations using cups and tiles to represent variables and constants respectively. A typical example on how a teacher can use algebra tiles to enhance pupils' understanding of linear equations is given in Figure 1 below:

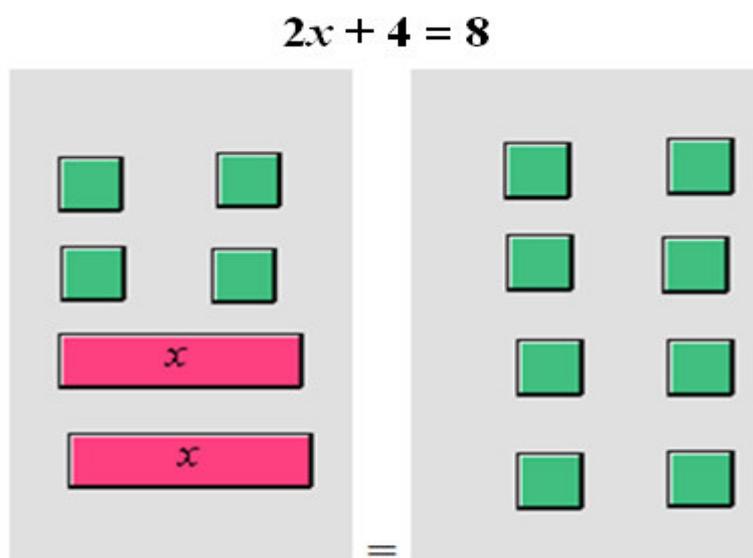


Figure 1: Equation represented with algebra tiles

[Source: Robin L. Magruder, 2012. *University of Kentucky*, robin.magruder@uky.edu]

Using manipulatives such as the one shown in Figure 1 could be helpful in enhancing pupils' understanding of solving linear equations. However, without necessary background knowledge, some students are unable to make expected connections.

3. METHODOLOGY AND PROCEDURES

To explore the three research questions, data were collected from 80 grade 11 pupils and 15 teachers by means of an assessment test for pupils on algebraic linear equations as well as separate interviews for both teachers and pupils. Those pupils and teachers were randomly selected from four government schools in Mufulira district of Zambia, Central Africa. At the time of data collection, grade 11 pupils were assumed to be the best holders of the required information owing to the fact that grade 12 pupils were busy preparing for their final examinations while grades 8, 9 and 10 were assumed to have received less instruction on algebraic linear equations as compared to grade 11 pupils. The assessment test for pupils was done in two stages. Firstly we wanted to find out whether pupils had challenges in solving algebraic linear equations by asking them to solve different algebraic linear equations. Those problems were identified through a qualitative and careful analysis of the pupils' marked scripts. After identifying pupils' problems in solving algebraic linear equations, another assessment test that focused on testing them on each of the identified areas of need was administered. The extent to which each one of those needy areas contributed to pupils' failure to solve linear equations was determined using a multiple regression model. Thereafter, separate interviews for pupils and teachers respectively were conducted to find out more on the causes of the identified problems and how best such challenges could be mitigated. The responses from pupils and teachers on related interview items were paired and a rank correlation coefficient was computed to determine the degree of agreement between pupils' and teachers' responses.

In this respect, we wish to acknowledge the following people who contributed positively to a successful completion of this study: Mr. Chimuka Head teacher, Mr. Chewe D/head and Mr. Kauseni Head of Mathematics Department from Kantanshi secondary school in Mufulira District, Zambia. Others include head teachers, deputy head teachers, HODs and teachers of mathematics from Ipusukilo secondary school and Pamodzi Girls' secondary school in Mufulira district, Zambia.

4. RESULTS AND FINDINGS

We present here a sample of results and findings.

4.1. *Pupils' difficulties in solving algebraic linear equations*

As stated earlier, the main purpose of the study was to investigate the challenges that secondary school teachers and their pupils face regarding the teaching and learning of algebraic linear equations. Eighty pupils were presented with algebraic linear equations (both symbolic and word statements), and were asked to solve them. After going through pupils' written solutions, we found that the mean mark obtained was 13.8%, an indication that the pupils' performance was very low. After a careful analysis of their written solutions, we came to conclude that pupils' failure to solve linear equations correctly was attributed to their inadequate pre-requisite knowledge. This inadequacy in pupils' pre requisite knowledge was then measured using three explanatory variables: (i) Pupils' ability to formulate algebraic equations (ii) Pupils' mastery of mathematical language (I.e.

Translation and interpretation of word statements into symbolic algebraic conjectures as well as reading and interpreting symbolic algebraic statements) and (iii) Pupils' manipulation of algebraic expressions. Pupils were then assessed on each one of these three explanatory variables and simple descriptive statistics were obtained as shown in Table 1.

Table 1: Descriptive Statistics

Category	Mean	Std. Deviation	Count
Formulating algebraic equations	51.25	24.202	80
Mathematical language	52.75	25.307	80
Manipulating algebraic expressions	39.75	27.466	80

At a glance, one would see that the lowest average mark was recorded in manipulation of algebraic expressions. The interpretation of this could be that pupils' failure to solve algebraic linear equation was attributed more to their inadequate understanding of manipulating algebraic expressions and statements. In this sense, manipulation of algebraic expressions refers to simplification, factorisation and evaluation of algebraic expressions. However, we thought it unwise to conclude based on the summary statistics presented in Table 1. To assess the extent to which each of these three variables contributed to the inadequacy in pupils' pre requisite knowledge for solving algebraic linear equations (i.e. the criterion or dependent variable), the following multiple regression model was used:

$$Y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3$$

Where;

$Y =$ Pupils' pre – requisite knowledge in solving algebraic linear equations,

$x_1 =$ Pupils' ability to formulate algebraic equations

$x_2 =$ Pupils' mastery of mathematical language

$x_3 =$ Pupils' manipulation of algebraic expressions

The parameters $\beta_0, \beta_1, \beta_2$ and β_3 were estimated using SPSS as shown in Table 2. In order for us to measure how strong each predictor variable influence our criterion (dependent) variable, standardized regression coefficients were also generated.

Table 2: Regression coefficients

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
(Constant)	-7.973	4.031		-1.978	.052
Formulating algebraic equations	.060	.063	.089	.956	.342
Mathematical language	.114	.061	.176	1.869	.065
Manipulating algebraic expressions	.320	.056	.538	5.703	.000

a. Dependent Variable: Solving linear equations

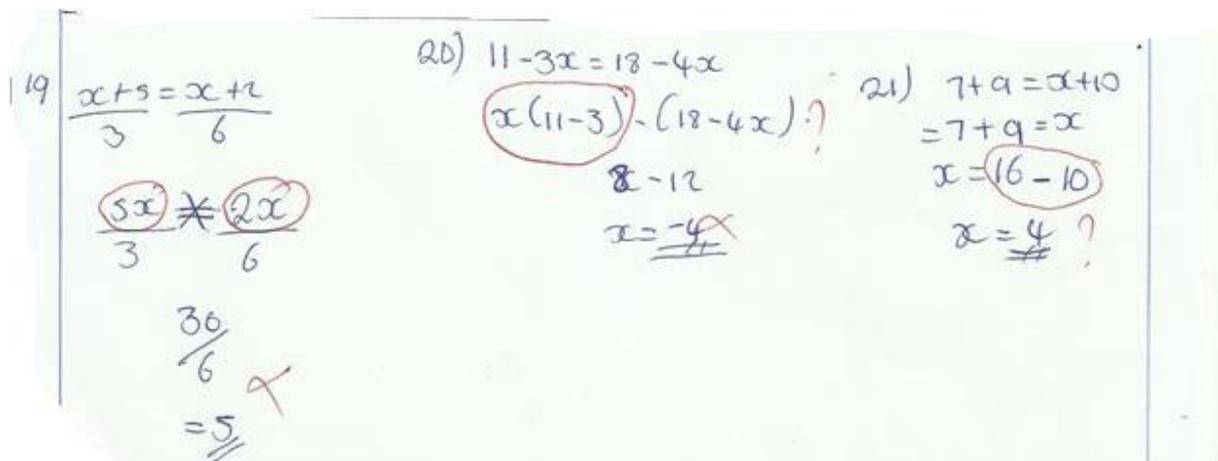
From Table 2, our fitted model could be written as

$$\hat{Y} = -7.973 + 0.060x_1 + 0.114x_2 + 0.320x_3$$

The standardized coefficient of 0.089 could be interpreted as; change of 1 standard deviation in pupils' ability to formulate algebraic equations resulted in 0.089 standard deviations in pupils' ability to solve linear equations, 1 standard change in pupils mastery of mathematical language could cause 0.176 standard increase in pupils' ability to solve linear equations and finally 1 unit change in pupils' ability to manipulate algebraic expression resulted in the standard change of 0.538 units in pupils' ability to solve linear equations. This confirms the results obtained in Table 1 that pupils' failure to solve linear equations was mostly affected by their failure to manipulate algebraic expressions. Furthermore, Table 2 also indicates that pupils' ability to manipulate algebraic expressions significantly affected their ability to solve linear equations in the sense that the p-value was 0.000 which is less than 0.05. In other words, we are 95% confident that this is true.

4.2. Pupils' opinions regarding difficulties they face in solving algebraic linear equations

After analyzing pupils' responses to the assessment tests that were administered to them, we conducted a short interview in trying to find out from them their opinions on challenges they encountered in solving algebraic linear equations. The most common issues that emerged from their responses included: (i) difficulties in grouping like terms, (ii) difficulties in manipulating algebraic signs and symbols and that (iii) their teachers were too fast in explaining algebraic concepts. These responses truly reflected some of the errors that those pupils made during their written responses to the assessment tests administered to them earlier. Figure 2 below illustrates such instances.



19) $\frac{x+5}{3} = \frac{x+2}{6}$
 $\frac{5x}{3} \neq \frac{2x}{6}$
 $\frac{36}{6} = 5$

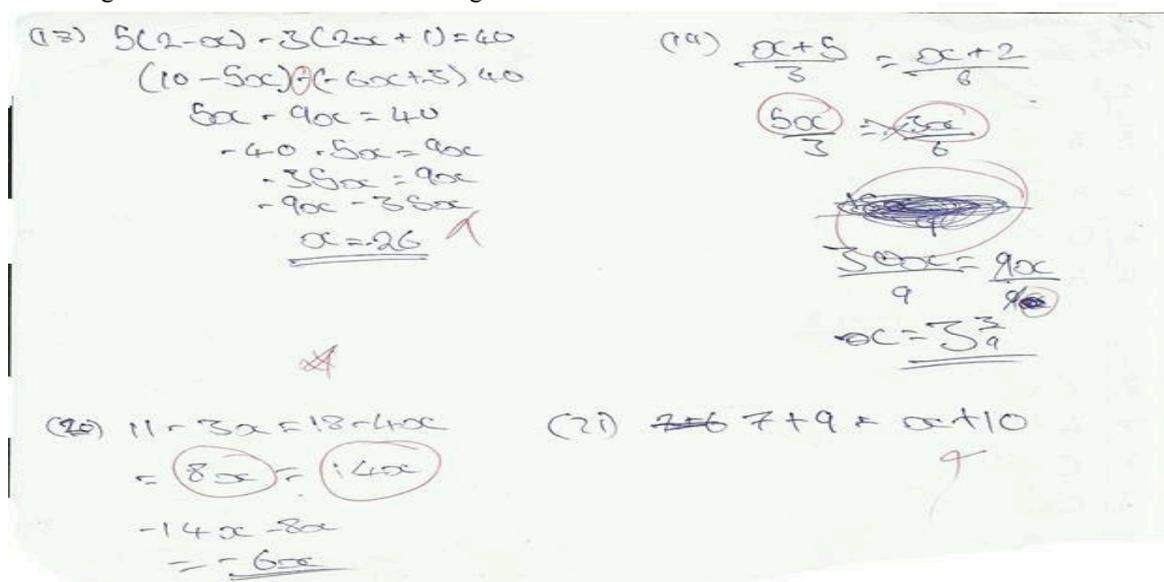
20) $11-3x = 18-4x$
 $x(11-3) = (18-4x)$
 $8-12$
 $x = \frac{-4}{-4}$

21) $7+9 = x+10$
 $= 7+9 = x$
 $x = 16-10$
 $x = \frac{4}{-4}$

Figure 2: First example of pupil's failure to simplify algebraic expressions correctly

The excerpt shown in Figure 2 above illustrates pupil's failure to solve algebraic linear equations due to lack of the necessary pre requisite knowledge such as simplification and manipulation of algebraic expressions and equations. This could be attributed to pupil's failure to deal with signs correctly and failure to group the like terms together. This is a true reflection of the responses given by pupils during an interview.

Figure 3 also reflects pupils' failure to solve algebraic equations due to lack of the pre requisite knowledge such as the ones exhibited in Figure 2.



(13) $5(2-x) - 3(2x+1) = 40$
 $(10-5x) - (6x+3) = 40$
 $5x + 9x = 40$
 $-40 + 5x = 9x$
 $-39x = 9x$
 $-9x - 39x$
 $x = 26$

(14) $\frac{x+5}{3} = \frac{x+2}{6}$
 $\frac{5x}{3} = \frac{2x}{6}$
 $50x = 9x$
 9
 $x = \frac{3}{9}$

(20) $11-3x = 18-4x$
 $= 8x = 14x$
 $-14x - 8x$
 $= -6x$

(21) $7+9 = x+10$

Figure 3: Second example of pupil's failure to simplify algebraic expressions correctly

4.3. Teachers' opinions regarding pupils difficulties in solving algebraic linear equations

Teachers were also asked to give their position on why pupils failed to solve algebraic linear equations especially that the questions that were given to them (pupils) were based on what they had learnt previously. In response teachers cited various reasons for the deficiencies that pupils exhibited such as the following:

- Simplification of algebraic expressions had always been problematic to the pupils
- Pupils are usually taught algebra in an abstract way without using concrete examples such as manipulative objects like algebra tiles.
- Pupils usually have challenges to read and understand mathematical statements. As a result, they tend to focus more on memorizing procedures instead of understanding the concepts.
- Pupils' failure to recognise and understand algebraic terms such as coefficients, constants, evaluate, simplify, expand, factorise and many more also contribute to their low achievement in algebraic linear equations.

When asked to give a comment on what should be done to improve and enhance pupils' understanding of algebraic linear equations, some teachers made the following submissions:

"Learners think mathematics is all about solving for x, y or z. Aspect of word problems analysis is not

emphasised. Learners have problems in solving word problems because of language barrier. There is need to emphasise and teach word analysis skills in learners”

“Teachers to use real life situations in relating to certain unknown variables”

“There is need to expose pupils to word problems through discussions on the skill of interpretation from word information to maths statements”

“Target variable identification and establishing relationships are always a challenge to pupils. More examples and explanations should be given to pupils on how to identify the target variables”

“Pupils need to be aware that English and Mathematics are two different subjects altogether and so teachers are expected to explain this to their learners”

The responses from teachers such as the ones above suggest that there is need to focus more on the language part when explaining algebraic concepts. There is need to emphasise to the pupils that Mathematics has got its own grammar which must be fully understood if meaningful learning of algebraic linear equations is to take place.

5. DISCUSSION OF THE FINDINGS

With reference to the responses gathered and the presented findings in the foregoing section, it suffices to state that our three research questions have been answered.

Results indicate that the majority of pupils failed to solve linear equations because they lack pre requisite knowledge. Pupils’ inability to formulate algebraic equations from given situations, inadequate mastery of mathematical language (i.e. Translation and interpretation of word statements into symbolic algebraic conjectures as well as reading and interpreting symbolic algebraic statements) and pupils’ failure to manipulate algebraic expressions are some of the key identified challenges that affected their ability to solve algebraic linear equations appropriately. Pupils’ failure to group the terms and manipulation of positive and negative signs were also identified as barriers to their effectiveness in solving algebraic linear equations. These findings do not only apply to the present study but past researchers have made similar observations. Among the identified common challenges that students often face when attempting to solve equations are the following:

- Lack of symbolic understanding of variables and coefficients within an equation (Kilpatrick & Izsak, 2008; Poon & Leung, 2010);
- Lack of understanding of the meaning of the equal sign (Knuth et al., 2006); and
- Reliance on procedural knowledge without conceptual understanding (Capraro & Joffrion, 2006; Siegler, 2003; Star, 2005).

In answering the second and third research questions, we found that there was an agreement in the responses that pupils and teachers gave on what made pupils to perform poorly in solving algebraic linear equations. In both pupils’ and teachers’ opinions, the most cited challenge was associated with pupils’ difficulties in translating algebraic word statements into symbolic statements and vice versa. Pupils’ inadequate understanding of algebraic terms such as coefficient, variable and many more were also cited by both teachers and pupils to be another challenge that secondary school pupils face in solving algebraic linear equations. A lack of symbolic understanding on the part of students is problematic. For example, students do not understand nuances such as the differing roles of 2 in the two expressions, 2 and $2x$. In the first example, 2 is a constant, in the second, 2 is a coefficient, but often students treat them the same (Poon & Leung, 2010).

On the other hand, one of the pupils made the following submission during interviews.

“Sometimes we find it difficult to understand mathematics because teachers are too fast when explaining mathematics”

The response above suggests that teachers need to take their time when explaining algebraic concepts to the pupils especially that algebra provide tools for learning and understanding other mathematical concepts. In fact this was complimented by one of the submissions that one of the teachers made to the researchers.

“There is need to expose pupils to word problems through discussions on the skill of interpretation from word information to maths statements”

In all, the responses that pupils and teachers gave to interview questions were similar. Researchers wanted to have an idea about the extent to which those responses were similar by pairing both pupils’ and teachers’ responses to the interview questions. A Spearman’s rank correlation coefficient of 0.738 indicated a strong agreement between teachers’ responses and pupils’ responses. It is therefore, proposed that the teaching and learning of secondary school mathematics need to focus on transforming the quality land scape that has been identified especially the transition from arithmetic to algebra. Pupils need to be exposed to formulating equations from situations with which they are familiar and that teachers get regular refresher courses to keep them up-dated with new developments in the teaching and learning of mathematics.

Edgar Dale’s cone of experiences and Jerome Bruner’s emphasis on the provision of hands on experiences need to be taken seriously by teachers of secondary school mathematics if the quality of teaching and learning algebraic linear equations is to improve significantly. Zoltan Dienes stated, “One of the first things we should do in trying to teach a learner any mathematics is to think of different concrete situations with a

common essence. Then . . . children will learn by acting on a situation” (Sriraman & Lesh, 2007, p. 61). This statement emphasized two salient beliefs of many constructivists regarding mathematics learners; students need to learn by doing and they need to understand mathematics in terms of real life (Gordon, 2009). This was also echoed in the present study by one of the teachers who emphasised the need to focus on providing students with real life situations to enhance students’ understanding of linear equations.

6. LIMITATIONS OF THE STUDY AND FUTURE DIRECTIONS

One of the possible limitations of this study is that only one topic “algebraic linear equations” was investigated out of all the topics that Zambian secondary school students find difficult to handle. Examinations performance reports in Zambia are littered with issues to do with low achievement levels in mathematics. This quality landscape is in most cases attributed to pupils’ failure to handle questions on earth geometry, linear programming, algebra, transformations, mensuration and many more. Therefore future research should take such into consideration.

All the participants were drawn from government schools in one district and so pupils’ failure to solve algebraic linear equations might have been influenced by some other factors that could not be identified in this study. In their study of *learning time of day and students’ academic achievement at secondary school level*, Mulenga and Mukuka (2016) stated that intelligence is not the only determinant of academic achievement of students but always associated with so many components of the learning environment and “time of day” when lessons are being delivered. Furthermore, it was a bit difficult getting pupils to work on the tasks as this took part of their school time. However, researchers were given greater support particularly from the mathematics teachers for the classes concerned.

7. CONCLUSION

From the above key findings it can be concluded that the mathematical language that was used in word problems appeared to be an academic obstacle to learners’ success in word problem solving. The results of this study also seem to suggest that pupils’ inability to manipulate algebraic expressions and their inadequate understanding of algebraic concepts contributed highly to their failure to solve algebraic linear equations. Part of the blame by pupils was shifted to their teachers whom they claimed did not create enough time and opportunities to understand the concepts fully. It is therefore essential that before pupils are exposed to the learning of linear equations, they are first equipped with the necessary background knowledge upon which new knowledge can be built. This prerequisite knowledge should include number sense and operations, algebraic laws, concept of variable, algebraic terms and expressions (the symbolic language of mathematics) and manipulation of algebraic expressions. Ausubel and Robinson (1969) once stated: “Meaningful learning takes place when we grasp the interrelationship between two or more ideas, old and new. A first pre requisite for meaningful learning the two gentlemen contended is that the material presented to the learner be capable of being related in some sensible fashion. Secondly, the new information must be fitted into a larger pattern or whole. Learners must possess relevant ideas to which the new idea can be related or anchored. Thirdly, the learner must have appropriate subsuming concepts in his or her cognitive structure. Finally, the learner must actually attempt to relate, in some sensible way, the new ideas to those which he presently possess (p.46)”. If any of these conditions is missing, the end result will be rote learning and challenges such as the ones exhibited in the present study will continue to be part and parcel of the mathematics education in Zambia and elsewhere.

Lack of knowledge on any of the above mentioned concepts may lead to serious hiccups in the pupil’s ability to understand linear equations. Our experience in teaching show that proficiency in dealing with operations with decimals, fractions, directed numbers lessens pupils’ problems in solving linear equations. When pupils are knowledgeable about laws governing algebraic manipulation, including order of operations, they are less likely to encounter any problems in solving linear equations. In order for them to be able to represent algebraic conjectures from words to their symbolic form, pupils need to have a sound knowledge of various uses of letters and algebraic symbols in mathematics. Like other studies in other parts of the world, the present study stresses on the need to provide hands on experiences for pupils if meaningful learning is to take place. Concrete and virtual manipulatives in mathematics education are a means of bridging the transition from concrete to abstract mathematics.

Nevertheless, we wish to state that meaningful learning in the teaching of algebraic linear equations can be achieved if teachers make use of the constructivist approaches. The essence of constructivism has been captured through the development of active learning, also known as learning by doing, learning by experience, learning through action, student-centered learning, peer collaboration and cooperative learning.

8. REFERENCES

Ausubel, D. P (1960). *The Use of advance organizers in the learning and retention of meaningful verbal materials*. Journal of Educational Psychology, 51(5), 267 - 272.

- Ausubel, D. P (1962). *A subsumption theory of meaningful verbal learning and retention*. The Journal of General Psychology, 51(5), 267 - 272.
- Bruner, J (1960). *The Process of Education*, Cambridge, Mass.: Harvard University Press.
- Caglayan, G., & Olive, J. (2010). Eighth grade students' representations of linear equations on a cups and tiles model. *Educational Studies in Mathematics*, 74, 143-162. Doi: 10.1007/s10649-010-9231-z.
- Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, 27, 147-164. doi: 10.1080/02702710600642467.
- Cockcroft, W.(1982).*Mathematics Counts: Report of the Committee Of Enquiry into the Teaching of Mathematics in Schools*. London: HMSO.
- Examinations Council of Zambia. (2012). *Chief examiners report*. Lusaka: Examinations Council of Zambia.
- Gordon, M. (2009). The misuses and effective uses of constructivist teaching. *Teachers And Teaching: Theory and Practice*, 15, 737-746.
- Kilpatrick, J., & Izsak, A. (2008). A history of algebra in the school curriculum. In C. E. Greenes (Ed.), *Algebra and algebraic thinking in school mathematics* (pp. 3-18). Reston, VA: National Council of Teachers of Mathematics.
- Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal of Research in Mathematics Education*, 37, 297-312.
- Magruder, Robin L (2012). Solving linear equations: A comparison of concrete and virtual manipulatives in middle school mathematics" *Theses and Dissertations--Curriculum and Instruction*. Paper 2.http://uknowledge.uky.edu/edc_etds/2
- Ministry of Education (MoE). (1996). *Educating Our Future: National Policy on Education*,Lusaka: Zambia Education Publishing House.
- Mulenga, H. M and Mukuka, A (2016). Learning Time of Day and Students' Academic Achievement at School Certificate Level: A Case Study of Chibote Girls' Secondary School. *Journal of Education and Practice*. ISSN 2222-1735 (Paper) ISSN 2222-288X (Online) Vol.7, No.20, 2016
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics* (Vol. 1). National Council of Teachers of Mathematics.
- Poon, K., & Leung, C. (2010). Pilot study on algebra learning among junior secondary students. *International Journal of Mathematics Education in Science and Technology*, 41, 49- 62.
- Siegler, R. S. (2003) Implications of cognitive science research for mathematics education. In J.
- Skemp, R (1971). *The Psychology of Learning Mathematics*. Penguin Books
- Sriraman, B., & Lesh, R. (2007). A conversation with Zoltan Dienes. *Mathematical Thinking and Learning*, 9(1), 59-75.
- Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching student-centered mathematics grades 5-8*. Boston: Pearson.