

# Teaching Arithmetic Combinations of Multiplication and Division to Students with Learning Disabilities or Mild Intellectual Disability: The Impact of Alternative Fact Grouping and the Role of Cognitive and Learning Factors

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## Abstract

The effectiveness of two instructional interventions was investigated in the context of teaching Arithmetic Combinations (ACs) of multiplication and division to students with Learning Disabilities (LD) or Mild Intellectual Disability (MID). The intervention for the control group (LD = 20, MID = 10) was based on principles of effective instruction, while the intervention for the experimental group (LD = 19, MID = 4) combined the intervention for the control group and an alternative grouping and presentation scheme of ACs. Correlations between cognitive and learning characteristics of the two disability categories and participants' performance in ACs learning were also investigated. Intra-group comparisons showed that post-intervention performance of both groups (control and experimental) was significantly higher than their pre-intervention performance. However, inter-group comparisons revealed that there was no significant difference between the results obtained through the two interventions. Students with LD outperformed their counterparts with MID. Differences of the two disability categories in domains such as speed of information processing and counting skills correlated with performance. Results are discussed in reference to the organization of effective intervention programs for supporting students with LD or MID in their effort to learn arithmetic combinations of multiplication and division.

**Keywords:** math disabilities, arithmetic combinations, learning disabilities, mild intellectual disabilities

## 1. Introduction

The multiplications of two one-digit numbers (e.g.,  $6 \times 9 = 54$ ) and the divisions created when using the products of these multiplications as dividends and the factors as dividers (e.g.,  $54 : 9 = 6$ ) are the arithmetic combinations of multiplication and division respectively (Agaliotis, 2011). Arithmetic Combinations (ACs) are components of complex mathematical tasks (e.g., use of algorithms in the context of word problem-solving). Considering that appropriate implementation of any complex task depends on the fluent use of individual structural components, it becomes obvious that students should be able to recall ACs as quickly and accurately as possible, in order to deal successfully with demanding mathematical tasks including ACs as one of their components (Baroody, Bajwa, & Eiland, 2009; Crawford, 2003).

Students of typical development usually manage to use fluently the ACs by the time they reach the 3<sup>rd</sup> grade (Robinson, Menchetti, & Torgesen, 2002). In contrast, students with Learning Disabilities (LD) and students with Mild Intellectual Disability (MID) often have significant difficulties in learning and directly recalling the ACs, until the end of primary school or even later in life (Bouck, Bassette, Taber-Doughty, Flanagan, & Szwed, 2009; Geary, Hoard, & Nugent, 2012; Gersten, Jordan, & Flojo, 2005).

The present research aims at the examination of the impact specific teaching practices might have on the acquisition of multiplication and division ACs by students with LD or MID. Furthermore, the present research seeks to identify correlations between specific cognitive and learning characteristics of the two disability categories and the result of their effort to acquire the ACs. Clarification of these issues would offer important information for organizing appropriate interventions for students of both categories of educational needs.

### *1.1 Approaches to the Learning and Effective Use of Arithmetic Combinations*

Effortless recall of ACs is a vital prerequisite for their effective use in daily school practice and everyday life. Researchers and practitioners cognizant of the difficulties faced by students with LD or MID in memorization and recall have been using, for decades now, various means and procedures in order to support them in their effort to learn the ACs. A widely used group of techniques for the facilitation of ACs' acquisition is grounded in the S-R tradition, i.e., utilizes the repeated and systematic presentation of individual ACs to students (e.g., use of flashcards). Researchers who have implemented such practices for teaching ACs to students of different categories of special needs (including students with LD and MID) have reported positive results (e.g., Cravalho, McLaughlin, Derby, & Waco, 2014; Hayter, Scott, McLaughlin, & Weber, 2007). However, there are also researchers, who emphasize that the repetitive display of individual ACs, with no interconnection between them, "forces" students to treat them as unrelated bits of information, the storage of which may create memory overload and difficulty in building a meaningful conceptual network of mathematical knowledge (e.g., Duhon, House, & Stinnett, 2012; Nelson, Burns, Kanive, & Ysseldyke, 2013).

In an effort to promote (a) the organization of ACs in groups of interrelated facts that are easy to memorize and recall, and (b) the understanding of the coherence of the number system, researchers have used alternative grouping and presentation schemes of ACs, based on mathematical relationships or characteristics (e.g., Agaliotis, 2011; Baroody, 2006; Garnett, 1992). Example of the application of this approach is to present "families" of multiplication and division ACs (e.g.,  $6 \times 9 = 54$ ,  $9 \times 6 = 54$  and  $54 : 9 = 6$ ,  $54 : 6 = 9$ ). Arithmetic combinations of multiplication are taught first, then the systematic connections and relationships between the ACs of multiplication and division are made clear, and finally all "members" of each AC "family", as a group, are committed to memory (Burns, Kanive, & DeGrande, 2010; Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009). Research shows that a main prerequisite for the effective use of this technique is that the multiplication ACs have been mastered by the participants to the degree of automaticity, prior to the commencement of the effort to learn the division ACs. Otherwise, students try to pursue simultaneously multiple objectives (i.e., use effectively multiplication ACs and learn division ACs), allocate unfavorably their mental resources, and are finally led to failure or suboptimal performance (Baroody, 2006; Dowker, 2009).

An approach that is oriented toward the creation of groups of interconnected ACs without, however, being so much dependent on a rigid sequence of prerequisite knowledge and skills, is the one utilizing common characteristics of the numbers included in the ACs or various arithmetic properties and principles (such as the principle of multiplying or dividing any number with "1" or "0"), as "criteria" for the formation of groups. In such an organization of facts the presentation sequence is not defined by the size of the result, but by the ease with which ACs can be integrated in discernable sets of data. For example, in traditional teaching the AC " $4 \times 6 = 24$ " is presented earlier than " $8 \times 8 = 64$ ", since  $24 < 64$ . In contrast, in alternative grouping of ACs " $8 \times 8 = 64$ " precedes " $4 \times 6 = 24$ ", since it is placed in the set of multiplications of "twin numbers" (together with  $3 \times 3$ ,  $4 \times 4$ ,  $5 \times 5$ , etc.), which are easy to memorize and thus they are presented at the beginning of alternative intervention programs (Agaliotis, 2011; Woodward, 2006).

The alternative grouping and presentation of ACs based on common arithmetic characteristic or arithmetic properties has been used successfully in a relatively small number of studies in various countries (e.g., Agaliotis et al., 2003; Bryant et al., 2016; Woodward, 2006). However, the interventions used in these studies were compositions of many specific teaching techniques; hence, the exact contribution of alternative grouping in improving student performance is not easily distinguishable. In other words, while the effect of the alternative grouping of ACs based on common features and properties is an ingredient of many effective interventions, it has not been adequately studied as a separate strategy.

In reference to the participants of the conducted studies on ACs learning, it should be mentioned that in most cases they belong to the LD category, while students with MID are clearly underrepresented among these participants (e.g., Agaliotis et al., 2003; Graham, Bellert, Thomas, & Pegg, 2007; Powell et al., 2009; Woodward, 2006). Moreover, Caffrey and Fuchs (2007) found only 1 study in 21 years, from 1982 to 2003, referring to the teaching of ACs to students of both disability categories (LD and MID). Nonetheless, students of these two categories of special needs are very often found in common training contexts, and follow similar educational programs in mathematics. Therefore, investigation of similarities and differences between these two disability groups in the way and ease with which they learn ACs, can provide researchers and practitioners with valuable information for improving the quality of educational services offered to them (Bouck et al., 2009; Caffrey & Fuchs, 2007).

### *1.2 Cognitive and Learning Factors of ACs' Learning*

Considering that math difficulties result from inadequacies and peculiarities in cognitive domains such as language, memory or executive functioning (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013), it becomes evident that these domains constitute adequate fields of exploration of the similarities and differences between students with LD and students with MID in learning ACs. A literature review by the present researchers revealed that the number of studies comparing the two groups in these domains is rather limited. In one of the few studies involving students from these two categories of special needs (LD, MID), along with other categories of struggling students (ADHD, anxiety disorder), Calhoun and Mayes (2005) found that students with LD had lower mean scores in processing speed than what would be expected considering their mean IQ score, while students with MID had low mean scores in all examined factors (processing speed, perception of information, verbal comprehension), as could be predicted by their compromised mean IQ score. However, in the aforementioned study no information is reported on whether the scores of the two groups differ significantly. In another study, Maehler and Schuchardt (2009) found that students with LD or MID differed significantly from students of typical development regarding the functionality of working memory, but did not differ significantly from each other in this perspective, despite their difference of 23 points in mean IQ score. In general, however, students with LD have been found to perform better than students with MID in direct recall of ACs (Parmar, Cawley, & Miller, 1994; Shin & Bryant, 2013; Van Luit & Naglieri, 1999). Nonetheless, more research is needed in order to identify similarities and differences between these two disability groups in domains like the counting skills and the knowledge of number (Caffrey & Fuchs, 2007).

### *1.3 Aim of the Study*

Based on what has been exposed previously, the first aim of the present research is to study the effect of an alternative grouping of ACs (utilizing structural characteristics, properties and principles of ACs formation) on the effort of students with LD and students with MID to learn the ACs of multiplication and division. The second aim of this study is to highlight the role played by cognitive and learning characteristics of the two disability groups in learning the ACs of multiplication and division.

## **2. Method**

### *2.1 Participant Characteristics*

The sample consisted of 53 students, aged 9-12 years ( $M = 10.18$ ,  $SD = 1.37$ ), who were enrolled in primary schools of Northern Greece, and had been diagnosed by qualified state agencies as presenting either LD or MID. These students were instructed in general classes, but they were also visiting the Resource Rooms of their schools for 2-3 hours per week.

#### *2.1.1 Sampling Procedures*

Selection of students was based on the following criteria: (1) the official diagnosis of the above-mentioned state agencies, which was issued less than a year before the beginning of the research, (2) the absence of other difficulties or disabilities, (3) the occurrence of difficulties in mathematics, which were so severe that students were classified by their teachers in the lower 25% of the performance spectrum of their class, and (4) the attendance of a Resource Room for at least one year before the commencement of the study. All students with LD were of average intelligence ( $M = 96.8$ ,  $SD = 9.6$ ), while students with MID were of sub-average intelligence, ranging from 60 to 75 ( $M = 71.8$ ,  $SD = 6.5$ ), as reported by the state agencies which categorized them.

#### *2.1.2 Sample Size*

The participants were randomly allocated to a control and an experimental group. The control group included a total of 30 students (LD = 20 and MID = 10) and the experimental group consisted of 23 students (LD = 19 and MID = 4). The experimental group included initially 30 participants. However, due to various reasons seven students (all with MID) were withdrawn or excluded from the final analyses. Parents of all students consented in written form to the participation of their children in the current research.

### *2.2 Procedures of the Study*

In order to distinguish the effect of the alternative ACs' grouping in the context of teaching students with LD and students with MID ACs of multiplication and division, the following procedure was employed: initially, an intervention in line with fundamental principles of effective instruction (Agaliotis, 2011; Griffin, 2004) was organized, and utilized for teaching the control group. To this intervention an alternative grouping of ACs (based

on distinct arithmetic characteristics and principles) was added, yielding the instruction used for the experimental group.

### 2.2.1 Duration, Content and Structure of the Interventions

The total duration of the intervention was 12 teaching hours (lessons), 6 hours for the ACs of multiplication and 6 for the ACs of division. Each lesson lasted 30 to 35 minutes, and there were 2 to 3 lessons per week (4 to 6 weeks). A hundred and twenty randomly selected ACs (60 multiplications and 60 divisions) out of the total of 190 existing ACs for both operations (100 multiplications and 90 divisions) made up the content of the intervention.

The intervention based on effective instruction principles (control group intervention) included 5 structural components or steps:

1<sup>st</sup> step: Review of prerequisite ACs that can facilitate the learning of the ACs of multiplication and division (e.g., review of ACs of addition from the group of “twin numbers” [e.g.,  $3 + 3$ ], before teaching ACs of multiplication with “2” as one of the factors [e.g.,  $2 \times 3$ ], because of the conceptual relationship and the same result [ $3 + 3 = 2 \times 3 = 6$ ]).

2<sup>nd</sup> step: Direct teaching of the new ACs through the use of the 3 modes of knowledge representation (three-dimensional materials, pictures and symbols), with clear demonstrations and explanations followed by students’ repetitions.

3<sup>rd</sup> step: Guided practice of students, under close teacher supervision and with frequent and short-term positive reinforcement, with the aim to achieve accurate and speedy recall of ACs.

4<sup>th</sup> step: Independent practice of students on accurate and speedy recall of ACs without any teacher support. Success criterion was the provision of at least 16 correct oral answers to a total of 20 ACs presented in written form, in one minute. Teachers provided corrective feedback to the students and, if necessary, repeated the former stage of guided practice.

5<sup>th</sup> step: Final assessment for controlling the achievement of the learning goal, and completion of student’s progress table.

In the context of this (control group) intervention, the presentation sequence of ACs was the one appearing in the school textbooks, which use the organization of the well-known multiplication tables.

In contrast, the presentation sequence used for the experimental group followed a different scheme, according to which the 60 multiplication ACs and the 60 division ACs were placed in 7 distinct groups each (14 groups in total). In each lesson one or two groups of ACs were presented, depending on the progress of the students. In other words, the intervention applied in the case of the experimental group was a combination of the principles used for the instruction of the control group and the alternative grouping of the ACs presented on Table 1.

Table 1. Groups of arithmetic combinations

ACs of multiplication		ACs of division	
Groups of ACs	Number of ACs	Groups of ACs	Number of ACs
Multiplication by 1	6	Division by 1	6
Multiplication by 2	8	Division by 2	8
Multiplication by 5	8	Division by 5	8
Multiplication of twin numbers (e.g. $3 \times 3$ , $8 \times 8$ )	10	Divisions with same divisor and quotient (e.g. $49:7 = 7$ , $64:8 = 8$ )	10
Multiplication by 3	8	Divisions by 3	8
Multiplication by 9	12	Division by 9	12
Remaining facts: $4 \times 7 - 7 \times 4$ , $6 \times 7 - 7 \times 6$ , $6 \times 8 - 8 \times 6$ , $7 \times 8 - 8 \times 7$	8	Remaining facts: $28:4 - 28:7$ , $42:6 - 42:7$ , $48:6 - 48:8$ , $56:7 - 56:8$	8

### 2.2.2 Measures of the Study

Participants' initial and final performance in providing oral answers to written multiplication and division ACs, was measured through informal tests. Twenty ACs, written with sizeable numerals, were presented to the participants in one minute, preceded by the question: "Can you please tell me how much is...?"

In order to investigate the possible effect of cognitive and learning characteristics of the two disability categories (LD vs. MID) on the results of the interventions, a series of comparisons focusing on key processing information factors of ACs' acquisition process was performed. The factors explored, some of the studies identifying their importance, and also the tools used in the present study for their investigation, were:

*Verbal skills* (Cirino, Fuchs, Elias, Powell, & Schumacher, 2015; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2011): The measures used for investigating the factor of verbal skills were two subscales (Verbal Analogies and Vocabulary) from the ATHINA Test, which is a Greek standardized and widely used tool for diagnosing students with mild disabilities (Paraskevopoulos, Kalantzi-Azizi, & Giannitsas, 1999). "Verbal analogies" include 32 couples of sentences, which the examinees have to complete orally, in order to prove that they understand the analogies of the referred concepts (e.g., "the table is square, the sun is ....."). In the "Vocabulary" subtest examinees are asked to provide oral definitions to 20 orally presented verbs and nouns (e.g., "neglect", "apple") ( $\alpha = .81$ ).

*Working memory* (Schuchardt, Maehler, & Hasselhorn, 2008): The capacity of working memory was examined through a measure that asks students to repeat in reverse order 10 orally presented and gradually increasing number sequences (e.g., 5-4-6, 6-6-7-9-5, 8-2-4-6-8-1-2) ( $\alpha = .80$ ).

*Processing speed* (Compton et al., 2011): Participants' processing speed was examined through a measure used also in the study of Compton et al., who presented students with 20 rows of six numbers each, and asked them in one minute to locate and circle the two identical numbers of each row (e.g., 9, 4, 6, 8, 9, 3) ( $\alpha = .79$ ).

*Phonological short-term memory* (Gathercole & Pickering, 2001): The "memory of verbal sequences" subscale of the ATHINA Test (Paraskevopoulos et al., 1999) was utilized in order to perform this examination. This sub-scale includes 16 gradually increasing digit sequences, which the examinee is asked to repeat orally, after the initial oral presentation by the examiner ( $\alpha = .81$ ).

*Counting skills* (Jordan, Kaplan, Locuniak, & Ramineni, 2007): Counting skills were controlled through the "Counting Skills" subscale of the ATHINA Test. The examinees were asked to skip count in direct and reverse order by 2 up to 12, by 3 up to 18, by 4 up to 24, by 5 up to 35, and by 6 up to 30 ( $\alpha = .78$ ).

*Properties of operations* (Cowan et al., 2011): Knowledge of the properties of operations was tested through a variation of a procedure used by Cowan et al. (2011), in order for the procedure to be better adapted to the curriculum by which the present participants were taught. The aim was to determine whether the participants could use known ACs to find unknown ones, based on operations properties. The task included a total of 20 couples of ACs, embedded in questions like: "If you know that  $5 \times 4 = 20$ , then how much is  $4 \times 5 = \dots$ ", "If you know that  $72 : 8 = 9$ , then how much is  $8 \times 9 = \dots$ " ( $\alpha = 0.78$ ).

*Fluency of ACs* (Cowan & Powell, 2014; Bryant et al., 2011): The fluency with which participants used ACs before and after the intervention was tested through a trial that included 20 multiplication ACs (e.g.  $2 \times 6 = \dots$ ,  $7 \times 7 = \dots$ ,  $8 \times 6 = \dots$ ) and 20 division ACs (e.g.  $20 : 4 = \dots$ ,  $18 : 2 = \dots$ ,  $24 : 6 = \dots$ ), to which students had to provide written answers in one minute ( $\alpha = 0.77$ ).

*Generalization of ACs* (Tournaki, 2003): Participants' ability to generalize the use of acquired ACs was tested through two measures, one for the multiplication ACs and one for the division ACs. Each measure contained 10 tasks with 3 one digit numbers of the form " $(3 \times 5) + 1 = \dots$ " and " $(12 : 4) + 1 = \dots$ ", which the participants were allowed to answer without time limit ( $\alpha = .79$ ).

### 2.3 Fidelity of Implementation

The interventions were implemented by a total of 18 Resource Room teachers, who taught the participants in groups of 2 to 3 students. All teachers had studies in special education at undergraduate or postgraduate level, and their average teaching experience in special, primary, educational settings was 10 years. Prior to the commencement of the program, the 18 teachers received 8 hours of training by the researchers on the implementation of the interventions. Moreover, every week they received feedback from the researchers on the quality of program application, during meetings that took place in the schools. Implementation fidelity was established by systematic observation and precision recordings, on a 4 point scale (1 = low to 4 = excellent), of at least four lessons of each intervention group. Conformity indicators were related to the quality of

implementing the 5 components of effective instruction and to the degree to which teachers complied with the instructions received by the researchers (Bryant et al., 2016). In all cases the results showed high quality of application.

### 3. Results

#### 3.1 Statistics and Data Analysis

Results of the Shapiro-Wilk test indicated that some variables did not satisfy the condition of normal distribution. For this reason, non-parametric tests were used (Mann-Whitney and Wilcoxon Signed Rank tests).

The differences in the impact of the interventions (differences between intervention groups and between students with LD and MID) were determined through the calculation of the effect size (Thalheimer & Cook, 2002). Calculation was performed through the formula:

$$r = Z / \sqrt{N} \quad (1)$$

Use of this formula is recommended when the research population is small and the analyses are made with non-parametric tests (Fritz, Morris, & Richler, 2012). The  $Z$  in the formula represents the conversion of the individual data in a standardized format, and shows how many standard deviations below or above the population mean a raw score is. The  $N$  is the total number of participants' data. For example, if an individual's score is 18 and the mean score is 15, with SD 3.5, the  $Z$  score is 0.85 [ $Z = (18-15)/3.5 = 0.85$ ]. If the control group includes 20 students, for each of whom 2 data are available (1 before and 1 after the intervention), then  $N = 20 + 20 = 40$  data (sqrt 40 = 6.32). Consequently, the  $r$  for this individual would be  $0.85/6.32 = 0.13$ . According to Fritz et al. (2012), the effect is considered small if  $r = 0.10-0.30$ , modest if  $r = 0.31-0.50$ , and large if  $r > 0.51$ .

Correlations between cognitive and learning variables of students with LD or MID, on the one hand, and fluency in the use of multiplication and division ACs, on the other, were calculated through Spearman's  $\rho$ .

#### 3.2 Pre-Intervention Data

##### 3.2.1 Demographic and Cognitive Characteristics of Participants

Results for demographic characteristics showed that differences between the two intervention groups were not statistically significant regarding gender ( $U = 312.0$ ,  $z = -.713$ ,  $p = .476$ ), age ( $U = 241.0$ ,  $z = -1.870$ ,  $p = .062$ ) and category of special educational needs ( $U = 290.0$ ,  $z = -1.292$ ,  $p = .196$ ) (Table 2).

Table 2. Characteristics of participants

	Gender			Age				SEN	
	Boys		Girls	Boys		Girls		LD	MID
	<i>N</i>	<i>N</i>	<i>N</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>N</i>	<i>N</i>
Control group	30	18	12	10.32	1.24	10.23	1.37	20	10
Exp. Group	23	16	7	9.66	1.6	9.25	0.84	19	4
Total	53	34	19	10.01	1.44	9.87	1.27	39	14

With regard to the investigation of cognitive factors and learning parameters of students with LD or MID, it was found that there were statistically significant differences in verbal skills abilities ( $U = 120.000$ ,  $z = -3.090$ ,  $p = .002$ ), processing speed ( $U = 84.500$ ,  $z = -2.579$ ,  $p = .010$ ) and counting ( $U = 162.000$ ,  $z = -2.623$ ,  $p = .009$ ); there were no significant differences in working memory ( $U = 142.000$ ,  $z = -.935$ ,  $p = .350$ ), phonological short-term memory ( $U = 251.500$ ,  $z = -.467$ ,  $p = .641$ ), and properties of operations ( $U = 221.5$ ,  $z = -.919$ ,  $p = .358$ ) (Table 3).

Table 3. Cognitive and learning factors of students with LD and students with MID

	LD (n = 39)		MID (n = 14)	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Verbal skills	4.28	1.94	2.64*	.92
Working memory	3.72	1.16	3.33	1.07
Phonological STM	1.97	1.15	2.21	1.42
Processing speed	12.03	3.98	8.83*	2.65
Counting skills	1.69	4.5	1.29*	.46
Properties of operations	13.47	4.5	11.99	4.81

Note. \* $p < .05$ .

### 3.2.2 Comparison between Pre- and Post-Intervention Results of the Intervention Groups

There were no significant differences between the control and the experimental group in the initial fluency of multiplication ACs ( $U = 333.5$ ,  $z = -.208$ ,  $p = .835$ ) and division ACs ( $U = 307.5$ ,  $z = -.712$ ,  $p = .476$ ) (Table 4).

Intra-group comparisons revealed that the control group presented a significant difference between initial and final fluency in the use of multiplication ACs ( $z = -4.713$ ,  $p = .000$ ) and division ACs ( $z = -4.541$ ,  $p = .000$ ). In the experimental group the differences between initial and final fluency were also statistically significant for multiplication ACs ( $z = -3.989$ ,  $p = .000$ ) and division ACs ( $z = -4.205$ ,  $p = .000$ ) (Table 4).

Inter-group comparisons between the experimental group and the control group showed that there was no statistically significant differences between the two categories regarding the post-intervention fluency ( $U = 319.5$ ,  $z = -.460$ ,  $p = .645$ ) and generalization ( $U = 339.00$ ,  $z = -.110$ ,  $p = .913$ ) of multiplication ACs. Moreover, the effect of alternative grouping was negligible ( $r = .05$ ). In the case of division ACs, post-intervention differences between the experimental group and the control group were not statistically significant on fluency ( $U = 269.0$ ,  $z = -.767$ ,  $p = .443$ ) and generalization ( $U = 260.00$ ,  $z = -.951$ ,  $p = .342$ ), while the effect of alternative grouping of ACs was negligible for fluency ( $r = .08$ ) and small for generalization ( $r = .19$ ) (Table 4).

Table 4. Results on multiplication and division ACs of research groups

	Control Group (N = 30)		Experimental Group (N = 23)		Effect size
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>r</i>
<b>Multiplication</b>					
Pre intervention	6.17	3.81	5.83	3.18	
Post intervention	11.5	5.02	12	4.04	.05
Generalization	7.4	2.67	7.65	2.24	.05
<b>Division</b>					
Pre intervention	5.14	4.48	3.91	3.89	
Post intervention	9.54	6.35	10.41	4.28	.08
Generalization	5.93	3.63	7.14	2.29	.19

In summary, both research groups showed after the intervention significant improvement in the fluent use of the ACs of both operations in relation to their initial performance; however, the post-intervention differences between the two groups were insignificant both for multiplication and division ACs.

### 3.2.3 Comparisons between Students with LD and MID

Results of comparisons between students with LD ( $n = 39$ ) and students with MID ( $n = 14$ ) showed that there was no significant difference in pre- intervention fluency in the use of multiplication ACs ( $U = 263.5$ ,  $z = -.193$ ,  $p = .847$ ), while the difference between the two groups in the fluency in division ACs was marginally not significant ( $U = 184.0$ ,  $z = -1.901$ ,  $p = .057$ ), with students with LD having higher mean scores (Table 5).

Intra-categorical comparisons showed that students with LD presented a significant difference between pre- and post-fluency in the use of multiplication ACs ( $z = 5.211$ ,  $p = .000$ ) and division ACs ( $z = -4.681$ ,  $p = .000$ ). Students with MID also presented statistically significant improvement in fluency of multiplication ACs ( $z = -3.375$ ,  $p = .001$ ) and division ACs ( $z = -2.437$ ,  $p = .015$ ) (Table 5).

Table 5. Results on multiplication and division ACs of students with LD and MID

	LD (n = 39)		MID (n = 14)		Effect size <i>r</i>
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	
<b>Multiplication</b>					
Pre intervention	6.21	3.6	5.5	3.39	
Post intervention	12.49*	4.48	9.57*	4.32	.31
Generalization	7.56	2.52	7.36	2.43	.04
<b>Division</b>					
Pre intervention	4.95	4.38	2.86	3.27	
Post intervention	10.54	5.1	8.15	6.4	.31
Generalization	6.54	3.07	6.23	3.46	.04

Note. \*  $p < .05$ .

Significant differences were found between students with LD and students with MID in the final fluency in the use of multiplication ACs ( $U = 174.5$ ,  $z = -1.998$ ,  $p = .046$ ) [modest effect ( $r = .31$ )], and in generalization ( $U = 254.5$ ,  $z = -3.81$ ,  $p = .703$ ) [negligible effect ( $r = .04$ )]. Students with LD significantly outperformed students with MID. In contrast, the differences were not significant for fluency of division ACs ( $U = 175.5$ ,  $z = -1.447$ ,  $p = .148$ ) [small effect ( $r = 0.20$ )] and for generalization ( $U = 229.5$ ,  $z = -.247$ ,  $p = .805$ ) [negligible effect ( $r = 0.04$ )] (Table 5).

In summary, students with LD and MID showed significant improvement in the fluent use of the ACs of both operations in relation to their initial performance. Moreover, there were no significant differences between students with LD and MID either in fluency or in generalization of division ACs.

### 3.2.4 Correlations between Cognitive-Learning Factors and Fluent Use of ACs

Table 6 presents the results of correlations between cognitive and learning factors, on the one hand, and fluency in the use of ACs of multiplication and division ACs, on the other, after the completion of interventions for students with LD and MID. It was found that in the case of students with LD information processing speed had a significant correlation with the final fluency in the use of ACs of both operations, while phonological short-term memory was correlated only with the fluency of division ACs. In students with MID, processing speed had a significant correlation with the final fluency in the use of multiplication ACs, while the fluency of division ACs had no significant correlation with any cognitive factor (higher correlation, but still statistically insignificant appeared with information processing speed).

Table 6. Correlations between cognitive and learning factors and fluency (post) of ACs

	LD (n = 39)		MID (n = 14)	
	Post intervention fluency of ACs			
	<i>Multiplication</i>	<i>Division</i>	<i>Multiplication</i>	<i>Division</i>
Language abilities	.183	.035	-.060	.156
Working memory	.116	.187	.172	.024
Phonological STM	.159	.349*	.273	.299
Processing speed	.496**	.469*	.671*	.402
Counting skills	.465**	.379*	.753**	.590*
Properties of operations	.655**	.563**	.709**	.569*

Note. \* $p < .05$ , \*\* $p < .01$ .

Regarding the two learning factors, counting skills and properties of operations, results showed a significant correlation with the fluency of ACs of both operations (multiplication and division) for students with LD and students with MID ( $p < .01$ ) (Table 6).

#### 4. Discussion

The present research compared the effectiveness of two teaching interventions for supporting students with LD or MID in achieving fluency and generalization in the use of multiplication and division ACs. The control group received an intervention based on principles of effective instruction. The experimental group was supported through a synthesis, which consisted of the intervention used for the control group and an alternative grouping of ACs based on distinct AC characteristics. Results of the comparisons between the two groups reflect the impact of alternative grouping, as it was the only instructional component differentiating the two interventions. Furthermore, this research compared students with LD and students with MID regarding the cognitive factors and the specific mathematical prerequisites affecting the learning of ACs, in order to reveal similarities and differences between the two disability groups.

##### 4.1 Effect of Alternative Grouping of ACs

According to the results, students in both groups showed significant improvement in the fluency of ACs, in comparison to their initial performance. This result is consistent with results from other studies, which have shown that interventions grounded in principles of effective instruction have a positive effect on the performance of students with severe learning difficulties or disabilities (Agaliotis et al., 2003; Bryant et al., 2016; Re, Pedron, Tressoldi, & Lucangeli, 2014). On the other hand, comparisons between the results obtained only through the application of effective instruction principles, and the results produced by the combination of effective instruction with the alternative grouping of ACs, showed no significant differences in fluency and generalization of ACs. This is in line with results of meta-analyses and comparative intervention studies (e.g., Carr, Taasooobshirazi, Stroud, & Royer, 2011; Codding et al., 2007; Kroesbergen & Van Luit, 2003; Methe, Kilgus, Neiman, & Riley-Tillman, 2012; Woodward, 2006) which have concluded that there are no significant performance differences between groups of students receiving comparable evidence-based practices for acquiring ACs. Regarding the present research, the view can be supported that the positive results obtained through the high-quality intervention used for instructing the control group was probably not easy to be significantly exceeded through the mere addition of the alternative ACs' grouping used for the experimental group.

An additional explanation for the absence of difference between the results of the two interventions applied in the present research might be found in the view of Garnett (1992) that alternative grouping may facilitate the memorization of ACs that are interrelated on the basis of a clear rule (such as the principle that "the product of any number multiplied by '1' is the same number"), but not of ACs that cannot be easily grouped under a particularly distinctive feature (e.g.,  $7 \times 8$ ,  $6 \times 4$ ). Consideration of this view should have led to the use of two distinct groups of ACs in the final assessment of the present study: one group consisting of combinations that are easy to memorize (e.g., multiplications and divisions of twin numbers, multiplications and divisions by 1 etc.) and another group including the more loosely connected ACs. Comparison of the fluency in the use of ACs from the two groups would yield a better estimate of the potential of alternative grouping. However, no such differentiation was used in the present study.

##### 4.2 Comparisons between the Disability Categories

In reference to the categories of special needs (students with LD and students with MID), the results showed that students of both groups significantly improved learning of multiplication and division ACs. Comparisons of the final performance showed that students with LD had significantly better performance than students with MID in the fluency of multiplication ACs, while the differences were not significant in the fluency of division ACs. These results agree with those obtained by Van Luit and Naglieri (1999), who found that students with LD and students with MID significantly improved in the fluent use of multiplication and division ACs, with students with LD outperforming students with MID. One possible explanation for the differences between the two disability groups may be found in the more extensive limitations in cognitive and learning factors characterizing students with MID in comparison to students with LD (Calhoun & Mayes, 2005; Kroesbergen & Van Luit, 2003).

The fact that the difference between students with LD and students with MID in the final fluency was significant for multiplication ACs, but insignificant for division ACs, may, at least partly, be explained by the characteristics of the strategies usually employed for finding the results of the two AC groups. Specifically,

multiplication ACs (e.g.,  $5 \times 9 = 45$ ) are usually recalled by most students directly from long-term memory (Baroody et al., 2009; Campbell, 2008), while division ACs are more often calculated on the basis of multiplication, and even subtraction or addition ACs. This stands also for typical students and is rather due to both idiosyncratic and acquired strategies traditionally used in the daily school practice (Crawford, 2003; Robinson et al., 2006). For example, the result of “ $21 : 3$ ” may be found by seeking the number that needs to be multiplied by 3, in order for the result “21” to be obtained (“7”). In another example, the result of  $32 : 8$  can be found by counting the times the divider (8) is added repeatedly until the sum reaches the dividend, i.e.,  $8 + 8 = 16 + 8 = 24 + 8 = 32$ , with the reply being “4”. In other words, it is not rare for students to stop trying to memorize division ACs, as soon as they learn a number of ACs of other operations, which they can utilize to find division ACs (Robinson et al., 2006). Nonetheless, no matter how effective they may be, these processes are certainly time-consuming, especially in early stages of learning ACs; hence, they may affect students’ performance, especially in timed trials. Although in the present study instruction was geared toward direct retrieval of division ACs from memory, it is possible that some students with LD used time-consuming techniques, which they acquired prior to this intervention, thus hindering the emergence of their superiority toward their peers with MID, in the timed tasks of the present study. The difficulty to test fluency in the case of ACs that may be found through the use of ACs from other operations has been identified by other researchers too (e.g., Campbell, 2008; Woodward, 2006). On the other hand, students with MID, who did not possess division ACs prior to the received instruction, may have exploited the implemented intervention to improve substantially their performance and, thus, diminish the gap to their peers with LD.

Regarding generalization of multiplication and division ACs, students with LD and MID showed, rather unexpectedly, similar performance. One possible explanation for this result can be found in the characteristics of generalization tasks used in the present study, which presented significant conceptual and procedural proximity to the tasks used in the main teaching phase [e.g., main teaching phase “ $3 \times 4$ ”, generalization phase “ $(3 \times 4) + 1$ ”]. Because of this proximity and regardless of their category of disability (LD or MID), students were perhaps able to transfer knowledge from acquired ACs to respond to the task of generalization. Generalization tasks with greater conceptual and procedural distance from the tasks of the main instruction could have produced different results. However, it should be noted that the obtained results show that students with MID seem to be able to transfer knowledge to new tasks, when those tasks differ slightly from the knowledge they already possess. Careful and gradual increase of the distance between acquired knowledge and new tasks may allow students with MID widen their opportunities for applying the gains they make at school, as also observed by Griffin (2004).

#### *4.3 Correlations between Cognitive-Learning Factors and ACs’ Learning*

Regarding the correlation between cognitive and learning factors of students with LD and MID, on the one hand, and exhibited progress, on the other, it was revealed that the common cognitive factor for both categories of educational needs that correlated with fluency of ACs was the processing speed of arithmetic information. The result is consistent with the research of Fuchs et al. (2006), who investigated third grade students with and without learning difficulties (e.g., learning disabilities, speech and language difficulties and behavioral problems) and found that the processing speed is a crucial factor for the fluency of ACs. The processing speed facilitates the concurrent processing of information constituting an AC, namely (a) the two numbers, (b) the operation involved and (3) the result. Outcome of the simultaneous presence and processing of these elements is the storage and maintenance of each AC as an integrated structure, which is easy to be recalled with all its constituent parts (Compton et al., 2011; Ellemor-Collins & Wright, 2009).

Results of the present research showed that working memory is not a main factor for fluency in the use of ACs both for students with LD and MID. This is not to say that working memory does not play an important role in ACs learning, but it should be taken to mean that this role is not as vital as it is probably in the case of algorithms or problem solving. Such claims have been made also by other researchers (e.g., Butterworth, 2005; Cirino et al., 2015; Fuchs et al., 2006; Maehler & Schuchardt, 2009).

The present study showed also that properties of operations and counting skills have significant correlations with the fluency of ACs, both for students with LD and with MID. The finding is consistent with results of other studies, which also showed the importance of these two basic arithmetic skills in learning ACs (e.g., Cowan et al., 2011; Jordan et al., 2007; Murphy, Mazzocco, Hanich, & Early, 2007; Toll & Van Luit, 2013).

Besides similarities, the present study showed some differences between students with LD and students with MID regarding the correlation between cognitive factors and learning of ACs. Specifically, results showed that in students with LD phonological short-term memory was associated with the fluency of division ACs, but not with multiplication ACs, while in students with MID phonological short-term memory was not associated with

learning ACs of any of the two operations. The small correlation of phonological short-term memory with the fluency of multiplication ACs (Alloway & Passolunghi, 2011; Shin & Bryant, 2013) and the significant correlation with the fluency of division ACs was an unexpected finding, the interpretation of which exceeds the aims of this research.

#### 4.4 Limitations of Research

This study has several limitations that should be considered when interpreting the results. One such limitation refers to the sample size, particularly to the category of students with MID. Future research should include larger populations with LD and MID to better examine existing trends.

Another limitation is associated with the small number of learning factors examined as to their correlation with participants' final performance. Considering that there are probably more learning factors associated with both the direct recall of ACs and the processes for finding the result (such as the number sense or the fact strategies), it is obvious that they should be examined in the context of future research.

#### 4.5 General Conclusion

Despite the limitations, the present findings may be regarded as supportive of the position that interventions for ACs learning based on the principles of effective instruction are beneficial for both students with LD or MID. Alternative grouping of ACs can contribute to the effectiveness of interventions, without necessarily producing significantly better results. Moreover, well-designed interventions for the teaching of ACs may reduce, but not completely eliminate, the effect of cognitive and learning factors differentiating students with LD and students with MID. Information processing speed seems to be a common decisive factor of ACs learning both for students with LD or MID, whereas knowledge of operation properties and counting skills, and to a lesser extent phonological short term memory, seem to differentiate them.

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