REDIMAT
Journal of Research in Mathematics Education
Hipatia Press
www.hipatiapress.com

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Date of publication: February 24th, 2012

To cite this article: Palhares, P. (2012). Mathematics Education and Ethnomathematics. A Connection in Need of Reinforcement. REDIMAT - Journal of Research in Mathematics Education, 1 (1), 79-92. doi: 10.4471/redimat.2012.04

To link this article: http://dx.doi.org/10.4471/redimat.2012.04

REDIMAT - Journal of Research in Mathematics Education Vol. 1 No. 1 February 2012 pp. 79-92.

Mathematics Education and Ethnomathematics. A Connection in Need of Reinforcement

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Abstract

In this article we analyze Ethnomathematics research in Portugal in the recent past and the way it has been used in Education. We divide our reflection in three main parts. One is about people's discourse and actions and how to use them to uncover mathematical thinking. The second is about artifacts and how to use them to uncover mathematical thinking. The third is about how to use ethnomathematics research to improve mathematics education. In each of these sections we reflect on some of what has been made and we propose paths we think are worth pursuing.

Keywords: Ethnomathematics, Culture, and Mathematics.

ISSN 2014-3621
DOI: 10.4471/redimat.2012.04

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In a debate between Popper and Eccles (Popper & Eccles, 1990), they both agreed on Popper's perspective of reality, consisting of three worlds: world 1, the world of the physical things, world 2, the world of mental states and world 3, the world of the products of the human mind, including problems, theories and culture, therefore where mathematics exists. Popper claimed that the latter started with the development of language and Eccles pointed to the arising of a tool culture. We think that both the development of language and the development of a tool culture seem to impact on the creation of mathematical thinking connected with people's lives. And both impact on the creation of cultural artifacts as well. These cultural artifacts may be of various types, such as hand-woven baskets or textiles, boats, rulers, or even systems to record numeric information as in the 'quipu' (Palhares, 2010).

As to the mathematics of people's lives, the work of Terezinha Nunes and colleagues on the capacities of mental Arithmetic evidenced by street children was such as to make us rethink the concepts connected with oral and written mathematics (Nunes, Schliemann & Carraher 1993). After that, an enormous effort has been put on the study of people's actions and discourse from the point of view of the mathematical thinking behind it. It is not the case of Portugal where a lot remains to be studied.

Gerdes (1991) has devised a methodology that allows for the discovery of the geometric thinking of the artifacts builders. It begins with the researcher learning the surviving techniques of manufacturing artifacts. In each phase of the manufacturing process the researcher places the question: 'Which considerations of geometric nature play a role to get to the following phase?' This way the elements of a geometric thinking (hidden or frozen) easily come up. From these elements it is possible to establish the path of the evolution of geometric thinking. Shapes of all kinds exist everywhere. Through labor work we recognize that certain shapes are more suitable, therefore become reproduced through apprenticeship within a community. Through continuous reproduction a sense of aesthetics becomes attached to that shape, which then becomes used for other purposes too. And we think this method may be used for a different kind of artifacts, such as texts of

traditional oral literature memorized using the body, and traditional games, where both spatial and numeric thinking may be uncovered and brought to light.

Brousseau (1997) sees the work of mathematicians as, first discovering, then communicating. In the second phase, they reorganize the results they obtained shaping it into something general, removing any context or personal characteristics that were present at the phase of discovery. Teachers should do the opposite, providing a context and personalizing, to give meaning to the mathematical knowledge being taught. He called this second process didactical transposition. Freudenthal (1973) sees mathematics education as a process, a human activity that produces mathematics. But in a world increasingly diverse culturally, we think it is ethnomathematics (the mathematics of identifiable cultural groups - D'Ambrósio, 2006) that can help us in the process of humanization of mathematics. By contextualizing tasks within the cultures of countries where Portuguese history is linked, we associate meaning that is easily connected by school children. In a recent research forum there has been an appraisal of all the efforts made so far in this area. One of the major points highlighted was the significant contribution from different areas and perspectives. In that forum, a historical-cultural approach was adopted, i.e., a look at how mathematical thinking is enabled and structured by cultural tools and practices. A set of 5 questions was brought up, one relating to how artifacts have been and can be used to uncover mathematical thinking, another on people's actions and discourse revelation of mathematical thinking, a third on the relevance to education, a fourth related to language issues and a fifth about the continuities and discontinuities of informal and scientific mathematical thinking (Nunes & Palhares, 2010). It came out that the three initial questions are the most directly relevant to the discipline of mathematics education and the ones in dire need of extensive effort.

These are the reasons why we think a project such an investment makes sense and is necessary. The main lines to be considered and along which we will organize this contribution include:

- People's actions and discourse outside school and academic contexts in Portugal as a way to uncover their mathematical thinking?

- Artifacts used by different communities as a way to uncover Portuguese communities' mathematical thinking.
- The use mathematics education can make of information regarding reasoning outside school to improve mathematics teaching and learning in school.

Mathematical thinking behind people's actions and discourse

As to the search for mathematical thinking behind people's action and discourse, some efforts have been made all over the world (e.g. Nunes, 1992). But some recent efforts have also been produced in Portugal worth mentioning and describing briefly.

Cadeia, Palhares and Sarmento (2008) have studied the mental arithmetic capacities of the Romani community connected with their major activity in Portugal, market fair trade. They found good ability, some community methods, and above all a prevalence of the oral methods not just as a support, but instead as the major way of thinking mathematically. They also found no differences between Romani girls and boys concerning their mental ability and the use of different methods when the questions were posed in a school environment (based on algorithms) or in a market environment (Cadeia, Palhares and Sarmento, 2010).

On this issue of the Romani Community, Pires (2008) has studied the interaction of Romani children in school with problem solving. She found that some children already knew some of the problems since their parents had presented them previously. This was an opportunity to improve motivation. Many Romani children find mathematics formal education apart both from their needs and practices. Finding that the problems the teacher was giving them were told within the community was a plus in terms of establishing a connection between school and their culture.

Sousa, Palhares and Sarmento have studied a caulker working on the isle of Madeira constructing boats in connection with the fishermen community. They found an impressive array of mathematical knowledge and processes being used, although not connected with formal education. For instance, the caulker understood scales and knew what it meant to have 1:20 in respect to boats, but did not know why it

was represented that way. As to the fishermen community, the notions of proportion or fraction were much used but again no formal education was ever involved (Sousa, Palhares and Sarmento, 2008; 2010).

Costa, Nascimento and Catarino have studied several professions in the region of Tras-os-Montes and Alto Douro. One of these was the coopers (Costa, Nascimento and Catarino, 2008a) another was the tinkers (Costa, Nascimento and Catarino, 2008b) and a third was the yoke makers (Costa, Nascimento, Catarino and Fernandes, 2010). In all of these cases, particular processes and knowledge were found, even a certain use of language that is interesting itself, like the expression 'working the roundness' which basically means that a certain degree of approximation is going to be needed because of the non-linearity.

Fernandes and Matos have studied the teaching and learning practices within a locksmith learning community. Contrarily to other profession like the tinkers or the caulkers, that are becoming extinct, locksmith is a thriving activity in Portugal and it is learnt in professional schools. There the master discusses with the apprentices, and if the title reveals a higher status, discussions within the community were solved by the use of a ruler that the master carries at all times, not by claiming authority. The apprentices were free to pursue their own projects and encouraged to appreciate and criticize both the product and the process used in the development.

Pires (2008) has studied masons' professional activity. She has found implicit mathematics used while the masons were unaware they were using pieces of mathematics. For them, school mathematics has almost no bearing for their profession. School mathematics is about formal mathematics and they use in their profession informal mathematical knowledge.

Mathematical thinking behind people's actions and discourse

As to the search of mathematical thinking through the use of artifacts, Gerdes theory already mentioned in the introduction is the basis for this kind of research. One interesting application of it is the Marrosula, which are small artifacts constructed by folding palm tree leafs and introducing seeds inside for a sound effect. It is a kind of artifact that can be used as a context for mathematical tasks in education also. But Gerdes has been using this method for a variety of artifacts, especially

linked with geometry, but with a great emphasis on baskets (Gerdes, 2007).

Vieira, Palhares and Sarmento (2008) have studied in the north of Portugal aspects connected with basket weaving. Symmetry is a mathematical concept that is extremely relevant to this activity. It was possible to find symmetrical groups of all sizes up to 44 (22 reflections and 22 rotations). It was also possible to find many different patterns when interweaving strips and in a small community a non-exhaustive search has uncovered five of the seven friezes in the decorations in bags locally produced. Baskets revealed a lot of mathematics that could be used in mathematics education.

Despite this small contribute, the exploration of artifacts in Portugal is almost non-existent and it is a field where there should be a major effort. Artifacts like sidewalks decoration with different colors stones, or tiling decoration at houses, or the iron decorative motifs in balconies or at doors, are just a few examples of interesting beginnings for ethnomathematics research in a country where those artifacts are everywhere. But other artifacts of a different nature exist to be researched which we will elaborate in the following paragraphs. One kind of cultural product that has not been explored is the texts of traditional oral literature. It is not difficult to explore certain literature texts we know have a connection to mathematics, especially of certain authors like Lewis Carroll (Palhares, 2006). But to get into other texts of other authors or of popular origin we need other means, like going into the gestures that go with the retelling or into hermeneutical analysis. As for the first kind, Gerdes has analyzed the drawings made in sand by the Chokwe when retelling a story. Those drawings function as mnemonic devices but at the same time they catch the listeners attention. It happens that the drawings can be classified mathematically and constitute a powerful source for mathematical educational tasks (Gerdes, 1999). With a different background perspective, Guimarães has been developing a connection between rhymes and small tales to mathematical knowledge. In fact there is a Portuguese tradition of rhyming with numbers and other mathematical objects that is also existent in Spanish ‘artitexts’ (Guimarães, 2006). Also promising appears to be the approach used in connecting traditional oral sayings in the area of health care (that were collected by Michel Giacometti a few

decades ago) by means of exam and commentary by medicine doctors of today (Almeida, Guimarães and Magalhães, 2009). Hermeneutical analysis looks at literary texts and interprets them in the light of a theoretical reference that incorporates the main authors of Criticism, seeking to identify symbols, intertextual dialogue relationships, stylistic images, and the main ideological and thematic roles in the text. This methodology of literary studies, naturally, respects codes and conventions of the literary semiotic system and the constraints arising from the point of view of literary reception by specific audiences (Carvalho and Azevedo, 2005).

Games are another kid of artifacts that should be explored in Portugal for the purpose of finding mathematical thinking behind their use. On what concerns mathematical games, we have established already some correlation between one particular mathematical game (chess) and problem solving involving patterns (Ferreira and Palhares, 2008). We have now preliminary results indicating even better correlation for other mathematical games like Traffic Lights or Hex (Ferreira, Palhares and Silva, 2008). Therefore we know that for modern games being played, there is mathematical thinking connected with them. It is however only preliminary work in a sense even if it is a promising line of research. We think we need to explore games artifacts as may be found in churches or other very old buildings and connect them with mathematical thinking of the people that were drawing or constructing them. That is a new line of research that can be very useful.

The design of mathematical tasks culturally contextualized

Let us recall what Brousseau (1997) says about mathematics teaching and learning. He acknowledges that mathematicians, when communicating mathematics, tend to depersonalize and remove both context and temporal marks. The objective is an important one, because by doing so generality and abstraction are then achieved. Teachers however have to give meaning to the mathematical content and so they have to personalize, contextualize and inscribe in a time where it was discovered or known. This he called didactic transposition. This process has some pitfalls as the teacher in order to have the correct answer from the student, may provide questions that are progressively less

demanding, making the use of context unnecessary to the eyes of students. That poses a problem for teachers one that some solve by teaching abstract material hoping that students learn it as it is and the need for contextualization is substituted by clues to correct answers. But teachers should do differently, and provide a context or contexts that give meaning to the mathematics being taught and at the same time use a sound pedagogy to bring students to understand the context, the content and making it their own. It is imperative in a good pedagogy that teachers promote decontextualization once students learn within a context or the mathematical content will be forever linked to a context and so not applicable to other situations.

Let us remember also the work of Freudenthal (1973) who saw teaching mathematics as a process that should provide students with situations that help them mathematize and abstract. Under the name of didactic phenomenology, he proposes the use of situations organized by mathematical objects that students are supposed to construct. These situations should be contextualized and should allow for the reinvention of mathematics. Furthermore they should lead to the mathematization of reality, and he considered history of mathematics as a powerful source of inspiration. Situations that serve as mere stories to include content will not be helping students to mathematize. In a way, we need contexts as a starting point relevant in itself so that students can model the situation mathematically. He thought that situations arising from the history of mathematics should be good situations to achieve the objective since they already gave rise once to the mathematics we are trying to teach.

We however think it is ethnomathematics (the mathematics of identifiable cultural groups - D'Ambrósio, 2006 or the other names that were given to the association between mathematics and culture) that can help us in the process of humanization of mathematics. We intend to build tasks that are grounded on the cultures of countries where Portuguese history is linked and by doing it we intend to associate meaning that is easily connected by school children.

Shirley also thinks that concepts and artifacts produced by some cultures should be incorporated in the classroom in order to allow multicultural mathematical connections and so meaningful constructions that pupils might elaborate (Shirley, 1995).

What we think this kind of research is still missing is a program where after experimentation follows analysis, reconstruction, retrial, which allows us to build tasks culturally relevant, mathematically rich and pedagogically sound that teachers may feel confident to use. The research conceptualization it implies is already there (Lesh and Sriraman, 2004). In their conceptualization, mathematics education research should emerge as a design science, where a series of iterative design cycles occur, and where reusable and sharable materials are produced.

As to the design of tasks, we can find the necessary help on the principles and processes to construct tasks (Mason and Johnston-Wilder, 2006). We may add that recently there has been a shift of attention to the design of tasks for work with teachers (e.g. Watson and Mason, 2007; Palhares, Gomes, Carvalho and Cebolo, 2009) or with preservice teachers, even using technological means for distance education (Palhares and Gomes, 2008). There is however still very little on culturally contextualized mathematical tasks either for students or future teachers.

Conclusion

There is mathematical thinking behind many people's actions and discourse and even behind all the different kind of products of human activity. The work we intend to start will build on all the research that has been made uncovering mathematical thinking, expand it to new areas, and use it to improve mathematics education. The key questions to be considered include:

- What can people's actions and discourse outside school and academic contexts tell us about their mathematical thinking?
- What can the artifacts used by different communities tell us about that communities' mathematical thinking?
- In what ways can mathematics education use information regarding reasoning outside school to improve mathematics teaching and learning in school particularly in order to engage pupils so as to have a positive disposition towards mathematics and an immersion in aspects related to their cultural history?

There has been much work done in ethnomathematics or, as it has also been called, everyday mathematics, street mathematics, situated

cognition, mathematics and culture. We claim this work as a consequence and an extension of that whole lot.

What has been a major difficulty is to bring these results to education. Some experiences have been made in other countries, but even elsewhere, the results are not systematic or based on solid research. We intend to use a new perspective and therefore to try to solve this issue at least in Portugal.

During the development of the project, there will be a continuous collection of processes and knowledge of culturally bound people (either professional or ethnic groups). This collection will be made by means of a series of case studies. Case studies are meant here as qualitative studies that try to catch both the particulars and the complexity, and in general small in scope, with a few weeks of fieldwork and a few months for analysis and writing (Stake, 2000).

Another thing to be carried out during the development of the project is the use of artifacts (games) and 'artitexts' (texts of oral traditional literature) already collected in several sources to uncover hidden mathematical thinking by communities of people most of which are no longer existent. We will use Gerdes' method of uncovering hidden geometrical thinking in Africa and adapt it to the nature of texts and games. 'Artitexts' will be analyzed from different angles and perspectives to give an ample specter of possibilities.

Another thing to be carried out during the development of the work is the testing of culturally contextualized mathematical tasks for the elementary education in Portugal. Design-based research will be the method to be used and we plan to be able to develop pedagogy to use with the tasks.

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