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Limited Intervention at Sub Concept of Fractions in the Object Conversion into Fractions

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Abstract

This research is an exploratory study with a qualitative approach, which is based on interviews with a task-based the purpose of this study is to describe the understanding of elementary school students in interpreting sub concept fractions in changing of the object is given to fractions with limit intervention. While intervention on problems solving mathematical in a fraction of this is an attempt to change behavior, thoughts and feelings of a person to develop the students' knowledge in achieving the objectives of the fractions learning experience.

Keywords: interpretation, fractions, object, limit intervention

1. Introduction

This article discusses the learning of mathematics in elementary school (elementary school) about the basic concepts of fractions by using media/props are believed to provide pleasure and understanding to students. Learning by using media/props greatly assist the creation of learning in accordance with the demands of the curriculum in 2013, namely: fun, contextual and meaningful through the steps of learning to observe, ask, experimental/discovery, process the information and summarizing the results which were consistent with the objectives.

Learning activities to recognize the concept of ordinary fractions will be more meaningful when it is preceded by a story about the use of real objects e.g. eggs, apples, tomatoes, tofu, pancakes, followed by fractional block or shaded paper. To avoid misunderstanding in assessing the concept of fractions and arithmetic operations on these numbers we will need to pay close attention to sub concept of fractions. Sub concept is useful to guide the understanding and give command of the facts, operations and principles of the fractions as an integral part of mathematical objects.

According to Psychology Bruner (1966) learning will be more meaningful and more quickly achieve the goal if it starts from the stage of concrete (enactive) that uses the real object, then semi-concrete (econic) the object that replaced the image, and the last is abstract (symbolic) the grain which is only in the form of emblem/symbol that only in the form of letters, or numbers only. According to Bruner if students are experienced math learning for each topic with treatment by the third stage, the students will be able to develop knowledge far beyond what they received from the teacher.

In the process of teaching the fifth grade students of SDN 2 Merjosari, teachers beginned by fractions ½ using paper one sheet is cut into two equal parts, and they told his students "a piece of paper divided into two parts, then the result is ½", herein if No students ask "why ½?, why not the paper into two pieces?" On learning activity it is seened that some of students do not understand the concept of fractions ½.

Then the learning continue after students acquired basic knowledge of the fractions concept, the teacher wanted to measure students' reasoning and the ability to apply the basic concepts of fractions. Those students were asked to determine the fractional value as congruent parts of a whole based on the colors red, yellow and white as in the Figure 1 below:

Figure 1. Fraction value as part group congruent part

From Figure 1 above students answered fractional value, is ½, 1/3 and ¼ without mentioning the color in the image. Although some students know part of the overall concept, but the interpretation of these students did not know that the fractional value as part group congruent part, is the value ¼ of the red, the value ¼ of the yellow and the value 2/4 of the white color.

This example raises the possibility of major problems/conflicts in learning fractions for the next. If this condition is not recognized by the teacher, then the conflict will lead to misconceptions students in learning fractions.

Students are said to have a thorough understanding of the fractions meaning if he can establish relationships between the various interpretations of fractions. In addition, he also must be able to establish a relationship between the representation of the different fractions (Cathcart, Pothier, Vance, & Bezuk, 2006). Representations may include oral or written symbol, object manipulation, image and real world situations. In addition to the interpretation and representation of fractions, other mathematical ideas that are important to be built by the students are the parts that states do not have to be congruent or fractional equal say (Fosnot & Dolk, 2002).

Based on this background, the research question is: "How does an understanding of elementary school students in interpret the sub concept of fractions in changing the object of which is given to fractions with finiting intervention?" While the purpose of this study is to describe the understanding of elementary school students in interpreting sub fractions in changing the concept of the object which is given to fractions with Limit intervention.

Once the research is done, the results of this study are expected to be useful for:

a) Contribute to the development of the interpretation of the students in understanding the sub concept of fractions.

b) Provide inputs to the readers who are interested in this topic for further research.

2. Literature Review

2.1 Concept of Fractions

Clarke, Roche and Mitchell (2007) states that the concept of fractions is not a simple concept; the uniqueness of the fractions, which is different from the natural numbers and integers, sometimes making it difficult to understand students' learning (Behr, Lesh, Post, & Silver, 1983; Kieren, 1976; Streefland, 1991) and makes it difficult to be introduced to the students.

Learning activities to recognize the concept of fractions ordinary will be more meaningful when preceded by the word problems using concrete objects e.g. apple and melon. As can be seen in Figure 2 below:

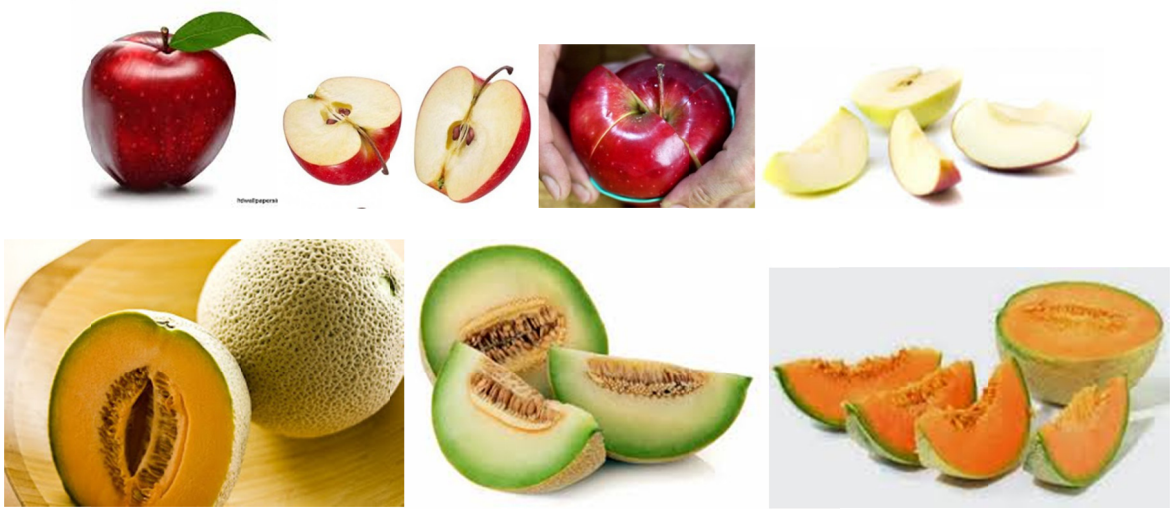


Figure 2. Introduce the concept of fractions using concrete objects

Figure 2 above, suppose an apple cut in two equal parts having a fractional value ½ and two equal parts had been cut again into each of the two parts together again, then the value of the pieces into ¼ and so on. So is the case with melon.

Then proceed to introduce the concept of fractions using fractional block as in Figure 3.

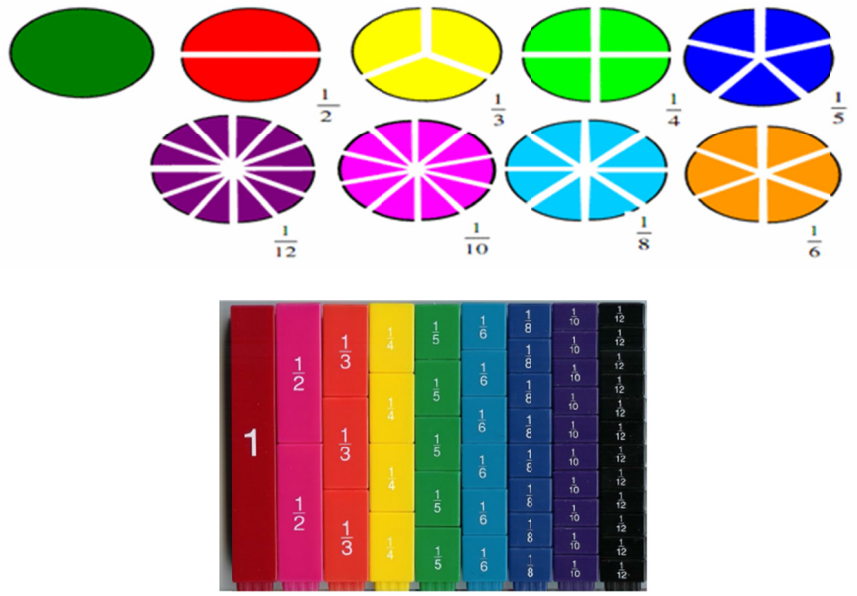


Figure 3. Introduce the concept of fractions using fractional beams (Husna, 2012)

Fractional beam is shaped props pieces beam or a circle divided into several sections. The size of the beam fractions starting at 1, 1/2, 1/3, 1/4, 1/5 and so on.

Furthermore shading paper that states the area. Such as fractional 1/2 can be demonstrated by folding paper circular or square, so the precise pleats covering each other. The part that is folded open and shaded as desired. This can be seen in Figure 4 below.

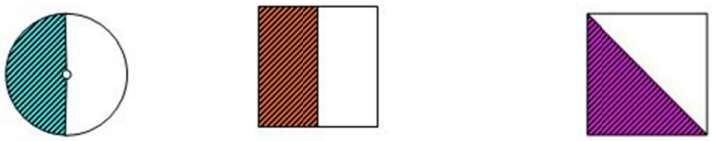


Figure 4. Stating the concept of fractional $\frac{1}{2}$ (Sukayati, 2003)

Demonstration for fractions $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{8}$ can be seen as in Figure 5 below.

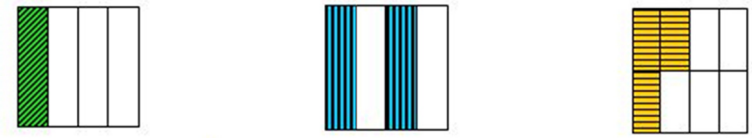


Figure 5. Stating the concept of fractional $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{8}$ (Sukayati, 2003)

2.2 Fraction Meaning

According to Musser, Burger, and Peterson (2006), the fraction has two meanings, namely (1) as quantity expressed relative amount (relative amount), and (2) as a number that has the emblem. As the meaning of the quantity, it states the number of fractional part of a number of equal parts. Quantities which are expressed, among others, can be a long, wide area, or volume. For example fractional 2/3, as the relative amount, fractional 2/3 can be shown or expressed as the area shaded or colored, regardless of size (small, medium, large), build (square, rectangle, parallelogram-parallelograms, triangles, circles, or other shapes), arrangement or location, and orientation or perspective (in view of shading or coloring parts: a flat, vertical, oblique, curved

Regardless of size (small, medium, large), shape, composition, or orientation, all of the following model numbers stated 2/3.

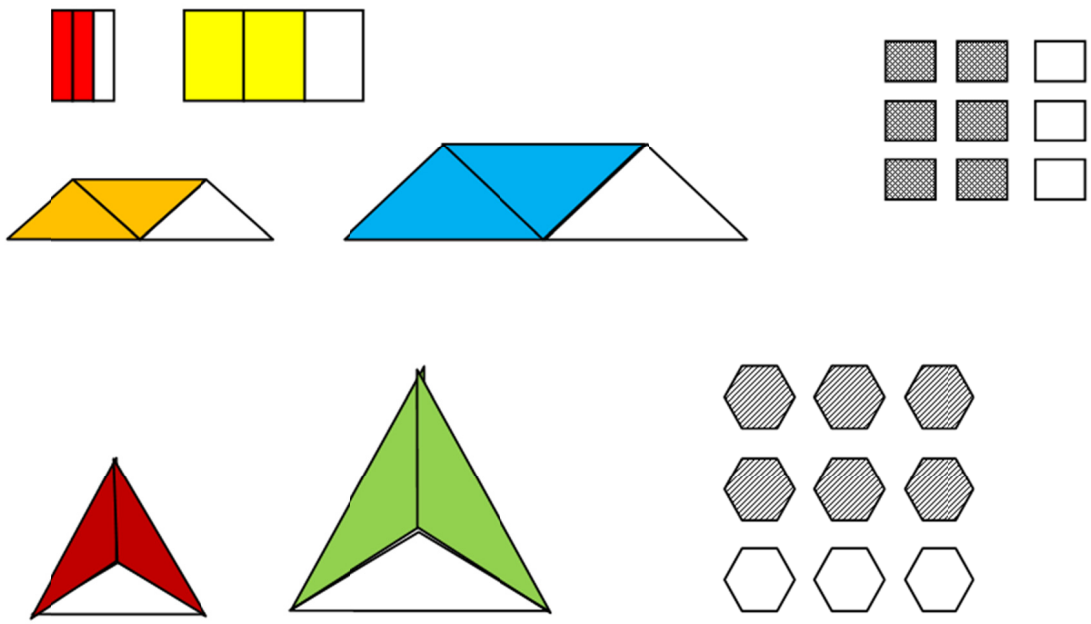


Figure 6. Fractions based on the quantity to stating the relative amount (Muhsetyo, 2014)

Fractions based on the quantity of stating the relative amount can be expressed as a **symbol** or **numeral** (or

name), to indicate a relationship "many parts of the whole part of the same" (*a part-to-whole relationship*).

If p and q are numbers chopped and q ≠ 0, then the fraction $\frac{p}{q}$ or p/q, showed p dividing by q equal parts (*equivalent*), p is called the numerator (*numerator*) and q is called the denominator (*the denominator*).

Several studies have been conducted to investigate the difficulties in understanding and learning the material fractions. Vale (2007) found that students will be more likely to make mistakes on a fractional operation if only a fraction of learning material focuses on memorizing formulas and procedures for operating without sufficient attention to the meaning of fractions. Part-whole relationship as a sub identifies key ideas and overall understanding of fractions which will be interpretation in fractions (Kieren, 1988). In addition, the complexity of the characteristics and stages of understanding the concept of fractions requires that made it can not be understood in a relatively short period (Yusof and Malone, 2003). Further research conducted Clarke, et al. (2007) found that the methods and learning strategies that are less precise can also contribute to the misconceptions students.

According to Bell, Castello, and Kucheman (1983) the concept of fractions can be interpreted into seven ways, namely:

a. Part group congruent part

Fractions as *part group congruent part*. Fraction $\frac{a}{b}$ is interpreted as an area divided into b congruent parts and pay attention to a part.

b. Part group non congruent part

Fractions as *part group non congruent part*. Fraction $\frac{a}{b}$ is interpreted as an area divided into sections that are not congruent b and pay attention to a part.

c. Part whole congruent part

Fractions as *part whole congruent part*. Fraction $\frac{a}{b}$ is interpreted as a set consisting of b objects are congruent and noticed an object

d. Part whole non congruent part

Fractions as *part whole not congruent part*. Fraction $\frac{a}{b}$ is interpreted as a set consisting of b objects that are not congruent and noticed a object.

e. Part group comparison

Fraction as *part group comparison*. Fraction $\frac{a}{b}$ is interpreted as a relative comparison of two areas A and B. The number of areas that are congruent to A is A, while the B as b.

f. Part whole comparison

Fraction *as part whole comparison*. Fraction $\frac{a}{b}$ is interpreted as the ratio of the bunch is the comparison of the relative number of objects in a set A with a lot of objects in a set B

g. Number line

Fractions as *the number line*. Fraction $\frac{a}{b}$ is interpreted as a point on the number line. In this case, a unit on the number line is divided into b equal parts, then point to note-a

The ability of representation is one of the standard math learning process should be developed and owned by the students. These process standards should not be submitted separately to the mathematical material. Unfortunately,

the representation is often taught and learned as if it stood alone without any connection in mathematics (Depkdiknas, 2005: 51). In fact, the representation is expected to support the students' understanding of mathematical concepts and relationships in communicating mathematics, arguments, and an understanding of the other ideas, in recognizing the relationship between the concepts of mathematics (NCTM, 2000, p. 206).

2.3 Limit Intervention

The term intervention in general is an attempt to change behavior, thoughts and feelings (Markam, 2003). While understanding the intervention by Samani and Hariyanto (2011) an action to develop an atmosphere of learning interaction that is designed to achieve the formation of character with the implementation of a structured learning experience (structured learning experience). So **intervention in solving math problems at a fraction** is an attempt to change behavior, thoughts and feelings of a person to develop the students' knowledge in achieving the objectives of the learning experience fractions.

3. Research Methods

This research includes exploratory research with qualitative approach conducted on 32 students grade V SD Negeri 2 Merjosari Malang academic year 2014/2015 in the odd semester, where the subject has been and has been learn about fractions. The main instrument in this study is the researchers themselves and supporting instruments are Student Task Sheet material fractions and completeness, interview, audio-visual, and transcript data.

Given the subject matter of the research through the student assignment sheet. When it was observed directly by making notes, also recorded with audiovisual. The matter obtained from students representing encountered an error in converting an object into fragments. So that the students will serve as research subjects.

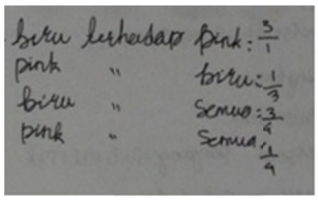
Based on the results of student work in completing the task to explore the subject of an interview about what, how and why related to a given problem and the results as well as other possibilities that arise from the impact questions. Interviews were used in this study can be categorized as unstructured interviews.

Selected unstructured interviews because researchers do not yet know what the answer will be obtained from the student and the answer will be the starting point of the development of the questions that will be followed up in the form of unstructured interviews.

4. Findings and Discussion

Results of this research was elected of five collected students who were taken from 32 students of SD Negeri 2 Merjosari grade 5 is used as the subject of the research is based on a wrong answer in transforming the object into fragments that represent fractions as a ratio and parts that are congruent and incongruent of the whole or a set of objects which are given.

Results of *interpretation fractional as part group congruent part* of the first subject (S1) can be seen from the results of the students' answers, as shown in Figure 7 below



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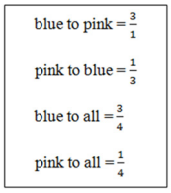


Figure 7. Question answer 1 of S1

Based on the results written by S1, the researchers conducted interviews with S1 as follows

Researcher : *Is the fractional value that you wrote is correct?*

S1 : *It seems correct, sir.*

Researcher : *How do you determine the value of such fractions in figure 7?*

S1 : *Figure 7, its value 3/1 blue to pink, pink to blue 1/3, blue against all 3/4 and 1/4 pink against all.*

Interpretation of the results of interviews with S1 can be seen in diagram 1.

From this diagram 1 S1 finish the answer is not in accordance with the desired answer to a *fraction as parts group congruent part*, because S1 provide initial answer as follows the number part of the whole was 4 (N) with a fractional value of blue (B) to pink (P) is 3/1 visible on furrow green arrow, a fractional value pink to blue is 1/3 look of the groove blue arrow, a fractional value blue to all is ¾ and pink to all is ¼ visible on furrow red arrows (red arrow groove that settlement should be).

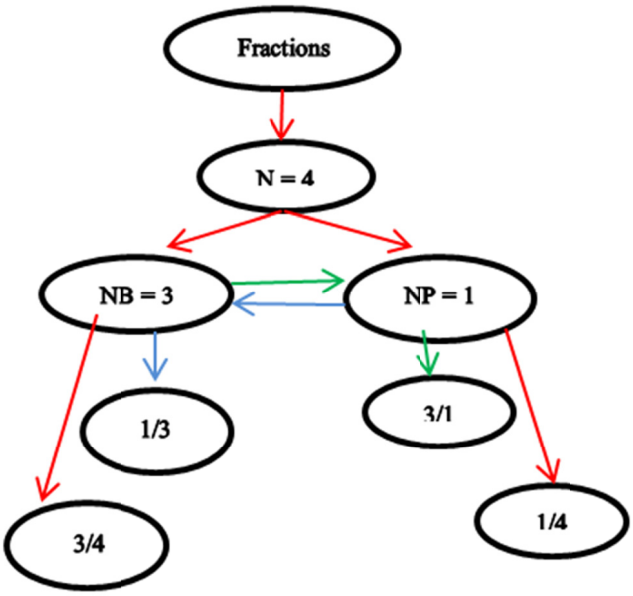


Diagram 1. Interpretation of the results of interviews with S1

Because S1 the answer is not in accordance with the settlement, the researchers conducted interviews with Limited intervention to S1 the following

Researcher : *How many part entirely?*

S1 : *4, sir.*

Researcher : *How many parts the blue ones? How many parts of pink ones?*

S1 : *3 blue and pink there is none.*

Researcher : *Yes, well, if the value of the blue shards how? And what is pink fractional value?*
(Limited intervention)

S1 : *Blue is 3/4 and 1/4 is pink.*

Researcher : *Very clever.*

From interviews researchers using Limited intervention by S1 can be seen in diagram 2 below:

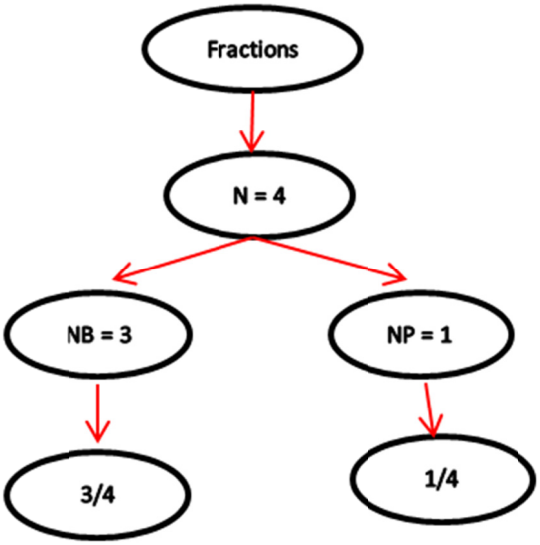
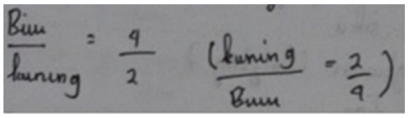


Diagram 2. The results of interviews S1 with limited intervention

Once the intervention is Limited to S1 by the researchers, the results are consistent with the desired answer of the question no one can see the plot on diagram 2 on the right side.

Next results *interpret fractions as part group non congruent part* of the second subject (S2) can be seen from Figure 8.



Translate version

$$\frac{blue}{yellow} = \frac{4}{2} \qquad \frac{yellow}{blue} = \frac{2}{4}$$

Figure 8. Question answer 2 of S2

Based on the results written by S2, the researchers conducted interviews with S2 as follows

Researcher : *Is the fractional value that you wrote is correct?*

S2 : *Yes, sir.*

Researcher : *How do you determine the value of such fractions in figure 8?*

S2 : *Fractions value in figure 8, No 4 is blue, the yellow are no 2. Then the blue fractional value is 4/2, while for the yellowis 2/4.*

Interpretation of the results of interviews with S2 can be seen in diagram 3.

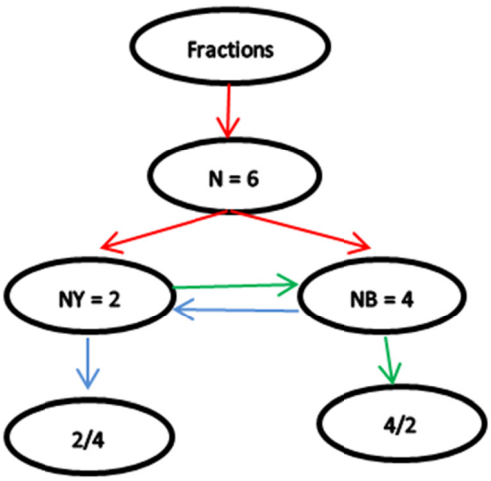


Diagram 3. Interpretation of the results of interviews with S2

From the diagram 3 looks S2 interpret many parts of the whole 6 (N) with a fractional value of blue (B) to yellow (Y) is 4/2 indicated by green arrows groove while the yellow to blue fractional value is 2/4 indicated by grooves blue arrows.

Because S2 wrong answer is not in accordance with the fractional value, the researchers conducted interviews with intervention limited to S2 as follows

Researcher : *How many of the value whole?*

S2 : *6, sir.*

Researcher : *Yes. Well if so, how is blue fractional value of the whole?* (Limited intervention)

S2 : *owh yes, it's 4/6. 2/6 for a fractional value that is yellow .... hehe*

Researcher : *Very good.*

Interventions conducted by researchers of the S2 shown in diagram 4 below:

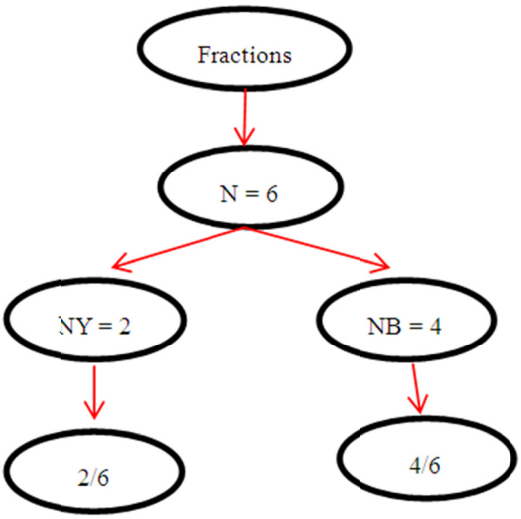
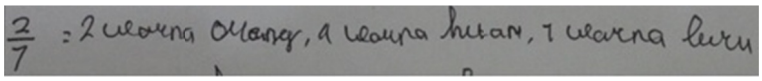


Diagram 4. The results of interviews S2 with limited intervention

From the results of the Limited interventions conducted by researchers of the S2 in question no 2 in diagram 4, it appears that the S2 begin to understand the value of *fractions as part group non congruent part* to the blue is 4/6 and yellow is 2/6.

Then the next result is *the interpretation of fractions as part whole congruent part* of S3 can be seen from the answer to question no 3 below



Translate version

$\frac{2}{7}$ = 2 Orange, 4 Black, 1 Blue

Figure 9. Question answer 3 of S3

Based on the results written by S3, the researchers conducted interviews with S3 as follows

Researcher : *Is the fractional value that you wrote is correct?*

S3 : *Yes, sir.*

Researcher : *How do you determine the fractional value?*

S3 : *its orange color was in front of black and blue while the total of number 7, then the value of fractions 2/7.*

Interpretation of the results of interviews with S3 can be seen in diagram 5.

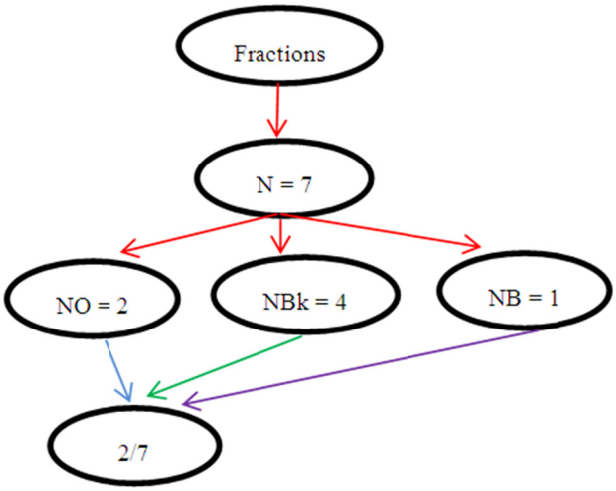


Diagram 5. Interpretation of the results of interviews S3

In the diagram 5 visible S3 interpret overall the number of parts is 7 (N) with a fractional value of orange (O), black (Bk) and blue (B) is 2/7 indicated by the flow arrows in blue, green and purple.

Because S3 wrong answer is not in accordance with the fractional value corresponding to the color, the researchers conducted interviews with Limited intervention to S3 as follows

Researcher : *2/7 the fractional value for the color orange or fractional value overall?*

S3 : *Hmmmm seems to be orange, sir.*

Researcher : *Yes, so how many fractional value of her black and blue?* (Limited intervention)

S3 : *Oh yes, can I fix it, sir?*

Researche : *Yes, Please*

S3 : *For the orange value fractions is 2/7, 4/7 black and blue colors 1/7*

Researcher : *That's right.*

Interventions conducted by researchers to S3 shown in diagram 6 below:

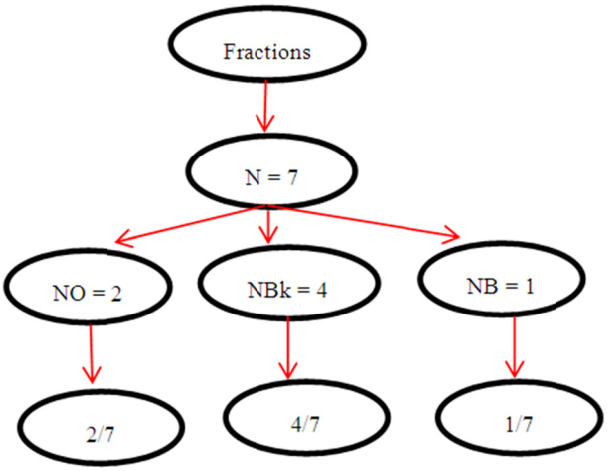
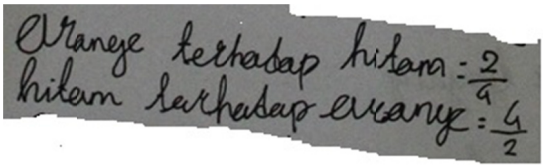


Diagram 6. The results of interviews S3 with limited intervention

From the results of the Limited interventions conducted by researchers of the S3 in question no 3 in the diagram 6, it appears that the S3 began to understand the value of *fractions as part whole congruent part* to the orange is 2/7, black is 4/7 and blue is 1/7.

Then proceed with *the interpretation of fractions as part whole not congruent part* from S4 shown in Figure 10 below:



Translate version

Orange to black = 2/4

Black to orange = 4/2

Figure 10. Question answer 4 of S4

Based on the results written by S4, researchers conducted interviews with S4 the following

Researcher : *How about the image 10?*

S4 : *Is it wrong, sir?*

Researcher : *Try to answer how much the value of the pieces*?

S4 : *2/4 Orange against black, black against the 4/2.*

Interpretation of the results of interviews with the S4 can be seen in diagram 7.

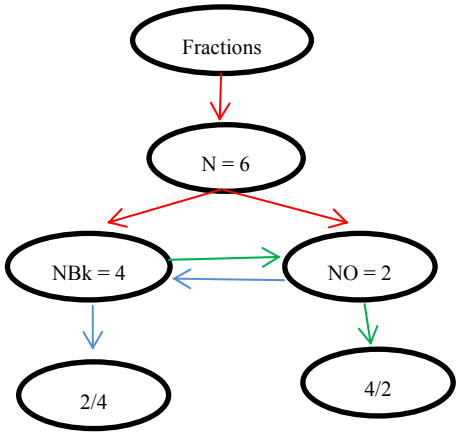


Diagram 7. Interpretation of the results of interviews S4

In the diagram 7 visible S4 interprets many parts of the whole 6 (N) with a fractional value of the orange (O) to black (Bk) is 2/4 indicated by the flow arrows in blue and fractional value of the black to orange is 4/2 indicated by a green arrow groove.

Because S4 wrong answer is not in accordance with the fractional value corresponding to the color, the researchers conducted interviews with intervention Limited to the following S4

Researcher : *What is the sum of that images a collection?*

S4 : *6, sir.*

Researcher : *Exactly. Then how many images are in black? And how many pictures are orange?*

(Limited intervention)

S4 : *The black is ones 4 and the orange is 2. The black fractions 4/6 and 2/6 orange, right sir?*

Researcher : *great.*

Interventions conducted by researchers to S4 shown in the diagram below 8:

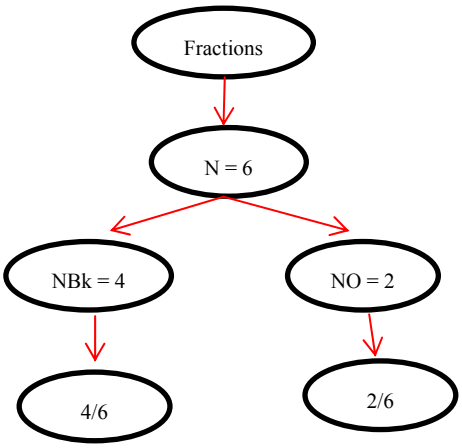
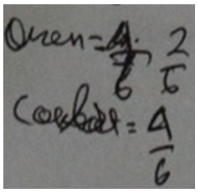


Diagram 8. The results of interviews S4 with limited intervention

From the results of the Limited interventions conducted by researchers of the S4 at no 4 on the diagram 8, it appears that the S4 begin to understand the value of *fractions as part whole non congruent part* to black is 4/6 and orange is 2/6.

While the next picture *interpret fractions as the part group comparition* of the S5 are shown in Figure 11 below:



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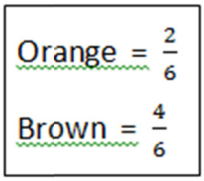


Figure 11. Question answer 5 of S5

Based on the results written by S5, the researchers conducted interviews S5 with the following

Researcher : *How is Figure 11? Is correct?*

S5 : *Yes, sir. Orange value is 2/6 and 4/6 is brown.*

Interpretation of the results of interviews with S5 can be seen in diagram 9:

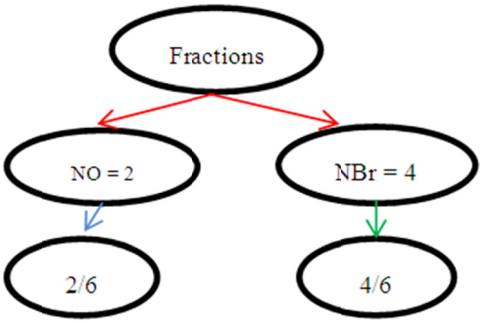


Diagram 9. Interpretation of the results of interviews S5

In the diagram 9 visible S5 interprets the fractions value of orange (O) is 2/6 indicated by the flow arrows in blue and the fractional value of brown (Br) is 4/6 indicated by green arrows groove.

Because S5 wrong answer is not in accordance with the fractional value, the researchers conducted interviews with intervention Limited to S5 the following

Researcher : *Do you know the part group comparison?*

S5 : *No, I don't sir.*

Researcher : *The ratio is a comparison of the overall, so how much is the image 11 fractional value?* (Limited intervention)

S5 : *owh, its value is 2/4 or ½ right sir?*

Researcher : *Exactly.*

Interventions conducted by researchers of the S5 shown in the diagram 10 below:

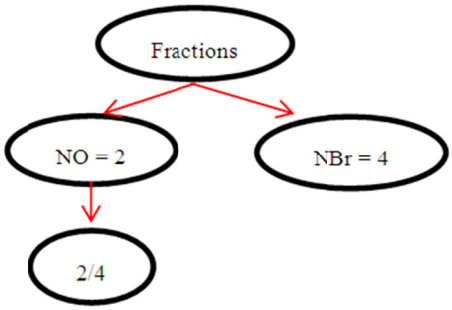
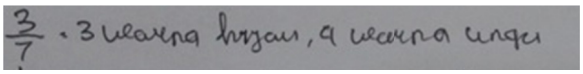


Diagram 10. The results of interviews S5 with limited intervention

From the results of the Limited interventions conducted by researchers of the S5 in question no 5 on the diagram 10, it appears that S5 begin to understand the value of *the part group comparition* based on the orange to brown is 2/4.

The next picture *interpret fractions as the part whole comparison* of S3 are shown in Figure 12 below:



Translate version

$\frac{3}{7}$, 3 Green, 4 Purple

Figure 12. Question answer 6 of S3

Based on the results written by S3, the researchers conducted interviews with S3 as follows

Researcher : *What is your answer in figure 12?*

S3 : *fractional value in Figure 12, its green color was on top of a purple color and a total number of 7, then the value of fractions is 3/7.*

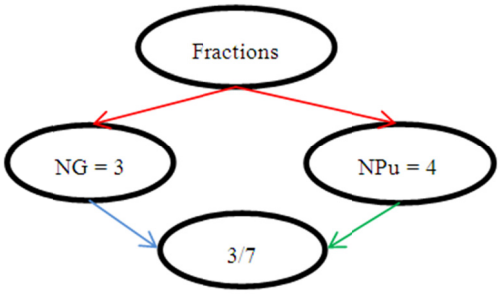


Diagram 11. Interpretation of the results of interviews S3

Interpretation of the results of interviews with S3 can be seen in the diagram 11.

In the diagram 11 looks S3 interpret the fractional value of green (G) is 3/7 indicated by the flow arrows in blue and the fractional value of purple (Pu) is 3/7 indicated by green arrows groove.

Because S3 wrong answer is not in accordance with the fractional value, the researchers conducted interviews with intervention Limited to S3 as follows

Researcher : *Do you know about the ratio?*

S3 : *It's a distribution, sir!*

Researcher : *True or with other words the ratio is of = comparisons.*

And what if the fractional value based on the ratio? (Limited intervention)

S3 : *Oh yes, sir. 3/4 of is the value*

Researcher : *You are very smart!*

Interventions conducted by researchers to S3 shown in the diagram 12 below:

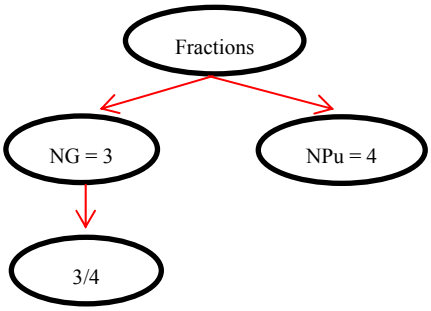


Diagram 12. The results of interviews S3 with limited intervention

From the results of the limited interventions conducted by researchers of the S3 in question no 6 on the diagram 12, it appears that the S3 began to understand the value of a *fraction as the part whole comparition* based on the green to purple is 3/4.

The last picture interpret *fractions as the number line* shown in the students' answers below:

Fractional $\frac{4}{6}$ as the number line

Figures 7 and 8 above error interprets the fractional value of the primary colors (first/previous) is above as the basis for determining the value of the fraction. Whereas in Figure 9 and 10 above error interprets the fractional value of the second color as the basis for determining the value of such fractions by looking at other parts. Later Figures 11 and 12 above interprets the fractional value in the second color as the basis for determining the value of a fraction without seeing the whole.

5. Conclusions

Completion of the process of student answers appeared that students cannot understand fractions as ratios and parts are congruent and incongruent of the whole or a set of objects which was given, it is seen from the inability of students to write answers to a fractional value of the three categories in the discussion. Students interpreted the fractional value soch as

1) The dominant colors are located on top of the other colors to see part of the overall
2) Colors are asked as part of other colors regardless of the whole.
3) The combination of No. 1 and 2.

After the intervention of the settlement of the problem of students, then the problem can be solved with the desired questions.

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