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"With Percentages the 100 is Always in the Denominator": From the Field to Pre-service Teachers

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Abstract

The ubiquity of the subject of percentages in our everyday life demands that math teachers and pre-service math teachers demonstrate a profound knowledge and thorough understanding of the concept of percentages. This work, which originated from one specific lesson in an 8th grade math class, studies the conceptual understanding and problem-solving strategies of 17 pre-service math teachers. We exhibit and analyze the pre-service teachers' misconceptions and their inefficient or false problem-solving strategies. We identify two main misconceptions. The first we name "In percentage problem is 100 always the denominator?" and the second "One can handle percentage problems using the same techniques as in numerical problems" (or Over-transferring of numerical problems). We hypothesize reasons and explanations for these misconceptions. The frequent use of percentages in everyday life and in sciences demands a sound knowledge of the subject. This paper aims to describe a process that began with a mathematics lesson in an 8th grade. As an outcome of attending the lesson and the discussions that followed, we investigated 17 pre-service mathematics teachers' conceptual understanding and problem solving strategies associated with percentages. The research sample is not random. These are 17 pre-service teachers that we met in the course of our work with them. Because of the small size of the sample, no statistical analyses were performed beyond calculating the frequency of the events.

Key words: Percent value; Calculating the percentage; Calculating the initial amount; The percentage as an operator; Table method; Misconceptions; Intuitions; Rational numbers; Percentage problems

Introduction

The mathematical topic of Percentage is highly prevalent in our daily life. Every day we can see an ad offering a discount of so-and-so percent; we compare groceries, like cheese and cream, based on having 3% or 30% fat content; we read in the newspaper about a survey claiming a 53% support for an important issue or an article about how nurses' wages became 39% higher than the wages of teachers; and we follow how the exchange rate for the US dollar has dropped by 0.7% in preparation for our trip abroad. However, do we really understand the subject of percentages? Is this term, the percent, indeed clear and well-understood by most of the population? Evidently not. Here is an example at an advertisement of a parking lot offering a special 50% discount sale:

Special Offer – 50% Discount!!!
Buy 10 parking vouchers
And get free additional 5 parking vouchers*
*A single entry voucher costs US$ 7.00

Is this truly a 50% discount or perhaps only a 33⅓% discount? After all, for the price of 10 vouchers we get 15, so the discount is for 5 out of 15 vouchers…

The extensive use of percentages in every-day life and especially in science follows from the common assumption that the general public has a good and well-founded understanding of percentages. And when it comes to junior-high math teachers – we expect them to have a flawless understanding of the subject of percentages. And so, it is our duty, as educators of pre-service teachers who will eventually teach percentages to the public, to consolidate and expand pre-service teachers' knowledge of percentages. To that end, we have to be familiar with their perceptions of terms associated with percentages. It is important, for example, to increase mathematics teachers' awareness of the initial perceptions of mathematical terms of their learners, perceptions which differ from those acceptable in math. This awareness can affect teaching in various ways such as planning

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of teaching sequence, nature of answers to learners' questions (Sheffet & Bassan-Cincinatus, 2004). This paper describes a process which began in an 8th grade mathematics class that the pre-service teachers witnessed as part of their practicum. The discussion that followed the lesson with the pre-service teachers revealed the pre-service teachers' perceptions of terms associated with percentages, the proportions between the numbers and the work procedures related to the topic. Next, we will present the various strategies, misconceptions and approaches to solving this percentage problem that we discovered. We will also try to come up with explanations in order to understand the reasons for these misconceptions and why they were made.

The Initial Question

In a math class of 8th graders dealing with percentage problems, the teacher presented the following word problem and asked the pupils to solve it.

The sum of two numbers is 464. **One is 32% bigger than the other.** **Find the numbers.**

The pupils solved the problem and presented their answers, which span a wide range. Some were routinely familiar and some were creative; some were correct and others – incorrect. Below are the answers suggested by the 8th graders.

Solution 1: $x+\frac{132}{100}x=464$

Solution 2: $x+\frac{100}{132}x=464$

Solution 3: $\frac{34}{100}x+\frac{66}{100}x=464$

Solution 4: $x+\frac{68}{100}x=464$

The discussion in class about each of the answers demonstrated the pupils' numerous hesitations as to whether they were correct or incorrect. Although the pre-service teachers were present in the lesson, witnessed the discussion in the class and heard the instructions of the teacher, the pre-service teachers too failed to identify the correct answers. The problem and the above-mentioned four suggested solutions were presented to 17 other pre-service teachers who did not take part in the lesson (see Appendix A). They were asked to (i) determine whether each solution was correct or incorrect, and (ii) to explain the approach they believed the pupils used when solving the problem using each solution. Then they were asked to write two explanations for 8th-grade pupils about the correct way to solve the above-mentioned problem, where in one explanation the variable *x* represents the smaller of the two numbers, and in the other explanation the variable *x* represents the bigger number (see Appendix B). We presented the problem in this way due to our wish to expose the pre-service teachers' Subject Matter Knowledge (SMK) of this topic, their degree of understanding the concepts and their misconceptions (if they exist). The discussion of the various solutions and the explanations given to the learners manifested the weak points, specifically in the Subject Matter Knowledge of the pre-service teachers.

Theoretical Background

Percent or percentage (from Latin "per centrum") means "out of 100" or "per 100". One percent of a quantity is $\frac{1}{100}$ of the quantity. Percentages are usually expressed by the sign: %. Since percentages describe part of a quantity, they are not numbers the same way fractions are. Fractions have multiple mathematical functions, and only one of them is describing a part of a given quantity (Koay, 1998). It is possible to substitute percentages with fractions (and vice versa) only when the fraction describes a part of a quantity. In contrast, one should not substitute the fraction with percentage when the fraction describes the result of a measurement or when adding or subtracting a fraction. Sometimes, percentages are described as merely a different name for fractions (simple or decimal). However, while fractions represent numbers, percentages represent part of a quantity. For example, it is fine to replace the phrase "$\frac{1}{4}$ of the square surface area" with "25% of the square surface area", yet it is false to substitute $\frac{1}{4}$ by 25% in the sentence "the surface area is $\frac{1}{4}$ squared meter." Similarly, it is false use "25%" instead of "$\frac{1}{4}$" in the exercise $3+\frac{1}{4}$, or to replace "25%" with the number $\frac{1}{4}$ on the axis of real numbers.

It is possible to add or subtract percentages only when they relate to the same quantity. For example, consider the following question. *"Yesterday I spent 20% of my money in shop A, and then 30% of the money in shop B. What was the overall percentage of my money spent yesterday?"* A valid solution to this question is "20%+30%=50%."[†] When describing part of a quantity, one may think of a smaller quantity. Percentages can describe a small part of the whole (e.g. 25% of the square surface area), and may sometimes be used to describe a quantity larger than a single unit (e.g. 150% of the square surface area). Below are two problems with fractions, illustrating the two different roles of fractions, as numbers and as operators (a part of a specific quantity). An operator is a function which creates a new number from every number.

Problem 1: The sum of two numbers is 2. One number is bigger than the other number by $\frac{1}{2}$**.** What are the two numbers?

Problem 2: The sum of two numbers is 2. One number is $\frac{1}{2}$ bigger than the other number. What are the two numbers?

When comparing the solutions of the two problems the different roles of "$\frac{1}{2}$" are evident.

Solution of problem 1: Denote the small number as *x* and the bigger number as $x+\frac{1}{2}$. We thus write the equation is $x+x+\frac{1}{2}=2$ and the two numbers are $\frac{3}{4}$ and $1\frac{1}{4}$.

Solution of problem 2: Denote the small number as *x* and the big number as $x+\frac{1}{2}x$. We thus write the equation is $x+x+\frac{1}{2}x=2$ and the two numbers are $\frac{4}{5}$ and $\frac{6}{5}$.

Suppose we substitute "$\frac{1}{2}$" by "50%" in the two problems. We end up with Problems 3 and 4:

Problem 3: The sum of two numbers is 2. One number is bigger than the other number by **50%.** What are the two numbers?

Problem 4: The sum of two numbers is 2. One number is **50%** bigger than the other number. What are the two numbers?

In this case the two mathematical problems have the same meaning and both have the same solution, i.e. $\frac{4}{5}$ and $\frac{6}{5}$. Percentages always describe part of a given quantity, and therefore once we substitute the number $\frac{1}{2}$ with 50% in the first problem, then problem has a different meaning.

Types of Percentage Problems

The elementary problems in the field of percentages are partitioned into three main types.

[†] If *x* represents the amount then according to the distribution law

$$\frac{20}{100}x+\frac{30}{100}x=\left(\frac{20}{100}+\frac{30}{100}\right)x=\frac{50}{100}x$$

Calculating the Percent Value

Here, one has to calculate a quantity which constitutes *p*% of the quantity *b*. E.g.: "Tom has in his saving box $250. He bought a book. What is the price of the book if we know that Tom spent 20% of his money on the book?" The respective formula is: $a = \frac{p}{100} \times b$ or $a = \frac{20}{100} \times 250$.

Calculating the Percentage

Here, one has to calculate what percentage *p,* is quantity *a* out of quantity *b*. For instance: "Tom has in his saving box $ 250. He bought a book for $50. What percentage of Tom's money did he spend buying the book?" The respective formula is: $p = \frac{a}{b} \times 100$ or $p = \frac{50}{250} \times 100$.

Calculating the Initial Amount

Here, one has to calculate the quantity *b* given that quantity *a* is *p* percent of *b*. For instance: "Tom spent $50 out of his saving box on a book. How much money did Tom have in his saving box before purchasing the book if we know that Tom spent 20% of his money on the book?" The respective formula is: $b = \frac{a \times 100}{p}$ or $b = \frac{50 \times 100}{20}$.

Sheffet & Bassan-Cincinatus (2006) showed that pupils find problems of the latter type (3.1.3) to be the most difficult, and that the success rate in solving these problems is lower than the former two types. Conversely, problems of calculating the percent value (3.1.1) are the easiest. Recall the leading question from Section 2. "The sum of two numbers is 464. One number is bigger by **32%** than the other number. What are the numbers?" If we categorise the type of this question, we find that when *x* represents the smaller number of the two, then this is a problem of calculating the percent value (3.1.1); and when *x* represents the bigger number – then it is a problem of calculating the initial amount. (3.1.3).

Strategies for Solving Percentage Problems

Finding Part of the Whole (Percentage as Operator)

Much like math problems involving fractions, in order to find part of the whole we perform multiplication between the part and the whole. The same holds for percentage problems. For example: "There are 40 pupils in the class. 35% of them study French. How many pupils study French?" (In this question we look for the percent value.) The solution is as follows.

The number of pupils learning French – *x.*

Using this strategy we get: $x = \frac{35}{100} \times 40$

Using Proportions

In this strategy we first determine the proportion between the part and the whole. The ratio of the percent value to the initial amount is equal to the ratio of the percentage to 100.

$$\frac{Part}{Whole} = \frac{The\ Percent\ Value}{Initial\ \ Amount} = \frac{p}{100}$$

For example: “There are 40 pupils in the class. 35% of them study French. How many pupils study French?” (In this question we look for the percent value.)
x – denotes the number of pupils learning French.

Using this strategy we get: $\frac{35}{100} = \frac{x}{40}$

Using a Method Table

Using method tables in order to solve typical percentage problems is recommended (Kachapova, 2011). Such tables take the form:

	Quantity	Percentages as a fraction
The whole	The initial amount	1
The part	The percent value	$\frac{p}{100}$

In the case of our example we will get:

	Quantity (pupils)	Percentages as a fraction
The whole	40	1
The part	*X*	$\frac{35}{100}$

The appropriate exercise will be: $x = \frac{35}{100} \times 40$

Calculating the Value of the Single Percentage

In this strategy we calculate the corresponding value for one percent, and then multiply by the number of percentage. One finds the value of the single percentage and from it one can calculate what one looks for.

$$y = \frac{The\ Initial\ \ Amount}{100}$$

$$p \times y = The\, percent\ value$$

In our example we obtain: $0.4 = \frac{40}{100}$ – value of a single percentage. The value of 35%: 35×0.4.

This strategy is presented for the purpose of solving problems. It is intuitive but is not prevalent in Israeli textbooks (Sheffet & Bassan-Cincinatus, 2006). In these examples we presented different strategies for solving a problem of calculating the percent value. These strategies can be used also for solving problems of calculating the percentage and calculating the initial amount. Strategies for solving percentage problems are based on proportion, which is defined as equality between two ratios. Ratio is defined as the relative sizes of two (or more) values. If two ratios $\frac{a}{b}$ and $\frac{c}{d}$ are given, a proportion will be equality between them: $\frac{a}{b} = \frac{c}{d}$. In percentage problems, when *a* represents the percent value, *b* represents the initial amount and *p* the percentage, the proportion is: $\frac{a}{b} = \frac{p}{100}$. That is, solving percentage problems is a special case of solving proportion problems.

The Development of Proportional Thinking

Inhelder & Piaget (1964) claim that the proportional scheme is developed only at the formal thinking stage (ages 12 and above). They argue that there are three stages in the development of proportional thinking.

a. The intuitive stage (ages 3 - 7/8). At this stage children do not understand the meaning of ratio and they relate only to one variable out of all the variables mentioned in the problem.
b. The concrete stage (ages 7/8 - 12). At this stage children begin to understand the meaning of proportion, yet they are not able to use the laws of proportion.
c. The formal stage (12 years and onward). At this stage children understand the laws of proportion and are capable of using them for solving problems.

Findings

Discussion of the Solutions Presented in Class by the 8th Graders

Let us go back to the solutions to the problem which the 8th graders proposed and examine these solutions. The problem was: The sum of two numbers is 464. One is 32% bigger than the other. Find the numbers.

Solution 1: $x+\frac{132}{100}x=464$

Correct solution. The variable *x* represents the small number.

Solution 2: $x+\frac{100}{132}x=464$

Correct solution. The variable *x* represents the big number. Consequently, the small number constitutes $\frac{100}{132}$ of the big number.

Solution 3: $\frac{34}{100}x+\frac{66}{100}x=464$

Incorrect solution. There is no match to the problem data. In this case the pupil related to the sum of the two numbers – 464 – as the initial amount and decided it is the 100%. He deducted 32% and divided the remainder into two equal parts, 34% each. To one of the parts he added the 32% and obtained 66%. The big part includes 32% more than the small part (32% of the entire sum). Let us calculate the present value of each number. Hence, the big number is 306.24 $(\frac{66}{100}\times 464=306.24)$ and the small number is 157.76 $(\frac{34}{100}\times 464=157.76)$. Thus, the big number is bigger by 94.1% than the small number – no match to the problem data.

In order to solve the problem the pupil calculated and solved in numbers[‡] and did not take into consideration that the problem involves percentages. He did not relate to the percentages as an operator. This is not an algebraic solution. However since he was asked to present an equation in algebra, he wrote the following equation and explained: $\frac{34}{100}x$ - the small number, $\frac{66}{100}x$ - the big number and the total is 464.

Solution 4: $x+\frac{68}{100}x=464$

Incorrect solution. There is no match to the problem data. In this case the pupil uses the variable *x* for representing the big number and calculates the small number – 68% of it. That is, the pupil erroneously thinks

[‡] The pupil solved the problem presented to him while substituting the percentages with numbers according to the following problem: the sum of two numbers is 464. One number is bigger by 32 than the other number. What are the numbers?

that there is symmetry here – if the larger is 32% more than the smaller, then the smaller is 32% less than the larger.

Identification by the pre-service teachers whether the answers are correct: Table 1 illustrates the distribution of the pre-service teachers' identification whether the answers are correct.

Table 1. Most of the pre-service teachers identified as correct only solution 1 of the pupils.

Pupils' solutions	**The problem solution**	**Correct recognition**	**Incorrect recognition**	**Did not know whether the solution was correct or incorrect**
Solution 1: $x+\frac{132}{100}x=464$	Correct	14 (82.4%)	3 (17.6%)	
Solution 2: $x+\frac{100}{132}x=464$	Correct	1 (5.9%)	16 (94.1%)	
Solution 3: $\frac{34}{100}x+\frac{66}{100}x=464$	Incorrect	5 (29.4%)	11 (64.7%)	1
Solution 4: $x+\frac{68}{100}x=464$	Incorrect	7 (41.2%)	10 (58.8%)	

Pre-service teachers' solution of the given problem: Further on in the questionnaire the pre-service teachers were asked to solve the problem twice, once when the variable *x* represents the smaller number and once when the variable *x* represents the bigger number. We wanted the pre-service teachers to provide two different ways of solving the problem because changing the representation of the variable *x* changes the problem from a problem of calculating the percent value to a problem of calculating the initial amount.

The solution whereby the variable x represents the smaller number: All the pre-service teachers solved this problem correctly. Three pre-service teachers solved it with two variables in the following way:

x – the smaller number, *y* - the bigger number
The equations set was: $x+y=464$

$$x+\frac{32}{100}x=y$$

The other pre-service teachers solved the problem with one variable and the equation was:

$$x+x+\frac{32}{100}x=464 \quad (or \;\; x+\frac{132}{100}x=464)$$

The solution whereby the variable x represents the bigger number: The three pre-service teachers who in section 4.3.1 solved the problem with two variables, switched between *x* and *y.* In fact they presented the same solution in both cases, only the function of the variables was changed. They wrote:

x – the bigger number, *y* - the smaller number
The equations set was: $x+y=464$

$$y+\frac{32}{100}y=x$$

This solution 'saves' the need for expressing the smaller number by means of the bigger number. When asked to solve the problem with one variable, the pre-service teachers failed to do it.

Nine pre-service teachers solved the problem in this way: $x+\frac{68}{100}x=464$. That is, if one number is bigger by 32% than the other, than the smaller number is smaller by 32% than the first number. Two pre-service teachers solved the equation they had presented. One of them realised that the solution is not identical to the problem solution when *x* represents the smaller number. He indicated that the solution was not good but could not indicate the reason. The other pre-service teacher realised that the solution was incorrect due to the obtained numbers. *"I did not receive an integer. It's wrong!"*.

Two other pre-service teachers who solved the equation did not relate to the fact that different results were obtained for the same problem. Five pre-service teachers only wrote the equation but did not solve it. Table 1 indicates that 10 pre-service teachers failed to identify that solution 4, $x+\frac{68}{100}x=464$, was incorrect. Among them nine pre-service teachers solved the problem in this way when the variable *x* represented the bigger number. The case of the pre-service teacher who claimed that solution 4 was correct is interesting. However, when asked to solve the problem by himself, he did not choose that solution but rather hesitated a lot. Eventually he presented a correct solution $x+\frac{100}{132}x=464$. Based on the correct solution he said: *"I want to change what I had written in Appendix A regarding the pupils' solutions on the previous page"*.

Only five pre-service teachers solved the problem correctly. They indicated the smaller number as a function of the variable *x* which represents the bigger number. One pre-service teacher wrote the solution immediately. He was also the only one to identify solution 2 as a correct one. The other four pre-service teachers did not come up with an immediate solution. They all deliberated, thought, tried, deleted and only after working for some time on the problem they achieved the correct answer. One of them refused to hand in the page before reaching a correct answer. He took the page home and said that he continued thinking about the problem until finding the correct solution. Another pre-service teacher replaced writing in percentages by writing in decimal fractions, yet his solutions attested to the fact that he understood the problem. He wrote: $x+\frac{100}{1.32}x=464$. All the three pre-service teachers who solved the problem correctly adopted the same strategy. They represented the bigger number (*x*) as an expression of the smaller number (*y*). They wrote: $x=\frac{132}{100}y$. From this they extracted *y* and found: $y=\frac{100}{132}x$. The fourth pre-service teacher failed to explain how he reached the answer. *"I thought and thought and if the bigger number is 1.32 then I need only the whole which is 100%"*.

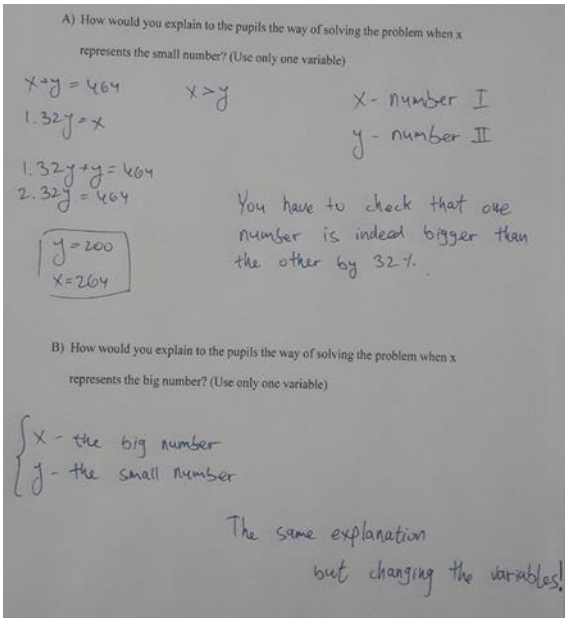
Discussion

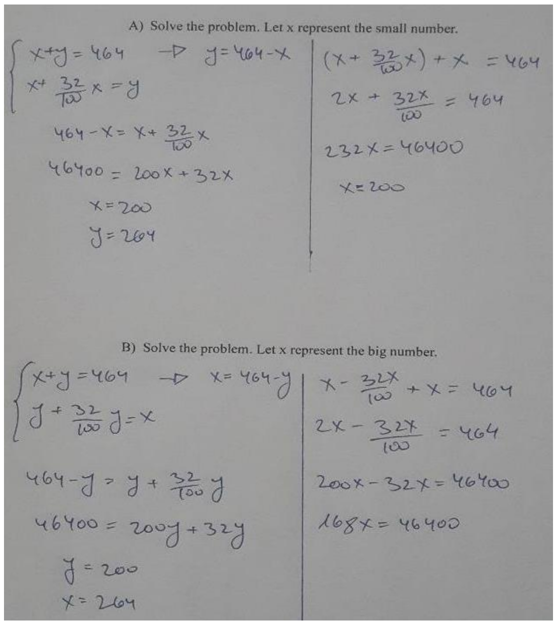
Discussion of the Pre-Service Teachers' Solutions

As indicated in the theoretical background so we also found that the problems which required calculating the percent value are the easiest and have a high rate of success. All the pre-service teachers solved correctly the problem of calculating the percent value when the variable *x* represents the smaller number. Conversely, only five pre-service teachers solved correctly the problem of calculating the initial amount, whereby the variable *x* represents the bigger number. The other pre-service teachers found the solution very difficult. Some of them overcame the difficulty by using two variables and in fact presented twice the same equation set with only a different position of the variables. See attached examples 1 and 2.

a. How would you explain to the pupils the way of solving the problem when *x* represents the smaller number?

b. How would you explain to the pupils the way of solving the problem when *x* represents the bigger number?





Example 1 **Example 2**

In example 1, the pre-service teacher forms a 'distinction' only by switching the variable. He does not show the continuation of the process at all because, *"the same explanation but changing the variables!"*. In example 2 one can see the same equations in the two items. Most of the pre-service teachers chose the incorrect solution: $x + \frac{68}{100}x = 464$. In this solution the percentages are referred to as numbers.

Five pre-service teachers managed to reach the correct solution. Please note that four of them did not identify solution 2 as a correct one, and only after solving the assignment, understood the pupil's solution. They comprehend the use of percentages for describing part of a quantity and what is the initial amount in each of the cases included in the problem. Therefore 32% of the smaller number are not equal to 32% of the bigger number. They rejected solutions similar to solution 3 and solution 4 given by the pupils and started a process of looking for the solution which was neither easy nor immediate. They had no intuition that oriented them towards a certain solution. They did not use solution strategies such as proportion or table method or an illustrating drawing or any other way which could lead them to the correct solution. Each of them 're-discovered' the problem of calculating the initial amount. It is worth noting that as a result of the demand to present another solution in another way (variable *x* representing the bigger number), four pre-service teachers attained the correct solution which led to a deeper insight.

Misconceptions

Perusal of the findings indicates three points which reflect misconceptions associated with the topic of percentages:

The first: In percentage problems, the 100 appears always in the denominator.
The second: If one part is 34% of the quantity and the second part is 66% of it, then the bigger part is bigger by 32% than the smaller part.
The third: If the bigger number is bigger by 32% than the smaller number, then the smaller number too is smaller by 32% than the bigger number.

Misconception: In percentage problems, the 100 appears always in the denominator.

Discussion of the first misconception raises the question: What prevented the pre-service teachers from identifying that solution 2 - $(x+\frac{100}{132}x=464)$ is correct? The pre-service teachers' arguments show that all those who were mistaken expected that in the solutions of percentage problems the 100 will always appear in the denominator. *"... in order to write percentage in a fraction it has to be divided by 100"*; or *"...the way that he wrote the percentages is incorrect. The pupil did not understand that the whole is represented by 100 in the denominator"*; or *"the pupil was confused between the numerator and the denominator"*. Only in problems of calculating the percent value the 100 is written in the denominator and in problems of calculating the percentage and calculating the initial amount the 100 is written in the numerator. Nevertheless, the pre-service teachers expect to find the 100 in the denominator. This expectation reflects intuition regarding percentage problems. Fishbein (1987) argued that,
...based on their experiences, individuals create a stable set of expectations regarding similar to whose which they have already experienced. The set of expectations is the basis for the emergence of intuition as a full and whole framework of a theory. Consequently, sometimes the intuitions include mistaken or wrong illustrations (pp. 88-89).

According to Fishbein (1987), intuition is characterized **Table 1**
Most of the pre-service teachers identified as correct only solution 1 of the pupils.
by self-evident and inner conviction, stability, obsessiveness, generalization, extrapolativeness and it has a hidden mechanism. Intuition develops the misconception that: *In percentage problems, the 100 appears always in the denominator.* When learners encounter a solution for a percentage problem that matches their intuition – the 100 in the denominator – the intuition 'coerces' them to accept it as correct. They immediately approve the solution and are certain that it is correct. However, when the solution does not match their intuition – the 100 in the numerator – intuition 'coerces' them to reject the solution. Fishbein (1987) maintained that the intuition is consolidated on the basis of learners' experiences. Most solved problems that deal with calculating the percent value (and in them the 100 is indeed in the denominator). This consolidates the intuition and perhaps this is also the reason for their higher degree of success. This might be due to the fact that when pupils start learning the topic of percentages, the percentage is emphasized as $\frac{1}{100}$. In any case, this intuition immediately rejected the possibility that solution 2 might be correct.

Misconception: If one part is 34% of the quantity and the second part is 66% of it, then the bigger part is bigger by 32% than the smaller part.

This misconception might stem from over-transferring of numerical problems. Below are some explanations for the reasons. When we look for two numbers, one is bigger by 32 than the other, pupils will express the smaller number as *x* and the bigger number as $x+32$. Adopting this technique to percentage problems will result in the bigger number being 66% of the total sum and the small number being 34% of the total sum. Hence, the bigger number is 32% bigger than 34%. We refer to percentages as numbers for all intents and purposes and as such we can add and subtract them. In our case, a person solving the problem does not distinguish between the two following problems:

Problem 5: the sum of two numbers is 464. One number is bigger by 32 than the other. Find the numbers.
Problem 6: the sum of two numbers is 464. One number is 32% bigger than the other. Find the numbers.
The person who over-transfers attempts to solve the second problem using the same technique used to solve the first problem. Such a solution demonstrates lack of comprehension of the concept of percentages.

Misconception: If the bigger number is bigger by 32% than the smaller number, then the smaller number too is smaller by 32% than the bigger number

This misconception too can be explained as an over-transferring from numerical problems. Those solving the problem by understanding (incorrectly) that if the bigger number is bigger by 32% than the smaller number, then the smaller number is smaller by 32% than the bigger number, based themselves on their experience with similar numerical problems. If $y=x+32$ then $x=y-32$.

Once more, adopting this approach to percentage problems attests that pupils have not internalized the fact that percentages always describe part of a quantity. Another option is that the solving person knows that percentages describe part of a quantity but does not consider the initial amount from which the percentages will be calculated. For example, consider a product whose price increased by 20% and then dropped by 20%. Some learners expect the price after the reduction to be the same as the price prior to the increase. The same applies in this case. The learner thinks that 32% of the smaller number is identical to 32% of the bigger number. However, a 32% value of one initial amount is not equal to a 32% value of another initial amount. When dealing with percentage problems it is important to find out from which whole the percentage is calculated.

Conclusions and Recommendations

In spite of the great importance of the topic of percentages and its extensive use in daily life, there are many difficulties and errors related to the solution of percentage problems. We started with an occurrence in a mathematics lesson in junior high school and identified misunderstandings and misconceptions about percentages not only among pupils but also among pre-service teachers. It is recommended to explore whether the events we indicated are manifested also among junior high school mathematics teachers. We discussed various reasons for the origin of the events we had traced. We are aware of the fact that in order to corroborate the reasons it is necessary to conduct an in-depth and comprehensive study. Nevertheless, we believe that the very description of these events should impact the way we teach. When teaching this topic, teachers should consider pupils' existing intuitions about it. The right intuition might help learners to absorb mathematical knowledge. Moreover, it is recommended to put more emphasis on the fact that percentages describe part of a quantity and in which cases they can or cannot be replaced by fractions. The similar and different features between percentage problems and numerical problems should be emphasized. Based on our experience, we intend to dedicate several lessons for engaging in the described occurrences. We maintain that the topic should be presented in a wide and comprehensive view, while relating to the mathematical aspects and the psychological-didactic aspects of this topic. We hope that later on these pre-service teachers will succeed in inculcating the correct terms in their pupils – the generation of the future.

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Appendix A

Numbers and percentages

The 8th grade pupils received the following problem:

The sum of two numbers is 464. **One is 32% bigger than the other.** **Find the numbers.**

Below are four different solutions to the problem. Please describe your opinion and each solution and give an explanation:

- Correct or incorrect solution?
- On what did the pupils based themselves when solving in this way?

Solution 1: $x+\frac{132}{100}x=464$

Solution 2: $x+\frac{100}{132}x=464$

Solution 3: $\frac{34}{100}x+\frac{66}{100}x=464$

Solution 4: $x+\frac{68}{100}x=464$

Appendix B

Numbers and percentages

Below is the problem:

The sum of two numbers is 464. **One is 32% bigger than the other.** **Find the numbers.**

A) Solve the problem. Let x represent the small number.

B) Solve the problem. Let x represent the big number.

Use only one variable

A) How would you explain to the pupils the way of solving the problem when x represents the small number? (Use only one variable)

B) How would you explain to the pupils the way of solving the problem when x represents the big number? (Use only one variable)