Racial Differences in Mathematics Test Scores for Advanced Mathematics Students

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Research on achievement gaps has found that achievement gaps are larger for students who take advanced mathematics courses compared to students who do not. Focusing on the advanced mathematics student achievement gap, this study found that African American advanced mathematics students have significantly lower test scores and are less likely to be proficient at all mathematics skill subdomains compared to White advanced mathematics students. Interestingly, African American students who take calculus as their highest level of mathematics in high school have similar achievement levels as White advanced mathematics students who have trigonometry/ pre-calculus as their highest level of mathematics in high school.

Achievement gaps have been a longstanding area of sociological research given the implications for social stratification. Through decades of study, researchers have identified student course taking as a mechanism that explains inequality in educational outcomes such as mathematics test scores (Gamoran, 1987; Lleras, 2008; Kelly, 2009; Schneider, Swanson, & Riegle-Crumb, 1997; Carbonaro & Covay, 2010). Research has found that students in more advanced mathematics courses tend to perform higher than students in lower level courses during their schooling careers (Bozick, Ingels, & Owings, 2008) even when controlling for ability (Gamoran, 1987). Yet, research has also shown there is inequality in access to advanced course taking. Researchers have found that African American students are less likely to take advanced courses as compared to White students within the same school (Kelly, 2009; Lleras, 2008). However, inequality with regard to access to higher course taking is not the only inequality that has adversely affected mathematics course taking for African American students.

Even when the examination of achievement gaps has been constrained to advanced mathematics students, the gaps continued to exist and were, in fact, larger than the gaps among students taking lower level courses (Riegle-Crumb & Grodsky, 2010).While research has shown that achievement gaps exist among our most advanced students, researchers know little about this gap. For example, little research has focused on achievement gaps *within* the courses (e.g., trigonometry, pre-calculus, calculus) themselves. Additionally, researchers know little about what the African American-White

gap among advanced mathematics students means in terms of the types of mathematics skill students possess. Do African American advanced mathematics students have lower achievement than White students on (a) basic mathematics knowledge and skills? (b) advanced mathematics knowledge and skills, or (c) both basic and advanced mathematics knowledge and skills? In other words, is the advanced mathematics student achievement gap present at various levels of mathematics knowledge? The current study explored the African American-White achievement gap among advanced mathematics students in order to gain a greater understanding of where the gap occurs not only in terms of the specific courses but also in terms of differences in mathematics skills. By having a fuller understanding of this gap, researchers, policymakers, and schools can help to reduce the achievement gap.

Focusing specifically on advanced mathematics students and mathematics outcomes, this study found that there are significant differences between African American and White advanced mathematics students in terms of their mathematics test scores. More specifically, African American advanced mathematics students were significantly less likely to be proficient in mathematics skills such as low level mathematics concepts as well as the use of multiple steps to solve problems compared to White advanced mathematics students. While student course taking, background characteristics, and prior achievement explained these differences for some skill levels, there remained statistically significant differences for the most advanced mathematics skills among advanced mathematics students.

Background

Advanced Course Taking
In a recent study examining high school mathematics course taking and achievement gaps, Riegle-Crumb and Grodsky (2010) separated their sample into two categories (students taking advanced courses and student taking non-advanced courses) to reduce heterogeneity as a result of differential exposure to the mathematics curriculum via tracking (Gamoran, 1987; Gamoran, 2004; Oakes, 1985; Oakes, Gamoran, & Page, 1992). In other words, Riegle-Crumb and Grodsky (2010) looked at achievement gaps within *kinds* of courses. They found that White students who took advanced mathematics courses scored 9.5 points higher than their African American peers on a mathematics test in 12th grade, whereas White students in the non-advanced mathematics courses scored 6.4 points higher than their African American peers on the same test (Riegle-Crumb & Grodsky, 2010). That is, the African American-White achievement gap was larger among advanced mathematics students than among non-advanced mathematics students. These differences among advanced mathematics students are striking, but they are consistent with past research that has found that the achievement gap is larger for higher achieving students (Hanushek & Rivkin, 2009).

Unfortunately, researchers have come to expect significant differences in achievement outcomes between students who have taken differing levels of courses given the years of research on tracking, which has found that students in lower level courses have fewer opportunities to learn (i.e., specific situations that cultivate learning) than students in more advanced level courses and thus lower achievement (Gamoran, 1987; Oakes, 1985; Oakes, Gamoran, & Page, 1992; Schneider, Swanson, & Riegle-Crumb, 1997). In other words, research has found that students in lower level courses have different learning opportunities within their courses than those students in more advanced level courses.

Past research has found variation in learning opportunities among the classrooms of advanced courses (Schmidt & McKnight, 2012; Covay Minor, 2015) analogous to the variation that is between levels of courses. Variation in student opportunities to learn

and outcomes within advanced courses (even courses with the same course title) may be explained by differences in teacher characteristics through teacher sorting (Ball, 1991; Ferguson, 1998; Goldhaber & Brewer, 1997; Hill, Rowan, & Ball, 2005; Kelly, 2004; Lankford, Loeb, & Wyckoff, 2002; National Science Board, 2008; Phillips, 2010), student sensitivity to teacher effects (Ferguson, 1998), content coverage (Metz, 1990; Porter, 1991; Sandholtz, Ogawa, & Scribner, 2004; Schmidt & McKnight, 2012; Spillane & Burch, 2006; Stecher, Hamilton, & Gonzalez, 2004; Covay Minor, 2015), classroom composition (Hoxby, 2002; Lomi, Snijders, Steglich, & Torlo, 2011; Covay Minor, 2015), student understanding of material/lesson (Ferguson, 2002), student identity (Tyson, 2011), or, most likely, a combination of these factors and others. However, before a thorough examination of *why* achievement gaps exist among advanced mathematics students, it is important to have a fuller understanding of the gap itself. The current study focused specifically on students whose highest mathematics course in high school involved advanced mathematics courses (i.e., trigonometry, pre-calculus, calculus) as a step to gain a better understanding of the achievement gap among the most advanced mathematics students.

Measuring Mathematics Outcomes

In addition to achievement gaps within levels of course taking, Riegle-Crumb and Grodsky (2010) found that those students who take advanced mathematics courses leave high school with higher overall test scores than those students who do not take advanced mathematics courses. While using overall test scores is one way to measure achievement outcomes, these scores tell us little about the types of mathematics skills that students possess. Indeed, Bozick, Ingels, and Owings (2008) argued that an aggregate measure of student achievement does not provide much information about "the *content* of that learning" (p. 2). In order to gain a better understanding of the achievement gap among advanced mathematics students, this study used a more nuanced measure of achievement.

The National Center for Education Statistics, the arm of the U.S. Department of Education that provides researchers with longitudinal education datasets, divided their administered achievement tests into skill subdomains based on the skills/knowledge necessary to correctly answer those questions (NCES, 2005; NCES, 2007). The use of mathematics skill subdomains or skill levels as a measure of student mathematics achievement provided a more nuanced examination or finer distinction of mathematics skills than an overall score and helped to provide researchers with more information about the content of achievement gaps. Examining aggregate test scores can mask important differences in skill levels of students (Hedges & Nowell, 1999; Rock & Pollack, 2002; West, Denton, & Reaney, 2001). Skill subdomains help us to get a better idea of where on the skills continuum learning takes place (NCES, 2005; NCES, 2006) and where along the continuum racial differences exist. For example, if students score as proficient in a skill level of using complex numbers, there would be more details regarding the mathematical concepts that the students know and what the students can do (NCES, 2005).

Like Riegle-Crumb and Grodsky's study (2010), most studies examining student achievement and racial gaps have relied on an overall or aggregate test score. However, recently researchers have begun to use student proficiency at various mathematics skill subdomains as a measure of student achievement (Bozick, Ingels, & Owings, 2008; Bozick & Dalton, 2013; Engel, Claessens, & Finch, 2013). For example, Bozick and Dalton's (2013) examination of the relationship between Career and Technical Education (CTE) courses and mathematics achievement used five skill subdomains to measure student achievement, with simple arithmetic as the lowest level

of mathematics skills and complex multistep problems as the highest level of mathematics skills. Based on tracking research, one would expect that those students taking an academic course track rather than a CTE course track would be more likely to increase their higher-level mathematics skills. However, Bozick and Dalton (2013) found that those students who concentrate their course taking in CTE courses and those who concentrate their course taking in academic courses have similar scores overall and for the five skill subdomains.

Bozick and Dalton's (2013) study helped to illustrate the additional information that skill subdomains can provide in terms of the skills students possess. In their study, the average 10th grade student correctly answered 46.2 test items. Based on this descriptive statistic, one knows little about student mathematics skill proficiency. However, Bozick and Dalton (2013) also reported that 93% of 10th grade students were proficient at basic mathematics skills and only 1% of students were proficient at advanced mathematics skills. From these reports, one can see that most students were proficient at simple arithmetic in 10th grade but few students were proficient at complex word problems. Therefore, it was highly likely that for the 46 test items answered correctly, few came from advanced mathematics skills.

Bozick, Ingels, and Owings (2008) also focused on student mathematics skill subdomains in their examination of student course taking sequences. They found that students with advanced mathematics course taking sequences tended to have the largest gains in mathematics skill subdomains such as complex word problems while students with course sequences that did not advance past geometry tended to have the largest gains in the basic mathematics skill subdomain. Stated differently, students taking advanced mathematics courses tended to make gains at the most advanced mathematics skills. Those students who took advanced mathematics courses also made the greatest gains in number of test items answered correctly; however, the mathematics skill subdomains helped to show where those gains in skills occurred along the mathematics knowledge continuum.

While the studies that have used skill subdomains as a measure of achievement have provided additional information about students' mathematics skills, they have not focused on achievement gaps. The use of the skill subdomains can provide a better understanding of whether and how the African American-White mathematics achievement gap may vary by mathematics skills.

Research Questions
Few studies have taken advantage of mathematics skills subdomains as a measure of achievement outcomes and few studies have focused on achievement gaps within courses. This study asked:

1. To what extent were there racial differences in mathematics knowledge and skills within course taking categories?
2. To what extent was the highest course taken in high school related to mathematics knowledge and skills for African American and White advanced mathematics students?

Data and Methods
The Education Longitudinal Study (ELS) was a rich source of data to examine course taking differences between African American advanced mathematics students and White advanced mathematics students. ELS was a nationally representative dataset collected in 2002, when students were 10th-graders, and 2004, when students were 12th-graders; it includes data from students, parents, teachers, school administrators,

and librarians, along with transcripts. Among the data were information about the highest level of mathematics course taken as well as mathematics test scores in 10th and 12th grade, which allowed for the examination of the relationship between taking advanced mathematics courses[1] and student mathematics outcomes. The analytic sample in this study (N=2,570)[2] included African American (N=410) and White (N=2,150) public school advanced mathematics students in general or college preparatory tracks who have 10th and 12th grade mathematics test scores.[3] As previous research found (Kelly, 2009; Lleras, 2008), African American students were less likely to be enrolled in advanced mathematics courses, which was likely why the African American sample is smaller than the White sample. Since the academic rigor and course taking patterns differ for public and private schools (Bryk, Lee, & Holland, 1993; Coleman, Kilgore, & Hoffer, 1982; Carbonaro & Covay, 2010), this study focused on one sector—public schools.[4]

Dependent Variables

Mathematics test scores were measured using the 12th grade mathematics item response theory (IRT) scores (see Table 1). In addition to the aggregate test scores, ELS divided the mathematics scores into five subdomains of mathematics skills. Each mathematics skills subdomain was measured as the student's probability of proficiency at a given skill; thus it ranges from 0 to 1 where 0 is no mastery and 1 is full mastery. The proficiency probability scores were NELS:88 (National Education Longitudinal Study:1988) equated criterion referenced scores. Each of the five mathematics skill subdomains was measured by a cluster of 4 items. In NELS:88, a student was considered proficient in a given skill subdomain if s/he answered at least three out of the four items correctly (NCES, 2005). However, because IRT was used, students did not necessarily receive the 4 items within a skill subdomain cluster. The proficiency level for each student was determined based on the pattern of answers on the items s/he received, not on his/her actual responses. The proficiency probability values were computed using IRT and reflect overall performance rather than a particular number of correct items (NCES, 2007) and indicate the probability that the student would get 3 out of 4 items correct (NCES, 2005).

The ELS mathematics assessments covered arithmetic (number sense and operations), algebra, geometry, data analysis and probability, and advanced topics (pre-calculus and analytic geometry). There were no calculus items in the ELS mathematics test. Most of the items on the tests covered arithmetic (19 items in 10th grade and 15 items in 12th grade), algebra (17 items in both grades), and geometry (20 items in 10th grade and 17 items in 12th grade). There were fewer items for data analysis and probability (9 items in 10th grade and 4 items in 12th grade) and advanced topics (8 items in 10th grade and 6 items in 12th grade). Given the limited number of items on advanced content, the relationship between advanced course taking and advanced topics may have been underestimated (Bozick, Ingels, & Owings, 2008).

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[1] It should be noted that the relationship between advanced mathematics course taking and student achievement likely includes other mathematics courses that the student took earlier in their mathematics course-taking career.

[2] When using restricted release data, the sample size values are rounded to the nearest ten, which may result in the rounded total sample size being slightly different than the sum of the two rounded subsamples.

[3] Since the analytic sample is limited to those students with test scores, this means that dropouts are not included in the sample. In other words, the students in the sample would tend to have higher test scores when excluding the dropouts and this is likely to affect the African American sample more than the White sample.

[4] All of the scale measures created for the analyses use private and public school African American and White students.

Table 1: *Descriptive Statistics for Advanced Mathematics Students*

	African American (N=410)		White (N=2,150)		Total (N=2,570)			
	Mean	SD	Mean	SD	Mean	SD	Min	Max
African American					.160	–	0	1
Mathematics Course Taking								
Trigonometry/Pre-Calculus	.823***	–	.651	–	.674	–	0	1
Calculus	.177***	–	.349	–	.326	–	0	1
Academic								
Mathematics-10th grade	41.63***	11.54	54.49	10.38	52.54	11.53	16.83	79.78
Mathematics-12th grade	47.50***	12.63	60.60	10.68	58.61	11.96	18.21	80.53
Mathematics Level 1	.975***	.090	.997	.029	.994	.045	0	1
Mathematics Level 2	.821***	.331	.976	.128	.953	.183	0	1
Mathematics Level 3	.625***	.437	.930	.221	.884	.287	0	1
Mathematics Level 4	.289***	.363	.684	.368	.624	.394	0	1
Mathematics Level 5	.012***	.068	.091	.202	.079	.190	0	.98
Reading-10th grade	28.67***	8.81	36.73	7.32	35.44	8.13	10.42	49.09
Background								
SES	–.047***	.721	.348	.632	.285	.663	–1.77	1.79
Female	.529	–	.549	–	.546	–	0	1
Native English Speaker	.964	–	.977	–	.975	–	0	1
Number of Siblings	2.45***	1.61	1.97	1.31	2.05	1.37	0	6
Family Resource Scale	3.57***	1.14	4.24	.745	4.13	.859	0	5.07
Intact Family	.398***	–	.717	–	.666	–	0	1

Table 1: *Continued*

	African American (N=410)		White (N=2,150)		Total (N=2,570)			
	Mean	SD	Mean	SD	Mean	SD	Min	Max
Expectations								
Student								
Don't Know	.049	–	.047	–	.047	–	0	1
BA	.364	–	.367	–	.366	–	0	1
BA Plus	.517	–	.538	–	.535	–	0	1
Parents								
Don't Know	.002	–	.003	–	.003	–	0	1
BA	.391***	–	.509	–	.490	–	0	1
BA Plus	.464*	–	.399	–	.409	–	0	1
Mathematics Teacher								
Don't Know	.051	–	.040	–	.042	–	0	1
BA	.454*	–	.519	–	.508	–	0	1
BA Plus	.167***	–	.245	–	.233	–	0	1
School Characteristics-Centered								
% Minority	35.64***	29.92	–9.40	19.50	–2.17	27.13	–29	71
% Free Lunch Eligible	14.47***	22.52	–6.06	12.75	–2.77	16.56	–21	75.2

Note: $*p<.05$, $**p<.01$, $***p<.001$ comparing African American and White students; Sample sizes decrease for achievement variables and course taking variables as they were not imputed. $N_{White}=2,090$; $N_{African\ American}=380$ for achievement and $N_{White}=1,900$ $N_{African\ American}=290$ for course taking

For both the 10th and 12th grade assessments, there were three forms (low difficulty, medium difficulty, and high difficulty) with 40–42 items on the 10th grade assessment and 32 items on the 12th grade assessment. In 10th grade, the students taking the low difficulty form had most of their items come from the arithmetic section at 40%. Students taking the medium difficulty form had most of their items coming from algebra at 26%, and students taking the high difficulty form had most of their items coming from geometry at 38%. For the advanced topic items, students taking the low difficulty form had 2 items, the medium difficulty form had 7, and the high difficulty form had 5. For the 12th grade assessment, students who took the low difficulty form had no items from advanced topics and most of their items from arithmetic at 44%. Most of the items on the medium difficulty form of the 12th grade assessment covered items relating to algebra at 38%, but it included 3 items from advanced topics. Finally, the high difficulty form of the 12th grade assessment had 5 items on advanced topics, but most of its items covered geometry at 47% (Bozick, Ingels, & Owings, 2008).

The five levels of mathematics skill subdomains increased in difficulty, starting with Level 1 as basic mathematics skills using whole numbers (e.g., basic computation). For example, Level 1 skills involved completing multiplication or division problems using whole numbers (Bozick, Ingels, & Owings, 2008). Proficiency at Level 2 required the ability to use "decimals, fractions, powers, and roots, such as comparing expressions, given information about exponents" (Bozick, Ingels, & Owings, 2008, p. 7). Level 3 involved simple problem solving and using low-level mathematics concepts. Problems that fall into Level 3 required students to solve an algebraic equation. Level 4 required students to use multiple steps in solving problems and the use of intermediate mathematics concepts. For example, students may have been asked to draw conclusions based on an algebraic equation or inequality. The last level, Level 5, used advanced mathematics concepts and complex, multiple-step word problems such as evaluating functions (Bozick, Ingels, & Owings, 2008). Level 5 captured the skills that are in the curriculum for advanced mathematics courses (NCES, 2005). The levels were hierarchical; that is, students were likely proficient at lower levels before they worked toward proficiency at higher levels (NCES, 2007). However, this did not mean that they lacked any knowledge of higher-level mathematics if they reached proficiency only at lower levels.

Independent Variables
To examine the mechanisms related to the longstanding African American-White achievement gap, race was used as a main independent variable. Race was self-reported by the student. In these analyses, race was interpreted to be a marker of a social location (O'Connor, Lewis, & Mueller, 2007). As such, in this study, race captured past discrimination experiences along with the social identity that the student formed based on micro-interactions around race (O'Connor, Mueller, Rivas, Lewis, & Rosenberg, 2011) and how the student responded and interacted with the socially constructed racial hierarchy (Martin, 2009).

Advanced mathematics course taking was defined from the student transcript data, which indicates the highest level mathematics course that the student took in high school. Those students considered to be advanced mathematics students were those that have taken trigonometry/pre-calculus or calculus as their highest mathematics course taken in high school.

Background Variables
To account for other student characteristics related to student learning, student background characteristics were included as control variables: reading scores, gender, family structure, family socioeconomic status (SES), and educational expectations.

To account for academic ability beyond math, tenth grade reading scores were included as a control variable. SES was a composite variable that included family income, mother/female guardian's education and occupation, and father/male guardian's education and occupation. To account for the family home environment, a family resource scale was constructed using (a) family receives a magazine, (b) family has a computer, (c) family has internet access, (d) family has a DVD player, (e) family has an electric dishwasher, (f) family has a clothes dryer, (g) family has at least 50 books, (h) family has a fax machine, (i) family receives a newspaper, and (j) the student has his/her own room. A factor analysis of the 10 variables indicated that a one-factor solution is appropriate for the variables. The scale was weighted for each item based on the factor loading and has an alpha reliability of 0.662. Finally, student, parent, and mathematics teacher educational expectations for the student (don't know, less than bachelor's degree [reference], bachelor's degree, and more than bachelor's degree) were included since educational expectations are likely related to students' course taking pattern. Expectations have been a common control variable when examining achievement outcomes (e.g. Carbonaro & Covay, 2010).

School Composition
In order to account for the school context, school variables were included, such as racial composition of the student body. The ELS data contained measures of percentage of minority students from the Common Core of Data. Percent minority came from the 2001–2002 school year, when the sample students were in the 10th grade. School SES composition (percentage of students eligible for free lunch) was also included. Both school composition variables were centered at their means.

Missing Data
Multiple imputation (MI) was used to deal with missing data for the independent variables. MI created multiple data sets ($m= 5$), which allowed more randomness to enter into the model for the prediction of standard errors than does single imputation (Royston, 2004). The use of MI allowed this study to maintain larger sample sizes and statistical power in the analyses. The use of MI accepted the assumption that the values on imputed variable were missing at random, conditional on other observed characteristics included in the imputation model.

Analyses
One of the first steps in examining achievement gaps was to look at bivariate relationships to see how African American and White students differ descriptively. For the main analytic technique, ordinary least squared regression was used when the 12th grade test score was the outcome of interest. When mathematics skill subdomains were the outcome measures, generalized linear modeling with logit as the link function was used. While the proficiency probabilities were continuous from 0 to 1, they are also bounded, making Ordinary Least Squares Regression (OLS) not the appropriate method. All regressions were clustered by school to account for students being nested within schools (see Downey, Ainsworth, & Qian, 2009).

Results

To what extent were there racial differences in mathematics outcomes within course taking categories?
African American advanced mathematics students had significantly lower scores on multiple measures of mathematics outcomes. In 10th grade, the average African American advanced mathematics student had a mathematics test score of 41.63, which was almost 13 points lower than the test score for the average White advanced mathematics student in 10th grade (see Table 1). In 12th grade, the African American-White

mathematics test score gap among advanced mathematics students remained 13 points, which means that African American and White advanced mathematics students were making similar gains in mathematics between 10th and 12th grades. However, African American advanced mathematics students continued to have significantly lower mathematics test scores in 12th grade.

There were also statistically significant differences when examining the 12th grade mathematics test by mathematics skill subdomain. White advanced mathematics students were significantly more likely to be proficient for each of the mathematics skill subdomains. For example, for the use of fractions, decimals, powers, and roots (Level 2) White advanced mathematics students were 8.9 times as likely to be proficient in this subdomain. White advanced mathematics students were also 5.3 times as likely to be proficient at using multiple steps when solving problems and using intermediate mathematics concepts (Level 4) compared to African American advanced mathematics students. The other mathematics skill subdomains fell between these two in terms of the proficiency differences between White and African American advanced mathematics students.

When students were divided specifically by their course, the same pattern existed (see Table 2). African American students who took trigonometry/pre-calculus as their highest mathematics course gained 5.85 points between their 10th grade and 12th grade mathematics tests, and White students who took trigonometry/pre-calculus as their highest mathematics course gained 5.84 points. Both African American and White calculus students gained about 7 points between the two tests. Despite similar gains in test scores, African American students still had lower test scores in 12th grade compared to White students whether they took trigonometry/pre-calculus or calculus as their highest mathematics course. Interestingly, African American students who took calculus as their highest level of mathematics had a similar 12th grade mathematics test score to White students who stopped their course taking with trigonometry/pre-calculus. The average African American student who took calculus as his/her highest mathematics course had a 12th grade test score of 58.69, which was only slightly higher than the 58.00 score of the average White student who took trigonometry/pre-calculus as his/her highest mathematics course.

A similar pattern existed for student mathematics skill subdomains. Ninety-four percent of African American students taking calculus as their highest mathematics course were proficient at Level 3 compared to 92% of White students who took trigonometry/pre-calculus as their highest mathematics course who were proficient at Level 3. Another way to compare the racial differences in mathematics skill subdomain proficiency was to compare the odds of being proficient. White students who took trigonometry/pre-calculus were 7.5 times as likely to be proficient at Level 3 than African American students taking trigonometry/pre-calculus. Similar percentages of African American students who took calculus and White students who took trigonometry/pre-calculus were proficient at more advanced mathematics skills (i.e., Levels 4 and 5). Moreover, White students were 4.5 to 6 times as likely than African American students to be proficient at these advanced mathematics skill subdomains depending on the highest mathematics course taken.

To what extent was the highest course taken in high school related to mathematics outcomes for African American and White advanced mathematics students?
In general, for advanced mathematics students included in this study, more advanced mathematics course taking was related to higher test scores and increased odds of being proficient at mathematics skill subdomains. Students who took calculus as their highest level of mathematics course taking scored 3 points more on the 12th grade

Table 2: *Racial Differences in Mathematics Knowledge by Advanced Mathematics Course Taking*

	Gain		Mathematics Test 12th grade		Level 1 12th grade		Level 2 12th grade		Level 3 12th grade		Level 4 12th grade		Level 5 12th grade	
	African American	White	African American	White	African American	White	African American	White	African American	White	African American	White	African American	White
Trigonometry/ Pre-Calculus	5.85	5.84	47.00***	58.00	.982§	.999	.834§	.980	.618***	.924	.255***	.607	.007***	.033
Calculus	7.52	7.21	58.69***	67.98	.999§	1.00	.985§	1.00	.944§	.996	.604***	.903	.053***	.217

Note: $N_{White} = 2000$ $N_{African\ American} = 620$ for Non-Advanced Mathematics; $N_{White} = 1220$ $N_{African\ American} = 240$ for Trigonometry/Pre-Calculus; $N_{White} = 660$ $N_{African\ American} = 50$ for Calculus; *p<.05, **p<.01, ***p<.001 comparing African American and White students; §The variance for these means are extremely low, thus significance levels for these differences are not noted.

mathematics test compared to those students who took trigonometry/pre-calculus as their highest level of mathematics course taking (see Model 3.1 in Table 3). In terms of intermediate mathematics skills for students who took calculus, the odds of being proficient at Level 3 were 5.9 times as great as the odds for those students who took trigonometry/pre-calculus (Model 3.2). For Level 4 skills, the odds of being proficient at Level 4 were 2.6 times as great as the odds for students who took calculus than those who took trigonometry/pre-calculus (Model 3.3), and for the Level 5 the odds of being proficient were 4.3 times as great as the odds for those who took calculus compared to those who took trigonometry/pre-calculus (Model 3.4).

Table 1 shows that African American advanced mathematics students scored 13 points lower on the 12th grade mathematics test than White advanced mathematics students. Model 3.1 showed that much of the initial significant difference was related to student prior achievement and background characteristics. Additionally, the initial significant racial difference in proficiency at Level 3 mathematics skill subdomains was related to prior achievement and student background (Model 3.2). However, even when student course taking, prior achievement, and background characteristics were taken into account, African American advanced mathematics students were significantly less likely to be proficient at both Levels 4 and 5 mathematics skill subdomains. For African American advanced mathematics students the odds of being proficient at Level 4 were decreased by a factor of 0.628 and by a factor of 0.365 for Level 5.

For each model in Table 3, there were also results for the same model with the inclusion of an interaction term to examine whether the relationship between test scores and calculus depended on student race. None of the interaction terms were statistically significant. Indeed, the odds ratio for the interaction in Model 3.3a was 0.986. An odds ratio of 1 indicates that there was no difference between African American and White students on the relationship between calculus and achievement, specifically at Level 4. This suggests that the average calculus experience was not different for African American and White students.

Discussion
As Riegle-Crumb & Grodsky (2010) found, the current study also found that the African American-White achievement gap existed among advanced mathematics students. More specifically, this study found that African American advanced mathematics students had significantly lower mathematics test scores both in the middle and at the end of high school. Additionally, African American advanced mathematics students were less likely to be proficient at all mathematics skill subdomains or skills levels. As almost all advanced mathematics students were proficient at basic and low level mathematics skills, racial differences in mathematics skills becomes most apparent for intermediate and advanced mathematics skills, which include both simple and complex problem solving. African American advanced mathematics students tended to leave high school with lower proficiency at mathematics skills, but especially intermediate and advanced skills, compared to White advanced mathematics students.

While there are significant differences in the mathematics test scores when advanced mathematics students leave high school, there are not significant racial differences in the gains that advanced mathematics students make between 10th and 12th grades. Trigonometry/pre-calculus students gained about 6 points on their mathematics test between 10th and 12th grades, while calculus students gained about 7 points. On the surface, non-significant racial differences in mathematics gains appears to be a positive finding since it would suggest that there are not racial differences in gains and thus more equality. Even though African American and White advanced students made similar mathematics gains over the last two years of high school, African

Table 3: *Racial Difference in Mathematics Knowledge in 12th Grade Mathematics for Advanced Mathematics Students*

	Model 3.1 Mathematics Test	Model 3.1a Mathematics Test	Model 3.2 Level 3	Model 3.2a Level 3	Model 3.3 Level 4	Model 3.3a Level 4	Model 3.4 Level 5	Model 3.4a Level 5
African American	−1.28*	−1.40*	.766	.783	.628***	.630**	.365**	.406
	(.491)	(.542)	(.246)	(.254)	(.124)	(.133)	(.371)	(.478)
Calculus	3.04***	2.99***	5.87***	6.77***	2.55***	2.63***	4.26***	4.31***
	(.291)	(.304)	(.324)	(.327)	(.104)	(.160)	(.609)	(.129)
African American by Calculus		.592		.711		.986		.832
		(.959)		(.678)		(.279)		(.659)
Control Variables Included	Y	Y	Y	Y	Y	Y	Y	Y
Prior Achievement Included	Y	Y	Y	Y	Y	Y	Y	Y

Note: N=2,170; *p<.05, **p<.01, ***p<.001; Coefficients are unstandardized for OLS regression and presented as odds ratio for the proficiency probabilities. Variables included in the models but not in table include 10th grade mathematics achievement, 10th reading achievement, SES, gender, native English speaker, sibship, family resource scale, family structure, teacher expectations, student expectations, parent expectations, percentage minority, and percentage free lunch.

American advanced mathematics students had both significantly lower 10th grade mathematics test scores and 12th grade mathematics test scores. This suggests that the types of mathematics skills African American and White advanced mathematics students gained across these two years were different. Thus, even though African American advanced mathematics students are keeping pace with their White peers, the African American-White achievement gap is not closing.

The significant racial differences in mathematics test scores and mathematics skill levels existed among trigonometry/pre-calculus students as well as calculus students. African American students who took trigonometry/pre-calculus or calculus as their highest mathematics had significantly lower 12th grade mathematics test scores and were significantly less likely to be proficient at each mathematics skill level than White students whose highest mathematics course taken in high school was the same. However, both African American and White students who took calculus as their highest mathematics course tended to have larger mathematics gains, higher test scores, and were more likely to be proficient at each mathematics skill level than their same-race peers who took trigonometry/pre-calculus as their highest mathematics course.

Interestingly, African American advanced mathematics students who took calculus as their highest mathematics course in high school tended to have mathematics test scores and proficiency in mathematics skill levels similar to White students who took trigonometry/pre-calculus as their highest mathematics course. In other words, White students who took a less advanced mathematics course tended to leave high school with the skill level of African American students who took a more advanced course as their highest mathematics course. This implies that White students who end their high school mathematics course-taking career with trigonometry/pre-calculus are similarly prepared for the next level of mathematics courses in college as African American students who take calculus.

Part of the African American-White gap in mathematics outcomes is explained by student prior achievement and background characteristics. When African American advanced mathematics students have the same prior achievement and background as White advanced mathematics students, there are no differences in their mathematics outcomes. However, this is only the case for some of the mathematics outcomes, specifically the 12th grade mathematics test and Level 3 mathematics skills subdomain. For both Levels 4 and 5 of the mathematics skills subdomains, African American students were significantly less likely to be proficient than similar White students.

These findings from the multivariate analyses raise two important issues. First, once again the lack of robust significant differences between African American and White students on some of the mathematics outcomes, once prior achievement and other control variables were included, appeared to be a positive finding since it would suggest a lack of racial achievement gap. When one thinks about this more closely, it means that when African American and White advanced mathematics students are similar in other ways, there are not significant differences in test scores. However, Table 1 clearly shows that the average African American advanced mathematics student was significantly different from the average White advanced mathematics student on several key factors. For example, the average African American advanced mathematics student had significantly lower prior test scores than the average White advanced mathematics student. Moreover, the average African American advanced mathematics student came from a family with lower socioeconomic status, fewer family resources, and a home less likely to be intact than the average White advanced mathematics student. Put differently, the average African American advanced mathematics student did not have similar prior achievement or background characteristics as the average White

advanced mathematics student. Few African American advanced mathematics students are actually similar to the average White advanced mathematics student, which suggests that there is still racial inequality in high school mathematics outcomes.

The second issue raised by the findings of the multivariate analyses also indicates that there is racial inequality in high school mathematics outcomes. Even when accounting for prior achievement and student background characteristics, African American advanced mathematics students were less likely to be proficient at Levels 4 and 5 of the mathematics skills subdomains. Despite having similar course taking and background characteristics, African American advanced mathematics students leave high school with less proficiency at solving problems and intermediate and advanced mathematics skills.

While this study focuses on the last years of high school, past research has shown that the achievement gap is present when students enter school (Lee & Burkam, 2002). The current study adds to the existing evidence that the achievement gap does not close during K-12 schooling but also offers suggestions for how to help reduce the African-American-White achievement gap. Based on the results, one knows that there are significant gaps in skills level for advanced mathematics students in simple problem solving and using low-level mathematics concepts and beyond. If more attention and remediation were focused on these skills earlier in middle school or high school and/or there was an increase in the use of standards-based grading, one may find that the achievement gap is reduced upon exit from high school.

The findings of this study make an important contribution to the research on the African American-White achievement gap and call into question why this gaps exists among our most promising mathematics students, particularly promising African American students. Perhaps the increased focus on standards and teacher effectiveness in education will equalize students' opportunities to learn within the classroom. However, past research has found that policies that promote standards do not always increase equality within the classroom (Rowan, Correnti, Miller & Camburn, 2009). In order to have a better understanding of why racial differences remain, it would be important to know whether or not these differences remain when focusing on students within the same classroom or between classrooms. This would allow for a better understanding of student opportunities to learn as well as teacher effectiveness. Unfortunately, this question cannot be answered with this data. The Measures of Effective Teaching data may be able to provide insight into these questions as it includes extensive measures of teacher effectiveness for many teachers and classroom content coverage for a smaller sample (White & Rowan, 2012).

Despite not being able to capture what is occurring in the classrooms, it is important to consider the mechanisms that may explain these patterns. It is likely that the actual explanation is the result of multiple mechanisms in combination, which may include but are not limited to: students' experiences within the classroom (e.g. teacher characteristics [Ball, 1991; Ferguson, 1998; Goldhaber & Brewer, 1997; Hill, Rowan, & Ball, 2005; Kelly, 2004; Lankford, Loeb, & Wyckoff, 2002; National Science Board, 2008; Phillips, 2010], student sensitivity to teacher effects [Ferguson, 1998], content coverage [Metz, 1990; Porter, 1991; Sandholtz, Ogawa, & Scribner, 2004; Spillane & Burch, 2006; Stecher, Hamilton, & Gonzalez, 2004; Covay Minor, 2015], classroom composition [Hoxby, 2002; Lomi et al., 2011; Covay Minor, 2015]), student prior knowledge, and student understanding of content. However, the interaction terms in Table 3, especially for Model 3.3a, suggest that African American and White students are not differentially experiencing the calculus classroom. While other work has found significant differences in calculus classrooms by the racial composition of the classroom (Covay

Minor, 2015), that does not appear to be the case here. Instead, the significant racial differences in mathematics levels are likely an artifact of student prior characteristics.

In addition to student classroom experiences and student prior characteristics, African American and White students may differ on social psychological characteristics such as feelings of isolation. Tyson's (2011) careful qualitative analysis of racialized tracking found that high-achieving African American students have to deal with isolation that comes from being in advanced classes, and that how students deal with the isolation depends on their sense of self. The interaction terms in this study suggest that African American and White students are experiencing the classroom in similar ways, but this does not capture how their non-advanced mathematics class peers treat them in other situations or how the students respond to their peers. Unfortunately, the current study cannot capture this isolation or sense of self to test whether this can explain any of the remaining racial differences in mathematics scores. However, this would be an important area of future research, not only in terms of gaining a better understanding of the isolation and sense of self for African American advanced mathematics students and the relationship to achievement, but also in terms of what can be done at the classroom and school level to decrease isolation and increase sense of self.

While this study adds to our understanding of the African American-White achievement gap, the data used for the study is not without limitations. In addition to not being able to include measures for the mechanisms that may help explain the gap, the data also focuses on high school students from the early 2000s. Nationally representative data for more recent cohorts of high school students' transcript data have not been released (NCES, n.d.). While it will be important to continue to examine these questions with more recent cohorts of students, this study does draw attention to the need for such research.

Conclusion

While advanced mathematics course taking is related to students being more likely to complete college (Adelman, 1999), the advantage of course taking may be symbolic rather than an actual in terms of skills. In other words, there is likely a disconnect between student course-taking records and the resulting skill level. African American advanced mathematics students leave high school with a smaller range of mathematics knowledge than do White advanced mathematics students, perpetuating an African American-White mathematics skills gap. Mathematics achievement can and should be measured not only by overall test scores but also by skill levels to provide more insight into where along the mathematics knowledge continuum the achievement gap exists so that gaps in knowledge can more specifically be addressed. In order to be able to reduce these inequalities, one needs to have a better understanding of where to focus interventions (Crosnoe & Schneider, 2010). To be able to focus interventions more effectively, more research should attend to the achievement gaps at the high end of the achievement distribution as these students have great promise.

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