

The Effect of Dynamic Geometry Software and Physical Manipulatives on Candidate Teachers' Transformational Geometry Success

Gül Kaleli Yılmaz^a

Bayburt University

Abstract

This study aims to investigate the effects of using Dynamic Geometry Software (DGS) Cabri II Plus and physical manipulatives on the transformational geometry achievement of candidate teachers. In this study, the semi-experimental method was used, consisting of two experimental and one control groups. The samples of this study were 117 students. A 30-question test which was prepared based on the relative literature and expert opinion was used as a data collection tool. The test includes sections on recognition, features, and construction, and each section consists of ten questions. The data obtained from the pre- and post-tests were analyzed using the SPSS program. As a result of the statistical analysis, success levels of all groups were found to be the same before the applications, but after the applications, students' transformational geometry success was found to significantly increase. When group success was analyzed, it was seen that the Computer Group placed first, the Manipulatives Group placed second, and the Traditional Group placed third in the sections of recognition and features. In the construction section, the Computer and Manipulatives Groups' success levels were equal, and both groups were significantly more successful than the Traditional Group. The exercises performed with all groups increased their success levels significantly, thus showing that the applications were effective.

Keywords: Cabri II Plus • Candidate teachers • Geometry success • Physical manipulative • Transformational geometry

a Correspondence

Assist. Prof. Gül Kaleli Yılmaz (PhD), Bayburt University, Faculty of Education, Department of Elementary Education, Bayburt Turkey

Research areas: Technology-assisted mathematics; Geometry teaching; In-service training

Email: gyilmaz@bayburt.edu.tr

Mathematics, having an important place in human life and playing a great role in developing many human cognitive capabilities, is divided into various subject areas. One of these subject areas is geometry (Kurak, 2009). Geometry consists of geometric objects, shapes, and their features and relationships to each other (Toptaş, 2008). Geometry helps students become closely acquainted with the world they live in. For example, the shapes of rooms, their construction and trim work forms are geometrical (Baykul, 2002). According to Struchens, Harris, and Martin (2003), students start to understand the world around of them, can analyze problems, and in order to understand intangible symbols better, define them by shapes (as cited in Gülten & Gülten, 2004). On the other hand, the National Council of Teachers of Mathematics (NCTM, 2000) dwells on the importance of geometry for the principles and standards of school mathematics, and it focuses on the fact that geometry develops the reasoning and proof-finding abilities of students. Jones (2002), on the other hand, states that geometry includes interesting problems and surprising theorems, and this supports students in developing their abilities of visualization, critical thought, instinctive reasoning, perspective, estimation, logical inference, deductive reasoning, and proof-finding. For this reason, geometry is an important subject area which should be addressed from pre-school throughout higher education (Goos & Spencer, 2003). With the implementation of the mathematics program in 2005, some subject areas started to become more prominent. One of these subjects is transformational geometry (displacement, reflection, and rotational transformation) (Güven & Kaleli-Yılmaz, 2012).

Transformational geometry, which improves students' geometric experimentation, imagination, reasoning, and three-dimensional perception skills, consists of reflection, displacement, and rotation (Fletcher, 1973; Gürbüz, 2008; Milli Eğitim Bakanlığı [MEB], 2005; NCTM, 2000; Soon, 1989). According to Knuchel (2004), people need knowledge of transformational geometry in order to develop qualitative senses about the external world as well as to organize objects and events. Students can establish a connection between art and mathematics thanks to the information they receive on transformational geometry; they can realize the importance of mathematics in daily life. Additionally, seeing geometric figures rotated, translated, and repeated (as in carpet patterns) helps the student who has knowledge about this topic to view things differently (Duatepe & Ersoy, 2001). For this reason the topic of transformational geometry should be taught to students from childhood, and it

should be emphasized that reflection, displacement, and rotation can be seen in many natural structures and events.

It is very important for primary school students to learn basic knowledge about transformational geometry and continue their education successfully in the years that follow. As Carroll (1998) stated, students who gain effective experience with geometry in primary school are able to apply reasoning to situations which contain geometry in secondary school. For example, reflection transformation relative to a line is used to teach analytical geometry, the following years' topic, and rotational transformation is used to teach solid-body volume. Moreover, transformational geometry basically forms a basis for the concepts of functions, a concrete foundation for vectors, and the formulation of the similarity theorem, making the world mathematical (Schuester, 1973). Transformational geometry should be taught to students beginning at childhood in order to transform their knowledge into conceptual and concrete understandings; teachers should help students understand the topics of reflection, symmetry, and rotational transformation correctly.

The topic of transformational geometry is not only in mathematics but is also included in other disciplines. A trace of transformational geometry is seen in the physics topic of optics and waves, the medical science of human anatomy, and in biology with the symmetrical structure of DNA as well as mitosis, or symmetrical cell division (Aksoy & Bayazit, 2010). As transformational geometry sees considerable use in mathematics, physics, biology, and so on, it is a part of daily life. Because of its importance, the topic of transformational geometry has been included in primary school curriculum and teachers should be responsible for teaching this topic effectively.

The Place of Transformational Geometry in Primary School Mathematics Curriculum

When one investigates the first primary education mathematics program implemented in Turkey in 2005, the basic topic of primary school transformational geometry was structured for second graders around the concept of symmetry. As is known, recognizing symmetry is the basis for further transformational geometry studies. Within the standards of geometry from pre-school through to the end of high school, it can also be understood from this statement that the application of transformation and use of symmetry is intended for

analyzing mathematical conditions, hence studies on symmetry have been discussed in detail within transformational geometry (NCTM, 2000). During middle school, details and mirroring are discussed

Table 1
Distribution of the Subject of Transformational Geometry in Public School from Second to Eighth Grade and Samples related to it (Güven, 2012, pp. 367-368; MEB, 2005)

Class	Learning Area	Sub-Learning Area	Acquired Skill	Samples
2.	Geometry	Symmetry	To determine whether a shape can be divided into two identical parts or not, and how to divide appropriate shapes into identical halves. Symmetry is explained through models.	
3.	Geometry	Symmetry	Determining symmetry with respect to a line on a shape's plane and what constitutes a symmetrical shape.	
4.	Geometry	Symmetry	Determining symmetrical lines on planar shapes as well as drawing them.	
5.	Geometry	Symmetry	Drawing the symmetry of a planar shape with respect to a given line of symmetry. Determining the lines of symmetry for polygons as well as drawing them.	
6.	Geometry	Transformation Geometry	Explaining Displacement Movement. How to structure the new shape after it has been displaced.	
				3 unities right translation 3 unities right and 1 unity below translation
7.	Geometry	Transformation Geometry	Explaining reflection. Explaining rotational movements. Making drawings by rotating shapes around a point on the plane according to a defined angle.	
8.	Geometry	Transformation Geometry	Drawing many views of polygonal reflection from an axis on the coordinate plane; defining displacement along the line and rotating around the origin. Determining the symmetries of geometric objects. Determining displaced reflection of shapes and structuring them.	

in transformational geometry; displacement and rotational transformations are given priority. Table 1 shows how the subject of transformational geometry is taught between second and eighth grades.

Transformational Geometry and Primary School Teachers

Students' geometric mentality and its improvement is closely related to the education received in primary school. There are many factors involved in this education, but the most important one is teachers. Teachers play the role of implementer and they have the greatest responsibility for reaching the intended goals of education. At this point, a teacher's knowledge, skills, and abilities come to the forefront. As is known, teachers who lack knowledge have a negative effect on students (Ball, 1990). No matter how well educational goals are determined or how functional and organized the topics are, it is obvious that these goals cannot be achieved unless they are carried out by discerning teachers (Köseoğlu, 1994). For this reason, teachers should first have wide knowledge about transformational geometry and know different methods of approach in order to teach the topic of transformational geometry effectively.

Primary school mathematics curriculum was renewed in Turkey in 2005, and the topic of transformational geometry was added to the curriculum. Naturally, most teachers who are now teaching transformational geometry were not trained for the current primary school curriculum; they were not educated for teaching transformational geometry in primary school. We can say, namely, that the topic of transformational geometry is new to both mathematics classes and class teachers. It has become important for teachers to reach a sufficient level of experience on this topic during their undergraduate education. Because of this importance, the current study aims to determine what effect transformational geometry, the DGS Cabri II Plus program, physical manipulatives, and traditional methods have on the success of students studying to teach transformational geometry in primary schools.

Dynamic Geometry Software (DGS) and Transformational Geometry

NCTM (2000) accepted the use of technology in mathematics education to be one of the principles and standards of mathematics education, and established this by saying "technology affects

mathematics learning and teaching, and mathematics which is taught with technology improves student learning" (p. 11). In Turkey, particular importance is given to the place of technology in the renewed primary school curriculum and it is emphasized that technology, especially DGS, should be effectively used at every level (MEB, 2005). Using DGS is an especially popular technology in geometry education because this kind of software encourages learning by discovery, contributing to the problem-solving skills of students (Ubuş, Üstün, & Erbaş, 2009).

The most important feature of DGS is that it allows shapes to be dragged while protecting the basic shape's structure, their points and lines (Hazzan & Goldenberg, 1997). When an original shape is dragged, the resulting transformations and formations which were implemented on these shapes can be immediately reviewed on screen. Thus, students have an opportunity to discover practically anything by being able to easily plug into the search environment, hypothesizing, testing, formulating, and explaining (Güven & Karataş, 2005). NCTM states the role of DGS in understanding transformations as follows (Güven, 2012, p. 365):

"Dynamic geometry software allows students to visualize a transformation by manipulating a shape and observing the effect of each manipulation on its image. By focusing on the positions, side lengths, and angle measurements of the original and resulting figures, middle-grades students can gain new insights into congruence. Transformations can become an object of study in their own right. Teachers can ask students to visualize and describe the relationship among lines of reflection, rotational centers, and the positions of pre-images and images. Using the interactive figure, students might see that the result of a reflection is the same distance from the line of reflection as the original shape (NCTM, 2000)."

Cabri II Plus is one of the first dynamic geometry software programs (Gillis, 2005). DGS Cabri strengthens mathematical thought by changing mathematical objects on screen like a tool. As one can define that some elements of geometry are changeable, some are stable and some can be defined according to another, this software gives one the opportunity to examine geometry dynamically when structures are moved accordingly (Baki, 2001). As the mobile structure of Cabri II Plus makes dragging and rotating geometrical shapes available, it is thought to be an effective tool for

teaching transformational geometry, a topic of the renewed primary school curriculum.

When research on transformational geometry is investigated, it can be seen that students have inefficient information on this topic and difficulty learning it (Battista, 1999; Küchemann, 1981; Yavuzsoy-Köse, 2008; Zembat, 2007). In the literature, DGS has been determined as an effective tool for overcoming these difficulties (NCTM, 2000; Van De Walle, 2004). It is specified that Cabri DGS has an effective role among the dynamic geometry programs for teaching transformational geometry (Dixon, 1997; Güven, 2012; Güven & Kaleli-Yılmaz, 2012; Hoyles & Healy, 1997; Kurak, 2009; Yavuzsoy-Köse, 2008).

DGS software for transformational geometry, especially Cabri II Plus, can be said to increase success and conceptual understanding from this point of view. Teachers should first, however, be informed about this software, and model implementation should be done in order to see how this holds up in practical applications. For that reason, one of the groups in this study participated as a computer group, and Cabri II Plus was firstly introduced to the teacher candidates; they carried out applications on the software, then the study investigated its effects on their transformational geometry success.

Why Cabri II Plus?

In this study, in order to proceed with the computer group's practice, the DGS Cabri II Plus was used. For all intents and purposes, different software programs such as GeoGebra or Geometry Sketchpad could have been used instead of Cabri II Plus. Hence, the many other studies that have been performed all found an increase in geometry achievement with software no matter which program was used (Dixon, 1997; Harper, 2002). In the scope of this study, the reason for selecting Cabri II Plus was because the researcher has comprehensive experience with the current software and it has a user-friendly Turkish version with easy-to-use menus.

Physical Manipulatives and Transformational Geometry

A manipulative is a concrete model that comprises mathematical concepts with respect to several aspects that can be touched and moved around by the learner (Absi & Nofal, 2010). Concrete models such as geometry rods, geo-boards, isometric paper,

symmetry mirrors, and so forth, are supposed to help students construct geometric ideas (Durmuş & Karakırık, 2006). Teaching activities in which manipulatives are used give students the chance to make observations and apply them to concrete models of the target and their representations (Heddens, 1997; Suydam & Higgins, 1984).

NCTM's (2000) standards and principles which were published on mathematics education show the importance of using a physical, concrete manipulative, which plays a role in structuring the mental process of students in learning mathematics. According to Moyer (2001), a physical manipulative is designed to represent mathematical terms, and it is a visual and movable object which can move the students' senses. From this point of view, we can say that physical manipulatives which can be used for transformational geometry are defined as activities such as symmetry mirrors, using dotted and lined paper, origami, and drawings on paper.

Studies on this Topic

When reviewing the literature, many studies performed on transformational geometry subject stand out. The important part of these studies is that they used different dynamic software such as Cabri II Plus, Geometry Sketchpad, and GeoGebra; according to the results of these studies, it was seen that the use of software is effective for increasing transformational geometry skills (Akgül, 2014; Dixon, 1997; Egelioglu, 2008; Gürbüz, 2008; Güven, 2012; Güven & Kaleli-Yılmaz, 2012; Harper, 2002; Karakuş, 2008; Yavuzsoy-Köse, 2008; Yazlık, 2011). These studies also generally preferred to use the experimental method consisting of unique experiments and a control group. In these studies, the experimental group used the dynamic software program and the control group is learned their lesson using conventional methods. An important part of these studies is that the students in experimental groups who were taught using dynamic software were more successful than students in the control groups who were taught using conventional methods (Egelioglu, 2008; Güven, 2012; Güven & Kaleli-Yılmaz, 2012; Karakuş, 2008; Yazlık, 2011). For example, Yazlık (2011) performed an experimental study with 135 seventh-grade students to find out whether teaching geometry with Cabri II Plus has any effect on how students learn transformational geometry. The experimental group lesson was taught with Cabri II Plus and the control group used traditional methods. At the end of this research

both experimental and control groups' success were seen to rise, but the experimental group had higher success levels than the control group. Similarly, Güven's study (2012) searched the effects of DGS (Cabri II Plus) on the success of eighth-grade students with transformational geometry. In that study, the experimental method consisted of using experiment and control groups. As a consequence of that study, it was determined that students in the experimental group who were taught with DGS (Cabri II Plus) were more successful than students in the control group who were taught their lesson with dotted paper and isometric worksheets.

In the body of literature, one frequently only encounters unique experiment-control group experimental studies; experimental studies consisting of two experimental groups are limited. Furthermore, it was noticed that routine achievement tests were used in those studies, and the tests did not focus on certain sections such as defining transformations, stating transformation features, or forming transformations. In this case, this study differs from others because it makes use of two experimental groups and one control group, and the achievement test consists of three separate sections: defining, stating features, and constructing.

The Importance and Aim of the Study

In Turkey, the mathematics education program for primary schools was revised in 2005, and the subject of transformational geometry was convolutedly added to the program with this revision. As a result, the teachers who now teach transformational geometry in schools have limited knowledge and experience with transformational geometry because they hadn't been educated in respect to the revised mathematics education program while they were studying to be teachers. When the mathematics education program is investigated, the first part of primary education (between first and fifth grade) forms the base of transformational geometry and is structured through the concept of symmetry. Recognition of symmetry is known to be a base for the study of transformational geometry. As such, the teachers who will teach symmetry should have adequate information and skills in that subject. For this reason, important duties fall on the Faculty of Education. The Faculty of Education should give the necessary experience on transformational geometry to teacher candidates through Basic Mathematics, Mathematics Education and other pre-services courses (as Teaching Methods, Teaching Practice). How to make this method more successful

should be determined by trying many kinds of methods instead of only one. Aside from the many studies performed that prove DGS is effective in increasing success in transformational geometry, it was not certain which section this impact was more effective on: defining transformations, stating the transformation features, or forming a transformation. This study is important for filling in the gap in the literature about this topic. Another important point of this study is to also find an answer to the question of which section of transformational geometry success is DGS (Cabri II Plus) and the use of tangible materials more effective.

This study intends to determine how teaching transformational geometry using different methods affects transformational geometry achievement. In this scope, the following questions are addressed:

- i) Does DGS-based instruction affect the academic achievement of pre-service primary school teachers as far as transformational geometry?*
- ii) Does instruction based on physical manipulatives affect the academic achievement of pre-service primary school teachers with regard to transformational geometry?*
- iii) Is there a significant difference among groups related to their academic achievement on transformational geometry?*
- iv) Are there any significant differences between the different groups' achievements with respect to recognition, features and construction?*

Method

Model of the Research

A quasi-experimental design was used to determine the effects of DGS-based instruction, physical manipulative-based instruction and traditional-based instruction on the transformational geometry academic achievement of freshman pre-service primary school teachers.

Participants

This study must be performed with teacher candidates who are preparing to teach 1st, 2nd, 3rd, or 4th grades due to the aim of studying the effect of dynamic geometry software and physical manipulatives on teacher candidates' transformational geometry success. In this study, freshman teacher candidates were selected to make up the practice groups via the purposive sampling

method. Freshman teacher candidates were chosen due to the requirements of this method. In research using this method, the experimental and control groups need to be selected at random. As a result, one of the other three grades has the probability of being selected as the Computer Group. For this reason, all participating teacher candidates in this study should be technologically literate and sufficiently able to use a computer. Freshman teacher candidates were considered to have more up-to-date information about computers because they had taken Computer I and Computer II classes. On the other hand, thanks to the Basic Mathematics I and Basic Mathematics II classes they had taken, they were seen to have gained the basic skills for understanding transformational geometry. As a result, 117 freshman teacher candidates selected through the purposive sampling method constituted the participants of this research.

In order to increase both internal and external validity in the scope of this study, groups were selected at random and any biased behaviors were not considered. After the random selection of teacher candidates, the Primary School Teacher Department Formal Training Class A was chosen as the Computer Group, the Night Training Class A was chosen as the Manipulatives Group, and Formal Training Class B was chosen as the control group. A total of 43 students comprised the DGS-based instruction group (Computer Group), 36 students comprised the physical manipulatives-based instruction group (Manipulatives Group), and 38 students were in the traditional instruction-based group (Traditional Group).

Instrument

The transformational geometry achievement test was used in this study as the data collection tool. While developing the achievement test, the body of literature was searched to form ideas for potential questions to ask about transformational geometry. Next, by collecting the current questions, an item pool was formed. Because transformational geometry success was requested to be investigated separately for the sections of recognition, features, and construction, 15 questions for each dimension were selected from the item pool. Selected questions were investigated by a specialist academician. According to his opinion, 10 questions were considered for each dimension. The Transformational Geometry Achievement Test (TGAT) with a total of 30 questions (short answer, simple illustrative, elective) over 3 dimensions, 10 questions per dimension, was

developed by eliminating questions that had similar features or that were abstruse. The developed TGAT was controlled in terms of comprehensibility and readability by an academician who is a Turkish language specialist. After incoherent and incorrect points were corrected, the TGAT was put into a final form. An explanation of each section of the TGAT is presented below.

Recognition: In this part, a total of ten shapes were given in mixed order. These shapes were about symmetry according to point, symmetry according to line, displacement, rotation, and symmetry axis; two shapes were given for each aspect that takes place in transformational geometry. Students were asked to write which transformation is being applied in the blanks under the shapes. In Figures 1 and 2, examples are given of the questions asked in the recognition section.

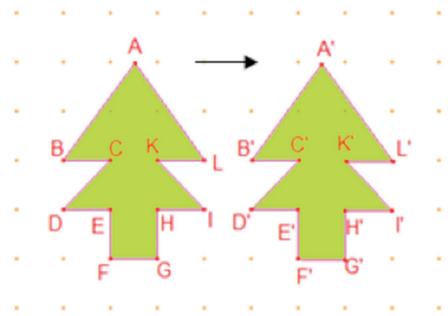


Figure 1: What is the transformation in figure 1?

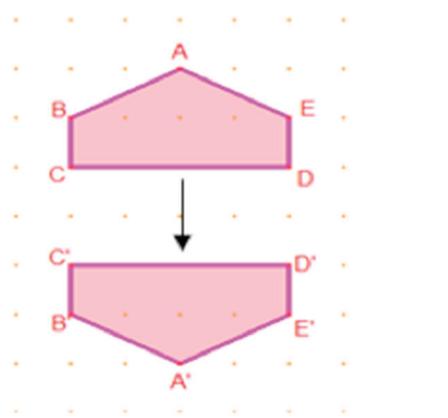


Figure 2: What is the transformation in figure 2?

Features: In this part, ten mixed features of transformational geometry are given and students were asked to write which feature belongs to which transformation in the space provided.

Transformation of..... direction is when the form, size, and shape are the same.

Transformation of..... is the image of the shape in the mirror.

Construction: In this part, ten transformations were given to students and they are asked to perform these transformations on the shapes under each question. Two examples which belong to this part are given in Figures 3 and 4.

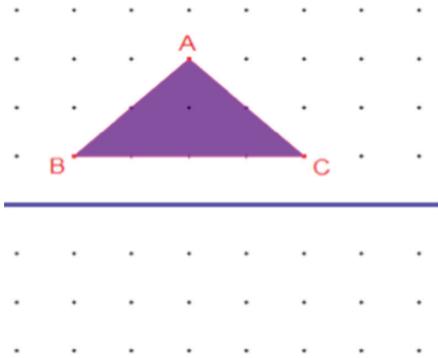


Figure 3: Reflection according to the line.

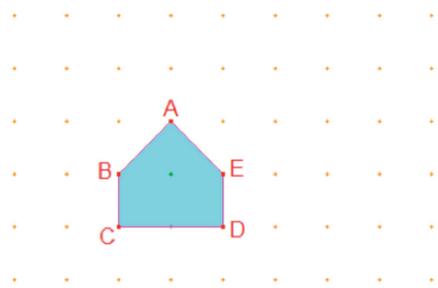


Figure 4: 2 units to the right displacement.

Procedure

Treatment of the Computer Group: Before treatment, students in the Computer Group were given six hours of training on Cabri II Plus because it was new to them. In this process, all toolbars in Cabri II Plus were introduced to the students and they were taught how to form structures using Cabri II Plus. During the treatment, the students received instruction in the computer laboratory. They individually studied transformational geometry topics by using Cabri II Plus with worksheets along with guidance from the teacher, who was also the researcher.

In the Computer Group, practice was performed by means of some work sheets as shown in Figure 5 using Cabri II Plus. The worksheets that were used in this group included extra directions that required the use of Cabri II Plus. In Appendix 1, there is an example of the worksheet used by the Computer Group.

Treatment of the Manipulatives Group: The students in the Manipulatives Group did not receive any special training before the treatment because using manipulatives is straight forward. Students in the Manipulatives Group studied transformational geometry in a classroom environment enriched with physical manipulatives such as symmetry mirror, dotted and lined papers, and origami. The students in this group also studied using worksheets. However, in contrast to the worksheets used in the Computer Group, the directions in these worksheets focused on the use of manipulatives. A total of five worksheets were used for each of the experimental groups. In Appendix 2, there is an example of the worksheet used by the Manipulatives Group.

Treatment of the Traditional Group: The Traditional Group (control group) received

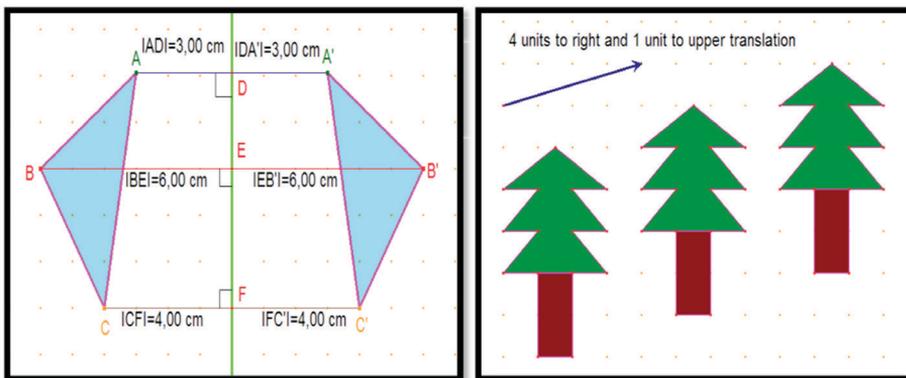


Figure 5: Explaining reflection according to line and displacement transformations on Cabri II Plus.

traditional-based instruction. Here, the students received teacher-centered instruction. The teacher drew and explained transformational geometry on the blackboard. The students tried to answer the teacher's questions in some parts of the lessons. As students were individually solving problems, the teacher would solve it on the blackboard. The contents of the course are presented in Table 2.

Table 2
Transformational Geometry Course Contents

Week	Course Content
1st week	<ul style="list-style-type: none"> - Identification of transformational geometry, informing students about its importance, and giving example from daily life. - Explaining the term <i>symmetry axis</i> and determining the axis of different geometric shapes.
2st week	<ul style="list-style-type: none"> - Expressing the topic of <i>reflection according to line</i>, giving examples from nature, and taking the reflections of different geometric shapes according to lines (vertical, horizontal and diagonal).
3st week	<ul style="list-style-type: none"> - Explaining <i>reflection according to point</i>, giving examples from nature, and taking the reflection of different geometric shapes according to different points.
4st week	<ul style="list-style-type: none"> - Explaining the term <i>displacement</i>, giving examples from nature, and performing displacements of different shapes for different sizes and directions.
5st week	<ul style="list-style-type: none"> - Explaining the term <i>rotation</i>, giving examples from nature, and performing rotations of different geometric shapes around stated points at stated angles, clockwise and counter-clockwise.

Students from all groups studied transformational geometry for five weeks, two hours per week. In both the experimental groups and control group, applications were managed by the same teacher, the researcher. Every group tried to solve almost the same number of examples. After completing all exercises, the TGAT was applied to the students.

Data Analysis

The TGAT consisted of 30 questions on transformational geometry. Ten of them were about recognition, another ten were about features, and the last ten were about construction. Every correctly answered question was scored as 1 point and every incorrectly answered question was

given zero points. For example, a student correctly answered five questions about recognition, three questions about features, and two questions about construction, he will receive 5 points for the recognition section, 3 points for the features section, and 2 points for the construction section, a total of 10 points. A student can receive a minimum score of 0 and a maximum score of 30.

The SPSS statistical package program was used to analyze the data obtained from the TGAT. Before performing exercises, variance analysis (ANOVA) was applied to data obtained from the pre-test to determine whether there was a difference between the groups' transformational geometry achievements. The t-test was applied to data obtained from the post-test. After the exercises were finished, the differences between the pre- and post-test scores of every group were examined. Then, by applying covariance analysis (ANCOVA) to the data obtained from the pre- and post-tests, the different groups' achievements for every section (recognition, features, and construction) were separately determined. The Bonferroni Test, one of the post hoc tests, investigated the differences between groups, looking for which group was favored by this difference. As is known, there are many choices in post hoc tests, and all of their basic functions are the same. In post hoc tests there are basic differences; however, they can give generally similar results. Tukey and Bonferroni are frequently preferred in studies (Kalaycı, 2009). Field (2009) states Bonferroni is the proper choice for ANCOVA. For this reason, the Bonferroni Test was selected as one of the post hoc tests in this study.

Findings

In this part, the findings acquired from analysis of the pre- and post-tests using the SPSS program are given. In Table 3, the result of variance analysis (ANOVA) acquired from the findings regarding the Computer, Manipulatives, and Traditional Groups' pre-tests are given.

Table 3
Results of Descriptive Statistics and Analysis of Variance on the Students' TGAT Scores Before Treatment

Groups	Computer Group			Manipulatives Group			Traditional Group			F
	N	Mean	SD	N	Mean	SD	N	Mean	SD	
Recognition	43	0.70	0.89	36	0.75	0.91	38	0.63	0.91	0.161'
Features		0.88	0.91		0.69	0.78		0.79	0.91	0.464'
Construction		0.54	0.67		0.50	0.61		0.55	0.69	0.061'
Total Test		2.12	2.00		1.94	1.69		1.97	2.26	0.085'

'p > .05.

Table 4
Descriptive Statistics of the Students' TGAT Scores after Treatment

	Computer Group			Manipulatives Group			Traditional Group		
	N	Mean	SD	N	Mean	SD	N	Mean	SD
Recognition	43	8.47	1.67	36	6.83	1.46	38	4.61	1.81
Features		7.77	1.54		6.17	1.38		4.29	1.58
Construction		6.67	1.52		6.42	1.36		2.89	1.29
Total Test		22.90	3.79		19.42	3.87		11.79	4.13

Note. SD: Standard deviation.

It can be observed from Table 3 that for the pre-test, the ANOVA results showed no significant difference in the mean scores among the groups for the recognition section of the test [$F(2,114) = .16, p > .05$], the features section of the test [$F(2,114) = .46, p > .05$], the construction section of the test [$F(2,114) = .61, p > .05$] or the overall test [$F(2,114) = .08, p > .05$]. This shows that there were no statistically significant differences between the transformational geometry achievement of students in the experimental and control groups at the beginning of the course.

The students took the same test again after the treatment. The descriptive statistics for the data obtained from the test after the treatment is presented in Table 4. It may be seen from Table 4 that the mean number of correct answers given by all the students in each section of the test increased in the post-test, regardless of treatment.

It can be seen from Table 4 that that the number of correct answers given by all the students in each section of the test increased in the post-test, regardless of the treatment. In order to determine whether or not differences in the averages of the scores of each group were statistically significant, a paired sample *t*-test was applied to the data obtained from the entire test and its sections, where a level of .05 shows significance. Table 5 summarizes the results of the paired sample *t*-test analysis for the pre- and post-tests.

Table 5
Paired Sample *t*-Test Results of Pre- and Post-Test Scores within Groups

Groups	Computer Group		Manipulatives Group		Traditional Group	
	df	<i>t</i>	df	<i>t</i>	df	<i>t</i>
Recognition	42	-26.288'	35	-21.098'	37	-13.120'
Features		-25.815'		-21.325'		-11.906'
Construction		-24.747'		-23.667'		-10.765'
Total Test		-30.759'		-25.050'		-13.588'

' $p < .001$.

This evaluation suggests that as a result of the treatments, there was a significant difference in the

students' transformational geometry achievement for all groups. This difference was observed with respect to not only the test results as a whole, but also for each section of the test results ($p < .001$ for the Computer Group, Manipulatives Group, and Traditional Group).

Analysis of covariance (ANCOVA) was performed to observe the potential differences between the mean of the post-test scores of the groups. In the application of ANCOVA, homogeneity of variances was first checked, being the basic hypothesis of ANCOVA. Variances were seen to be homogenous, so the basic hypothesis was met. In the next phase, ANCOVA analysis was performed. The data for the experiment was analyzed using a 3x1 (Computer Group, Manipulatives Group, Traditional Group) ANCOVA, with the pre-test as covariate. A

Table 6
Covariance Analysis Results of Each Section of the Test Group by Group

Measures	<i>F</i>	<i>df</i>	Bonferroni's pair-wise comparison	
			Mean Differences	Direction
'Recognition Section' overall	53.53'	2		
CG vs. MG			1.63'	CG>MG
CG vs. TG			3.83'	CG>TG
MG vs. TG			2.20'	MG>TG
'Features Section' overall	52.07'	2		
CG vs. MG			1.58'	CG>MG
CG vs. TG			3.47'	CG>TG
MG vs. TG			1.89'	MG>TG
'Construction Section' overall	86.92'	2		
CG vs. MG			0.25''	-
CG vs. TG			3.78'	CG>TG
MG vs. TG			3.52'	MG>TG
Total Test	81.73'	2		
CG vs. MG			3.50'	CG>MG
CG vs. TG			11.12'	CG>TG
MG vs. TG			7.62'	MG>TG

Note. CG: Computer group; MG: Manipulatives group; TG: Traditional group.
' $p < .001$;' $p > .05$.

Bonferroni's pairwise comparisons test was used to determine the direction of differentiation. Table 6 presents the results of the ANCOVA related to the sections of the test as well as the whole test, including both the overall (3×1) results and each of the pairwise comparisons.

The ANCOVA results showed that there was a significant mean difference in the gain scores of students in the Computer Group, Manipulatives Group, and Traditional Group with respect to the recognition section of the test [$F(2,111) = 53.53$, $p < .05$, $\eta^2 = .49$], where the recognition section of the pre-test was used as a covariate. Bonferroni's pairwise comparisons test revealed that the Computer Group's gain scores were significantly higher than those of the Manipulatives and Traditional Groups (mean difference = 1.63, $p < .001$; mean difference = 3.83, $p < .001$). In addition, the Manipulatives Group's gain scores were significantly higher than the Traditional Group's (mean difference = 2.20, $p < .001$).

There was a significant mean difference in the gain scores of students in the Computer Group, Manipulatives Group, and Traditional Group with respect to the features section of the test [$F(2,111) = 52.07$, $p < .001$, $\eta^2 = .48$], where the features section of the pre-test was used as a covariate. Bonferroni's pairwise comparisons test revealed that the Computer Group's gain scores were significantly higher than those of the Manipulatives and Traditional Groups (mean difference = 1.58, $p < .001$; mean difference = 3.47, $p < .001$, respectively). In addition, the Manipulatives Group scored significantly higher than the Traditional Group (mean difference = 1.89, $p < .001$).

There was a significant mean difference in the gain scores of students in the Computer Group, Manipulatives Group, and Traditional Group with respect to the construction section of the test [$F(2,111) = 86.92$, $p < .001$, $\eta^2 = .61$] where the construction section of the pre-test was used as a covariate. The Computer Group's gain scores were significantly higher than those of the Traditional Group (mean difference = 3.78, $p < .001$). Similarly, the Manipulatives Group scored significantly higher than the Traditional Group (mean difference = 3.52, $p < .001$). However, no significant mean difference was observed in the gain scores between the Computer Group and Manipulatives Group (mean difference = .25, $p > .05$).

There was a significant mean difference in the gain scores of students in the Computer Group, Manipulatives Group, and Traditional Group with

respect to the overall test [$F(2, 111) = 81.73$, $p < .001$, $\eta^2 = .60$], where the pre-test scores of the overall test were used as a covariate. Bonferroni's pairwise comparisons test revealed that the Computer Group's gain scores were significantly higher than those of the Manipulative and Traditional Groups (mean difference = 3.50, $p < .001$; mean difference = 11.12, $p < .001$). In addition, the Manipulatives Group scored significantly higher than the Traditional Group (mean difference = 7.62, $p < .001$).

Discussion and Conclusions

First of all, this study differs from other research studies in the body of literature because it includes two experimental groups and separately searches for transformational geometry achievement according to the sections of recognition, features, and construction. For this reason, it is thought that the results obtained from this research will bring important contributions to the body of literature.

In the scope of this research, the obtained results were discussed through research problems. When the performed analyses were researched, the transformational geometry achievements of the Computer, Manipulatives, and Traditional groups were seen to be approximately equal before the exercises. After the exercises, the transformational geometry achievement of each group was seen to increase significantly according to analyses of the performed tests. This situation is an expected result. Before the exercises, teachers performed the important part of the test at low achievement levels in the pre-test because they didn't have enough information about transformational geometry. During the exercises, all three groups performed with greater success than on the last test because transformational geometry was taught in detail by means of different methods and with many examples. When the literature is examined, some studies can be seen to present that performing exercises using different educational methods increased student achievement in transformational geometry (Egelioglu, 2008; Karakuş, 2008; Kurak, 2009; Şataf, 2010; Yazlık, 2011). In this context, this finding shows consistency with the studies in the literature.

When the achievements of each group in the sections of recognition, features, and construction were separately examined, the achievements when compared to the last test were determined to have increased significantly. This finding shows that performing exercises is very effective in increasing the achievement scores for transformational geometry. To determine which group was more

successful in the sections of recognition, features, and construction, ANOVA analysis was performed on the post-test data. As a result of ANOVA analysis in the recognition section, it can be seen that the Computer Group was more successful than the Manipulatives Group, and the Manipulatives Group was more successful than the Traditional Group. This result shows that the most successful group was the Computer Group, followed by the Manipulatives Group, with the Traditional Group last. In the Computer Group, many exercises were performed through DGS Cabri II Plus and worksheets. Students had the opportunity to see their alterations instantly and to manipulate them thanks to the dynamic structure of Cabri II Plus. For example, assume there is an activity to take the symmetry of a triangle with the respect to a line. During this activity, teacher candidates had the chance to see the reflection transformation onscreen by taking the triangle's symmetry with respect to the line thanks to Cabri II Plus. Next, by moving every corner of the triangle, views of how the *change on the reflection* occurs can be seen. This feature of Cabri II Plus provides important opportunities for teacher candidates in the Computer Group to define transformations. Students can see the results by applying reflection, displacement, and rotational transformation, instantly noticing how alterations occur when they move some corner, thanks to the many features of this software. In other words, Cabri's visual features and dynamism provided students in the Computer Group with more success in the section of recognition than students in the other groups. Also Yavuzsoy-Köse (2008) stated that Cabri is very effective for increasing transformational geometry achievement by means of its visual features and dynamism. In the Manipulatives Group, courses were conducted using tangible materials such a symmetry mirror and unit plotting paper. Even though teacher candidates in this group had the opportunity to see the reflection transformation instantly by means of the symmetry mirror, they did not have the opportunity to see instant transformations for displacement and rotation. They were obliged to manually make these transformations using unit plotting and isometric papers. For this reason they were less successful than the Computer Group as far as transformations. In the traditional group, courses were performed on the class board. Intangible materials were used. For this reason, teacher candidates had the least success as far as the section of recognition because they only had the chance to see transformations on the class board.

When the findings are examined, it is seen that for the section of features, the Computer Group was more successful than the Manipulatives Group, and the Manipulatives Group was more successful than the Traditional Group. So in the features division, the most successful group was the Computer Group, followed by the Manipulatives Group, with the Traditional Group in last regarding achievement in the section of recognition. This result can be interpreted as follows. For example, let us consider the features of displacement transformation. As is known, position of the figure, its form, and its dimensions are always the same with displacement transformation; only the figure is replaced. By using Cabri II Plus, students in the Computer Group could transform the screen as a unit-plotting page. In this way, they could measure length and area. Thus, they could easily determine whether the figure's features had changed or not by means of measuring the sides or surface area of a figure. Students in the Manipulative Group had the chance to measure the length of the figure by means of a ruler and unit-plotting papers. But they were at risk of misapplying the transformations. If students misapply the transformation on their unit-plotting pages, they will obtain an incorrect transformational view. For this reason, no matter how many times one measures, they will get incorrect features and results because of a misapplication. In the Traditional Group, applications were performed on the board by the teacher. Students wrote what was written on the board in their notebooks. For this reason, there was lost time and the possibility existed of writing or drawing incorrectly in their notebooks. Thus, the Traditional Group was the least successful group as far as the section on transformational features.

The results obtained from the section on construction differed from the others. In the previous two sections, the most successful group was the Computer Group; in the construction section, the successes of the Computer and Manipulative Groups were the same. The reason can be explained as follows. Thanks to the visual features and dynamism of Cabri II Plus, teacher candidates in the Computer Group had more success in the sections on recognition and features than the teacher candidates in the Manipulatives Group. But as far as construction, teacher candidates had to apply transformations by themselves, one-by-one. If a teacher has comprehended well the features of transformation, they will not encounter any problem in applying the requested transformation. But the contrary is possible. This is because many different abilities can be needed for the transformation application process, such as drawing and 3D conceptualization. In both the Computer and

Manipulatives Groups, because students applied transformations one-by-one, no significant difference was seen between the successes of these two groups. The Traditional Group came in last in the construction section just as with the other sections.

When the achievement of transformational geometry was generally evaluated, it was seen that the most successful group was the Computer Group, followed by the Manipulatives Group, with the Traditional Group last. This result shows that the applications performed by the Computer Group were more successful than the applications as performed by the other two groups. When the literature is examined, many studies are seen to show the result that DGS Cabri is very effective in increasing transformational geometry achievement (Dixon, 1997; Hoyles & Healy, 1997; Kurak, 2009; Yavuzsoy-Köse, 2008; Yazlık, 2011). In this context, it can be said that DGS Cabri is an effective tool for increasing the transformational geometry achievement of teacher candidates. As an aside, the question occurs, "If different dynamic software was used instead of Cabri, such as GeoGebra or Geometry Sketchpad, would there have been any change in the current results?" When the body of literature is examined, some studies are found which show that using a different DGS is also effective at increasing transformational geometry achievement (Dixon 1997; Güven & Kaleli-Yılmaz, 2012; Harper, 2002). For this reason, it is not possible to state the result that Cabri II Plus is more effective at increasing transformational geometry achievement than other software. No matter which software is used, it is obvious that transformational geometry achievement will increase. Researchers who want to study in this area can examine the differences in achievement between the different dynamic software programs by using them on different groups. This kind of study could probably give different contributions to the body of literature.

The achievement obtained by the Manipulatives Group is highly important. In the literature, many studies are available which show that education supported by tangible materials is effective at increasing success (Aydın-Ünal & İpek, 2009; Kutluca & Akın, 2013). Traditional methods in many studies are emphasized as having much less effect at increasing success in comparison to other methods (Egelioğlu, 2008; Yazlık, 2011). Dynamic software and then physical manipulatives were seen in this study to have a significant influence on increasing transformational geometry achievement. In this case, it is very important to increase the success of teachers

and academicians who will explain transformational geometry; this point should not be disregarded, and this subject should not be taught by traditional methods. Researchers who want to study this subject can benefit by performing similar research with different sample groups and then compare results in order to present more effective data.

Suggestions

The results of this study indicate that the Computer Group in which Cabri II Plus was used was more successful than the other two groups. When the related literature is scrutinized, however, GeoGebra, Geometry Sketchpad, and other similar software on Transformational Geometry were also proven to be useful for increasing achievement levels. As emphasized in the Discussion and Conclusion section, it would be useful for several experimental groups to compare the achievement levels of these groups using Cabri II plus, GeoGebra, and any other software which can be implemented for determining which software has a superior effect on success levels. Additionally, even though the Computer Group showed better success than the Manipulatives Group, the Manipulatives Group was noticed to have a considerable success rate. In this respect, it can be inferred that the use of concrete materials has a positive effect on transformational geometry success, and must be used in practical classroom instruction. In addition, no significant relationship was found between the Computer and Manipulative Groups in terms of the construction section of the achievement test. It must be noted that questions specifically involving construction should be focused on in further research as well as ways to increase construction-related success. Aside from all of this, another mixed group research different from the Computer and Manipulatives Groups from this research can be employed for a more robust achievement comparison to make useful contributions to the related literature. In summary, it is important to conduct similar longitudinal researches on how learners are taught transformational geometry via courses and compared in terms of success. In addition to the current research, supporting these studies with qualitative data, in-depth analysis of problem areas, and proposing solutions to these problems will be useful in increasing transformational geometry success. Finally, conducting a similar study with prospective math teachers and existing math teachers is necessary.

References

- Absi, M. M., & Nofal, M. B. (2010). The effect of using manipulatives on the mathematical achievement of the first grade students. *Damascus University Journal*, 26(4), 37–54.
- Akgül, M. B. (2014). *The effect of using dynamic geometry software on eight grade students' achievement in transformation geometry, geometric thinking and attitudes toward mathematics and technology* (Master's thesis, Middle East Technical University, Ankara, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Aksoy, Y., & Bayazit, İ. (2010). Simetri kavramının öğrenim ve öğretiminde karşılaşılan zorlukların analitik bir yaklaşımla incelenmesi. In E. Bingöballı & M. F. Özmantar (Eds.), *İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri* (pp. 187–215). Ankara: Pegem.
- Aydın-Ünal, Z., & İpek, A. S. (2009). The effect of realistic mathematics education on 7th grade students' achievements in multiplication of integers. *Education and Science*, 34(152), 60–70.
- Baki, A. (2001). Bilişim teknolojisi ışığı altında matematik eğitiminin değerlendirilmesi [Evaluation of mathematics education in the light of information technology]. *Journal of Milli Eğitim*, 149, 26–31.
- Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21(2), 132–144.
- Battista, M. T. (1999). Geometry results from the third international mathematics and science study. *Teaching Children Mathematics*, 5(6), 367–373.
- Baykul, Y. (2002). *İlköğretimde matematik öğretimi, 1-5. sınıflar için* [Mathematics teaching in primary schools 1-5th classes] (6th ed.). Ankara: Pegem.
- Carroll, W. M. (1998). Middle school students' reasoning about geometric situations. *Mathematics Teaching in the Middle School*, 3, 398–403.
- Dixon, J. K. (1997). Computer use and visualization in students' construction of reflection and rotation concepts. *School Science and Mathematics*, 97(7), 352–358.
- Duatepe, A., & Ersoy, Y. (2001). *Teknoloji destekli matematik öğretimi-I: Hesap makinesi ve okullarda geometri öğretimi* [Technology-based mathematics education I: Calculator and geometry teaching in schools]. Ankara: Milli Kütüphane.
- Durmuş, S., & Karakırık, E. (2006). Virtual manipulatives in mathematics education: A theoretical framework. *The Turkish Online Journal of Educational Technology-TOJET*, 5(1), 117–123.
- Egelioglu, H. C. (2008). *Computer based education has an influence on success and epistemological belief in teaching of sublearning zones of transformation geometry and areas of quadrangle zones* (Master's thesis, Marmara University, Institute of Educational Sciences, İstanbul, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Field, A. (2009). *Discovering statistics using SPSS* (3th ed.). London: Sage.
- Fletcher, T. J. (1973). *Some lessons in mathematics. A handbook on the teaching of modern mathematics*. Cambridge: The University Press.
- Gillis, J. M. (2005). *An investigation of students conjectures in static and dynamic geometry environments* (Doctoral dissertation, Auburn University, Alabama). Retrieved from https://etd.auburn.edu/bitstream/handle/10415/854/GILLIS_JOHN_2.pdf?sequence=1&ts=1442495041365
- Goos, M., & Spencer, T. (2003). Properties of shape mathematics-making waves, Spencer. In *Proceedings of The 19th Biennial Conference of The Australian Association of Mathematics Teachers* (pp. 424–434). Adelaide: AAMT.
- Gülten, D. Ç., & Gülten, İ. (2004). A study on the relationship between the learning styles and geometry of grade students. *Eurasian Journal of Educational Research (EJER)*, 4(16), 74–87.
- Gürbüz, K. (2008). *The qualification levels of in-service elementary school mathematics teachers on the sub-learning strands, namely transformational geometry, geometric objects, pattern and tessellations* (Master's thesis, Abant İzzet Baysal University, Institute of Social Sciences, Bolu). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Güven, B. (2012). Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. *Australasian Journal of Educational Technology*, 28(2), 364–382.
- Güven, B., & Kaleli-Yılmaz, G. (2012). Effect of dynamic geometry software on pre-service primary school teachers' achievement of transformation geometry. *E-Journal of New World Sciences Academy: Education Sciences*, 7(1), 442–452.
- Güven, B., & Karataş, İ. (2005). Dinamik geometri yazılımı Cabri ile oluşturmaya öğrenme ortamı tasarımı: Bir model. *İlköğretim-Online*, 4(1), 62–72.
- Harper, R. S. (2002). *Enhancing elementary pre-service teachers' knowledge of geometric transformations* (Doctoral dissertation, University of Virginia, Virginia). Dissertation Abstracts International: AAI3030678.
- Hazzan, O., & Goldenberg, E. P. (1997). Students' understanding of the notion of function in dynamic geometry environments. *International Journal of Computers for Mathematical Learning*, 1, 263–291.
- Heddens, J. W. (1997). *Improving mathematics teaching by using manipulatives*. Retrieved from <http://www.fed.cuhk.edu.hk/~flee/mathfor/edumath/9706/13hedden.html>
- Hoyle, C., & Healy, L. (1997). Unfolding meanings for reflective symmetry. *International Journal of Computers for Mathematical Learning*, 2, 27–59.
- Jones, K. (2002). Issues in the teaching and learning of geometry. In L. Haggarty (Ed.), *Aspects of teaching secondary mathematics: Perspectives on practice* (pp. 121–139). London: Routledge Falmer.
- Kalaycı, Ş. (2009). *SPSS uygulamalı çok değişkenli istatistik teknikleri* [SPSS Applied Multivariate Statistical Techniques] (4th ed.). Ankara: Asil Yayın Dağıtım.
- Karakuş, Ö. (2008). *The effects on the students' success of computer based transformation geometry learning* (Master's thesis, Osmangazi University, Institute of Science, Osmangazi, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Knuchel, C. (2004). Teaching symmetry in the elementary curriculum. *The Montana Mathematics Enthusiast*, 1(1), 3–8.
- Köseoğlu, K. (1994). *İlköğretime öğretmen yetiştiren kurumlarda öğretim elemanı yeterliklerinin değerlendirilmesi* (Master's thesis, Ankara University, Institute of Social Science, Ankara, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Küchemann, D. (1981). Reflection and rotation. In K.M. Hart (Ed.), *Children's understanding of mathematics: 11-18* (pp. 137–156). London: Murray
- Kurak, Y. (2009). *The effects of using dynamic geometry software on students' understanding levels of transformation geometry and their academic successes* (Master's thesis, Karadeniz Technical University, Institute of Science, Trabzon, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>

- Kutluca, T., & Akin, F. (2013). Somut materyallerle matematik öğretimi: Dört kefeli cebir terazisi kullanımı üzerine nitel bir çalışma. *Turkish Journal of Computer and Mathematics Education*, 4(1), 48–65.
- Milli Eğitim Bakanlığı. (2005). *İlköğretim matematik dersi (1-5. ve 6-8. sınıflar) öğretim programı*. Ankara: Devlet Kitapları Müdürlüğü Basımevi.
- Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, 47, 175–197.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Şataf, H. A. (2010). *Effect of computer-assisted mathematics teaching on achievement and attitude of 8th graders in transformation geometry and triangles sub-learning areas sample Isparta Primary School* (Master's thesis, Sakarya University, Institute of Social Sciences, Sakarya, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Schuester, S. (1973). *An evolutionary view. In geometry in the mathematics curriculum*. Thirty-six Yearbook. National Council of Teachers of Mathematics.
- Soon, Y. P. (1989). *An investigation of Van Hiele-like levels of learning in transformation geometry of secondary school students in Singapore*. Retrieved from <http://wwwlib.umi.com/dissertations/fullcit/8915764>
- Suydam, M. N., & Higgins, J. L. (1984). Manipulative materials. *Arithmetic Teacher*, 33, 27.
- Toptaş, V. (2008). Geometri öğretiminde sınıfta yapılan etkinlikler ile öğretme-öğrenme sürecinin incelenmesi (Examine the teaching – learning levels by the activities applied in the classroom during the teaching process of the Geometry). *Elementary Education Online*, 7(1), 91–110.
- Ubuz, B., Üstün, I., & Erbaş, A. K. (2009). Effect of dynamic geometry environment on immediate and retention level achievements of seventh grade students. *Eurasian Journal of Educational Research (EJER)*, 35, 147–164.
- Van de Walle, J. A. (2004). *Elementary and middle school mathematics*. Virginia Commonwealth University.
- Yavuzsoy-Köse, N. (2008). *Determining fifth grade primary school students' understanding of symmetry using dynamic geometry software cabri geometry: An action research* (Doctoral dissertation, Anadolu University, Institute of Educational Sciences, Eskişehir, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Yazlık, D. Ö. (2011). *Teaching transformation geometry with Cabri Geometry Plus II in seventh grade* (Master's thesis, Selçuk University, Institute of Educational Sciences, Konya, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- Zembat, İ. Ö. (2007). The main tenets of direct instruction and constructivism: The case of translations. *Gazi University Journal of Gazi Educational Faculty*, 27(1), 195–213.

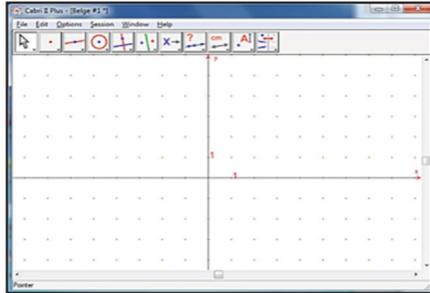
Appendix 1

Worksheet used in Computer Group-Reflection

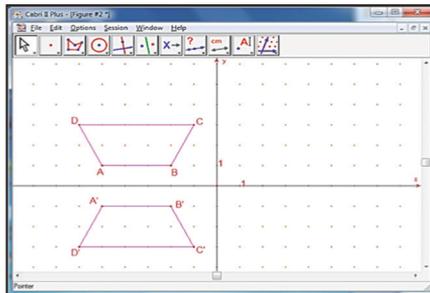


Dear Students! Today we will perform an enjoyable activity about Reflection by using Cabri Software. During the activity, it is required that you follow the steps below. Let us start. Are you ready?

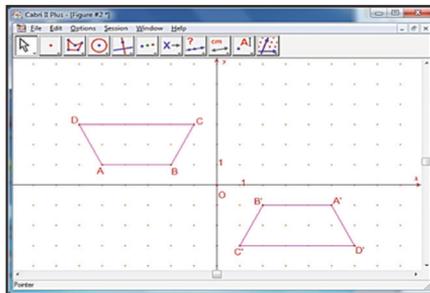
- Open the Cabri Program. Respectively, click *Show Axes-Define Grid* tool bars and then your work page becomes unit plotting.



- Draw the ABCD trapezoid whose corner points were given by means of coordinates in the following table. Name every corner. Take the reflection of the square relative to the X-axis. For this, activate the “Reflection” toolbar. After taking the reflection, move every corner of the polygon separately and observe how the reflected view of the polygon changes. Name the reflection of every corner coordinate of the polygon A^1 , B^1 , C^1 , and D^1 and note the value of every corner’s coordinate in their section on the following table.



- Take the reflection of ABCD trapezoid relative to the Y-axis and by following the above operations, write the values you found into their section on the following table.
- Lastly, take the reflection of ABCD trapezoid relative to the origin. (0, 0) To do this, use the “Symmetry” tool bar. Write the values you found in their section on the following table.



Point Coordinates	Reflections relative to the X-axis	Reflections relative to the Y-axis	Reflections relative to the Origin
A (-5, 1)			
B (-2, 1)			
C (-1, 3)			
D (-6, 3)			



Check the table you created. How is the change shown for the point coordinates while taking reflection of any point relative to the X-axis?

If the coordinates of a point are shown as (a, b) and reflection is taken relative to the X-axis, what kind of generalizations about corner coordinates can we make?

.....



If the coordinates of a point are shown as (a, b) and reflection is taken relative to the Y-axis, what kind of generalizations about corner coordinates can we make?

.....



If the coordinates of a point are shown as (a, b), and reflection is taken relative to the origin, what kind of generalizations about corner coordinates can we make?

.....

Appendix 2

Worksheet Example used in Manipulative Group-Displacement



Dear Students! Today we will perform an enjoyable activity about the subject of Displacement. During the activity, you are required to follow the steps below. Let us start. Are you ready?

- Take a plotting paper. Have the coordinate axis at the center.
- Draw a triangle which has the corner point coordinates as $A(-3, 3)$, $B(-4, 1)$, $C(-1, 2)$ on your plotting paper. Displace that triangle one by one by using the displacement values defined in the following tables.
- Name the ABC Triangle's coordinates as A' , B' , C' . Write the new values in the section on the following tables.
- Apply all displacements separately for all required conditions and write all found values into the section of the following tables.

Displace to the Right

Displacement Values	$A(-3, 3)$	$B(-4, 1)$	$C(-1, 2)$
	A'	B'	C'
1 unit to the right			
2 unit to the right			
3 unit to the right			
x unit to the right			

Displace to the Left

Displacement Values	$A(-3, 3)$	$B(-4, 1)$	$C(-1, 2)$
	A'	B'	C'
1 unit to the left			
2 units to the left			
3 units to the left			
x units to the left			

Displace Up

Displacement Values	$A(-3, 3)$	$B(-4, 1)$	$C(-1, 2)$
	A'	B'	C'
1 unit up			
2 units up			
3 units up			
y units up			

Displace Down

Displacement Values	$A(-3, 3)$	$B(-4, 1)$	$C(-1, 2)$
	A'	B'	C'
1 unit down			
2 units down			
3 units down			
y units down			

Displacements to the Right, Left, Up, and Down

Displacement Values	A(-3, 3)	B(-4, 1)	C(-1, 2)
	A'	B'	C'
2 units right, 1 unit up			
2 units right, 1 units down			
3 units left, 2 units up			
3 units left, 2 units down			
x units right, y units up			
x units right, y units down			
x units left, y units up			
x units left, y units down			



If the point coordinates are given as A(a, b), how do these coordinates change when displaced right x units? Can you make any generalizations?

.....



If the point coordinates are given as A(a, b), how do these coordinates change when displaced left x units? Can you make any generalizations?

.....

If the point coordinates are given as A(a, b), how do these coordinates change when displaced up y units? Can you make any generalizations?

.....

If the point coordinates are given as A(a, b), how do these coordinates change when displaced down y units? Can you make any generalizations?

.....