EXPLORING METACOGNITION IN PRESERVICE TEACHERS: PROBLEM SOLVING PROCESSES IN ELEMENTARY MATHEMATICS

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ABSTRACT

In Principles and Standards for School Mathematics (2000), the (U.S.) National Council of Teachers of Mathematics recommended that students communicate their mathematical thinking in a logical manner, and use the language of mathematics to express their thinking accurately and logically. Students should not only learn mathematics content, but should learn how to generate ideas, express them in multiple ways, and justify their thinking (Carpenter, Franke, & Levi, 2003). Journals are an effective way for students to communicate their understanding of mathematics content while using the process standards of problem solving, reasoning and proof, connections, and representation.

This article describes the use of mathematics journals by undergraduate Early Childhood Education majors in a mathematics methods course. In this course, pre-service teachers are asked to use journals to examine their own problem solving skills, through written and oral reflection. Samples of student reflections of their problem solving processes are included.

Keywords: Metacognition in Problem Solving, Problem Solving Processes, Metacognition in Mathematics.

INTRODUCTION

One of the reform efforts in mathematics education in the United States have been underway since the early 1980s, beginning with the release of *A Nation at Risk* (National Commission on Excellence in Education, 1983), which warned of the mediocrity of instruction taking place in America's classrooms. This report influenced the (U.S.) National Council of Teachers of Mathematics (NCTM) to develop standards for mathematics education (1989, 1991, 1995, 2000).

The NCTM standards documents, *Professional Standards for Teaching Mathematics* (1991) and *Principles and Standards for School Mathematics* (2000) described a vision for P-12 mathematics teaching and learning, including instruction that allows students to learn mathematics conceptually. “Students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward, mathematics are all shaped by the teaching they encounter in school” (NCTM, 2000, p. 17). With these recommendations, NCTM gave teachers a framework to design programs that reflect high-quality instruction.

Elementary school teachers should know mathematics ideas and processes, and how those are authenticated and generalized in other content and settings (NCTM, 1991). Teachers' classroom experience and knowledge of the subject-matter content were also found to impact student achievement (U.S. Department of Education, 2003). Therefore, teacher education programs are responsible for providing high-quality mathematics preparation for pre-service teachers (Capraro, Capraro, Parker, Kulm, & Raulerson, 2005).

The (U.S.) Mathematical Science Education Board (1990) suggested that few teachers have sufficient training in mathematics. Unfortunately, pre-service teachers may take only one of the four courses in mathematics recommended by the National Research Council (NRC) for training in teaching mathematics in elementary school (NRC, 1989), and this may leave them unprepared to teach children mathematics effectively. However, taking multiple mathematics courses does not guarantee students will attain a high level of mathematical knowledge and understanding (Wu, 1999). Pre-service teachers should recognize a connection between their coursework and the curriculum presented in their future classrooms (Cuoco, 1998). Therefore, it is important to evaluate the effectiveness of teacher education programs in developing beliefs in preservice teachers that promote current reform efforts in mathematics education (Hart, 2002).

Whether they take one or many mathematics courses, preservice teachers should be confident in their ability to teach math after their teacher preparation programs (NRC, 1989). However, many preservice teachers exhibit difficulty with even 6th grade mathematics activities (Lloyd & Frykholm, 2000). Therefore, although problem solving skills are more important than computational skills in many university-level mathematics courses (Kloosterman & Stage, 1992), pre-service teachers may be unable to demonstrate mastery of basic problem solving skills during their teacher preparation program.

Using Journals in P-5 Mathematics Education Classes

The first author taught a P-5 mathematics course that focused on mathematics pedagogy in preschool through 5th grade. Prior to this course, pre-service teachers in the early childhood education program were required to take a math content course (College Algebra or higher) and 4 classes defined as teacher preparatory classes in K-8 mathematics. The K-8 math courses for teacher preparation included Foundations of Number and Operations, Foundations of Data Analysis and Geometry, Statistics and Probability for K-8 Teachers, and Algebraic Connections for K-8 Teachers. The pre-service teachers were able to take the final math course (Algebraic Connections) in conjunction with the P-5 mathematics course.

To develop higher-level reasoning skills and to assist pre-service teachers in recognizing the connection between their coursework and the curriculum presented in their future classrooms, pre-service teachers were required to use problem solving journals in the P-5 mathematics courses. The goal of this learning activity was for the pre-service teachers to examine their own metacognitive processes they use during problem solving activities. The pre-service teachers were asked to engage in the process standards advocated by NCTM (2000):

- a problem solving approach to mathematics learning,
- communication of metacognitive processes and results with others,
- making connections across content areas and among skills,
- developing reasoning abilities and proving solutions, and
- representation of learning in multiple ways.

Each day in class, the first author provided a nonroutine, upper-elementary level problem for the pre-service teachers to solve. Nonroutine problems are those that "encourage logical thinking, expand students' understanding of concepts, develop mathematical reasoning power, develop students' abilities to think in more abstract ways, and allow for a transfer of mathematical skills to unfamiliar situations" (Daane & Lowry, 2004, p. 25). The selected problems required the pre-service teachers to use strategies that were identified in the literature as appropriate for use in P-8 classrooms, such as acting out the problem or using objects, drawing diagrams or pictures, looking for patterns, working backward, guessing and checking, using logical reasoning, making a table or an organized list, and creating a simpler problem (NCTM, 2000; Georgia Department of Education, 2006). The problems used during the semester were selected from various sources, including NCTM publications, elementary problem solving websites, and problem solving books. They were of varying difficulty and grade levels, and collectively, they required the use of a variety of strategies.

The pre-service teachers were asked to copy the problem at the beginning of each class, use whichever strategy they could to solve the problem, then document their thinking by writing the problem solving steps they went through while solving the problem. After that, the pre-service teachers were encouraged to share their answers and their thinking processes with their peers. After this small-group sharing occurred, solutions were shared with the entire class by individuals, partners, or small groups.

Samples of the problem solving work done by the pre-service teachers indicated that metacognitive processes were being done and recognized - by the pre-service teachers, particularly the NCTM process standard of reasoning and proof. Samples of problem solving activities and preservice teachers' documentation of their metacognitive processes follow.

Sample Problem 1: Use these numbers (0, 1, 2, 3, 4, 5) to find the value of these letters (A, E, H, L, R, T).

Clues: $E + T = E$

$L \times H = L$

$E + E = R$

$A - H = R$

Answer: A = 5, E = 2, H = 1, L = 3, R = 4, T = 0

Pre-service Teacher #1 thinking process:

To find the answer, first I noted that T has to equal 0, because any number plus zero equals that same number. Second, I knew H was equal to 1, because any number multiplied by 1 equals that number. Third, I knew that of the remaining numbers, only the number 2 could be added to itself an equal another one of the remaining numbers (4). So, E is equal to 2, and R is equal to 4. Once I knew R = 4, I figured out A equals 5, because if A minus 1 equals 4, A has to equal 5. Finally, if 3 is the only remaining number, I checked to see that L equaled 3.

Another student described her thinking in this way:

Pre-service Teacher #2 thinking process:

I started with the first clue, E + T = E. T has to be 0 since, when added to E, E is unchanged. Next, I looked at the clue L x H = L. If a number, (L), is multiplied by another number (H), and the product is L, you know that H is equal to 1. Next, for E + E = R, we know that E and R can only come from the numbers 1-5. Since 4 is the only number in that set that has a square root that is a whole number, E has to equal 2. Therefore, R = 4. The last clue is A - H = R. We know that R = 4, so 5 is the only number that can be substituted for A to end with the answer of 4.

Sample Problem 2: Penny had a bag of marbles. She gave 1/3 of them to Rebecca, then ¼ of the remaining marbles to John. Penny had 24 marbles left in the bag. How many marbles did she have in the bag at the beginning?

Answer: 48

This student used guess and check, beginning with a number she thought was reasonable.

Pre-service Teacher #1 thinking process:

Begin by selecting a reasonable number that would be divisible by three, which is 39. I then subtracted one-third (because she gave one-third away) from 39. One-third of 39 is 13, which yielded an answer of 26. Because 26 isn't divisible by three, I know I need to choose a higher number to work with. I chose 48. Divide 48 by three, and it produces a quotient of 16. Subtracting that from 48 equals 32. Next, subtract one-fourth of 32, which is 8, to get an answer of 24. Penny had 24 marbles left over, so we can say that there were 48 marbles in the bag to start with.

Another student also used guess and check, but was less precise about her thinking, and how she started.

Pre-service Teacher #2 thinking process:

I used guess and check to solve this problem. I knew from the problem that the beginning number has to be divisible by 3, so I tried different multiples of 3, then divided it by 4 to see if I get 24 left.

This student used the strategy of working backwards and using equations.

Pre-service Teacher #3 thinking process:

For this problem, I decided to work backwards. I asked myself: 24 is ¾ of what number? Set up the problem to find out: 24/x = ¾. To solve this, cross multiply (24 x 4 = 3x, which gives an answer of 32). Now, we know there were 32 marbles in the bag after 1/3 of them were given to Rebecca. Next think: 32 is 2/3 of what number? Set up this problem: 32/x 2/3. Cross multiply (32 x 3 2x, which gives an answer of 48). We know that there were 48 marbles in the bag to start with.

Discussion

As evidenced by the explanations of the metacognitive processes used in problem solving, as well as the variety of strategies used to solve the problems, math problem solving journals seem to provide positive outcomes related to mathematics learning. Not only were students solving problems similar to those they will eventually use in their own classrooms, they also examined their cognitive processes and documented those. Being aware of the processes they used to solve a problem themselves may allow them to teach those more effectively to children. Another benefit of using the math journals in this way is the collaborative aspect. In the process of sharing solutions and the step-by-step processes used, the pre-service teachers were exposed to a variety of ways to solve mathematics problems. It is hoped that the use of math journals as a metacognitive tool will promote higher-level, critical thinking in pre-service teachers, as well as providing the pre-service teachers with a model for using math journals in their classrooms with elementary school students.

Conclusion

The intent of this article was to provide a description of the metacognitive work done by the authors' undergraduate students during their mathematics pedagogy course. It is hoped that the use of math journals as a metacognitive tool will promote higher-level, critical thinking in pre-service teachers, as well as provide the pre-service teachers with a model for using math journals in their future classrooms with elementary school students.

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