Golden Proportion

Why the **Golden Proportion** really is golden

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Have you ever looked at an object and found yourself thinking how visually attractive that object appears to the eye? Interpersonal considerations to one side, we might be referring to the intrinsic properties which appear visibly in physical objects or images. For instance, your mind might implicitly declare, "That painting is just fabulous", "What an amazing looking building", or "What an interesting shape". It may have been a piece of furniture, an item of clothing or even a flower in your garden that virtually demanded your attention.

So just how can we have our attention suddenly hijacked? The most obvious factors must involve aspects such as size, colour, movement and discrepancy such as in looking at a Salvador Dali painting. Yet there is another subtle factor associated with shape that also can demand, and even attract, attention. This factor concerns the relationship between dimensions such as width and height. One such phenomenon is referred to as the Golden Proportion. Expressed mathematically, this represents a ratio coefficient of 1:1.62. Taken out of context, such a figure sounds strange. Indeed, it seems almost bizarre to inform someone that they like something because it is 1.6 times higher than it is wide. But in this article you can find many examples of phenomena that appear consistent with such a notion.

The Golden Proportion has been identified in products and constructions from ancient Egyptian times. So what has it got to do with teaching a mathematics class today? My argument is that the analysis of the Golden Proportion engages students in varied mathematical thinking. Specifically, such an analysis invokes measurement, ratio, rational number, and proportion. Most vitally, investigating the Golden Proportion, finding it within the world we live in, and being able to describe its dimensional properties, provides remarkably rich learning opportunities which can foster the awareness of proportional reasoning. As many teachers know, proportional reasoning represents an area in which many students express difficulty. Similarly, many teachers are known to find teaching in this area a difficult challenge.

Why is proportional reasoning important to teach?

For over three decades researchers have stressed the general importance of proportional reasoning (Che, 2009; Karplus, Pulos, & Stage, 1983). They have identified the mastery of proportional reasoning as a signpost to signal an understanding of elementary mathematical concepts (Lesh, Post, & Behr, 1988) and as a foundation for future learning of mathematics in secondary school (Bright, Joyner, & Wallis, 2003; Tabart, Skalicky, & Watson, 2005). Although there has been work undertaken in this area, there are strong suggestions that many adolescents and college students find proportional reasoning difficult (Fujimura, 2001; Lawton, 1993). It is further suggested that possibly 90% of adults do not reason proportionally (Lamon, 2005; Tipps, Johnson, & Kennedy, 2011). An emphasis by researchers and

mathematic associations to invest both time and effort into developing proportional reasoning (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; NCTM, 1989; Norton, 2006) reflects the overall importance proportional reasoning should be given in today's classrooms to improve the level of proportional reasoning competency.

What is the Golden Proportion?

The Golden Proportion really means the relationship between two entities or objects. To help understand what the Golden Proportion is, look at the picture below, representing the columns found in Durham Cathedral in England.

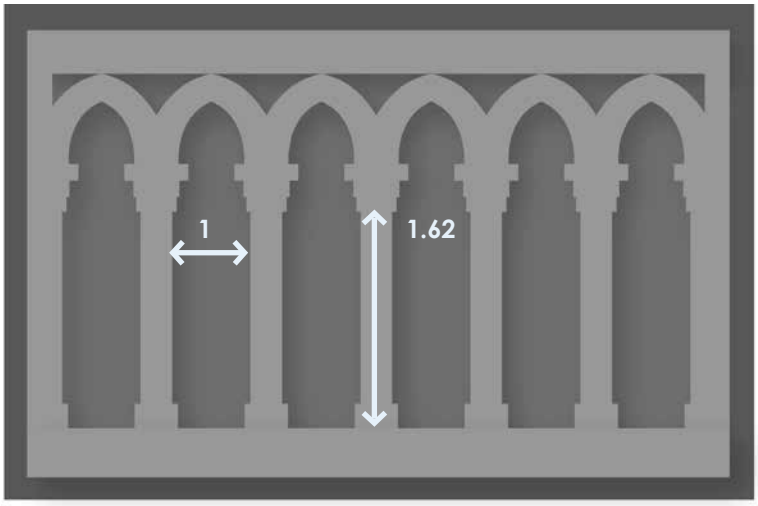

Figure 1. Diagram of the columns at Durham Cathedral.

When compared, there is a ratio between the column height and the distance between each of the respective columns. In this instance the relationship between the column height and distance between each column is the ratio 1: 1.62. This ratio is considered to be the Golden Proportion and can be found throughout the world. A second example of the Golden Proportion is the Parthenon (Hemenway, 2005), built by ancient Greeks in Athens. The Golden Proportion is observed when the height of the building is compared to the building's width.

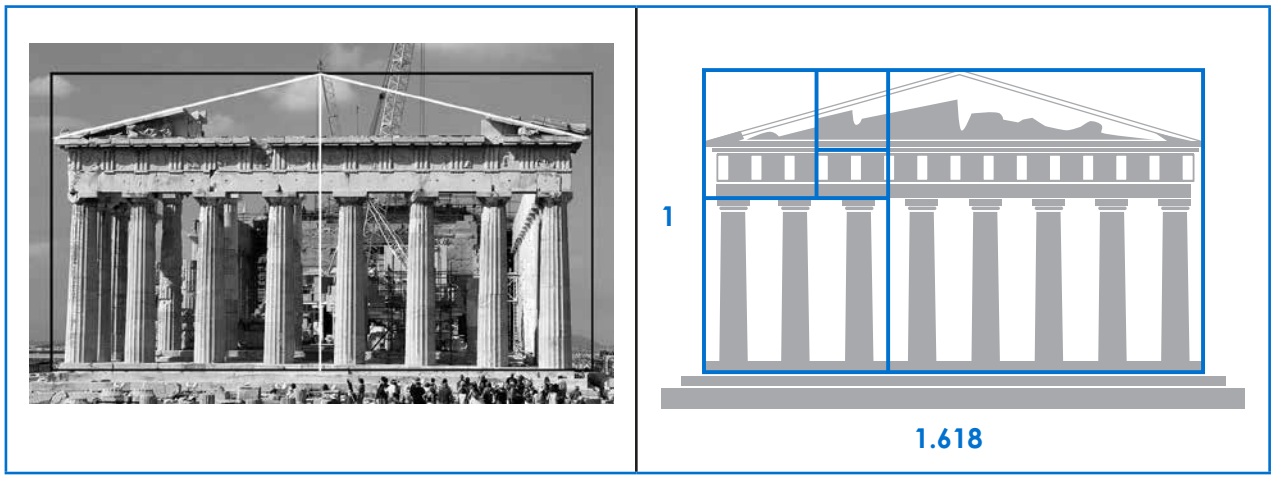

Figure 2. The Parthenon in Athens displaying the grandeur and beauty of the Golden Proportion.

How can the Golden Proportion be taught in a primary classroom?

The Golden Proportion can be used as a teaching tool in many curriculum areas. Although it can be found in many buildings and structures, it can also be found in nature. The Golden Proportion can be seen in the spirals of the Nautilus shell for example. The overlay on the image below outlines the relationship between height and width of the spirals and follows the ratio of 1:1.62.

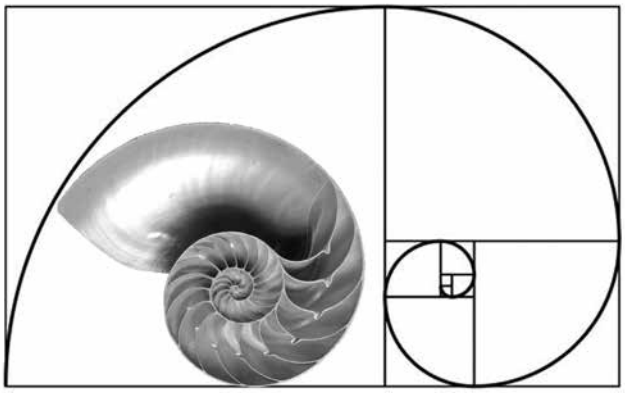

Figure 3. The Nautilus shell and Golden Proportion overlay.

Searching for this mathematical phenomena in a student's everyday world can be a great way to start to engage students in thinking about aspects of the mathematics curriculum, especially proportional reasoning.

Exploration of themselves, the school yard and classroom

An easy activity to commence a unit of work on the Golden Proportion is to establish if it can be found in the school environment. Begin with investigating parts of a student's own body which may follow the Golden Proportion. Exploration and measuring body parts such as faces, arms and fingers in search of the ratio provides an interesting activity.

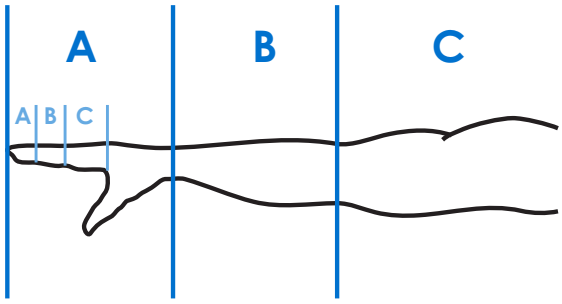

Figure 4. Fingers and arm defining the Golden Proportion.

Asking questions such as, "Can you find the Golden Proportion when looking at your fingers and arms, and does the same apply to everyone?", can invite discussion and thinking about rational numbers, and encourage further investigation of finding ratios between body parts.

Exploration in the classroom can invite students to measure the width and height of the classroom door, windows and furniture. Activities such as finding the ratio of a student's exercise book or school books, or going to the yard and finding the ratio of the length and width of a basketball or netball court, offers activities that investigate the notion of ratio in different contexts. These experiences can be extended to the natural environment, such as looking at the height and width of trees, bushes and shrubs.

Ancient Egypt

The busy and crowded primary curriculum offers a great opportunity to investigate the Golden Proportion in many cross curriculum activities. The study of the ancient world provides an insight into the intrigue and mystery of the Golden Proportion.

The earliest historical references of the Golden Proportion can be traced back to the Rhind or Ahmes papyrus (1650 B.C.E.) which details the use of proportion by the early Egyptian mathematicians from the Middle Kingdom (2000-1800 B.C.E). The Golden Proportion can be found in the Great Pyramid of Giza. By the viewing the diagram below, the Golden Proportion can be found by examining the relationship between the height AC and in this instance, half of the base CB where B bisects EF and the corresponding hypotenuse.

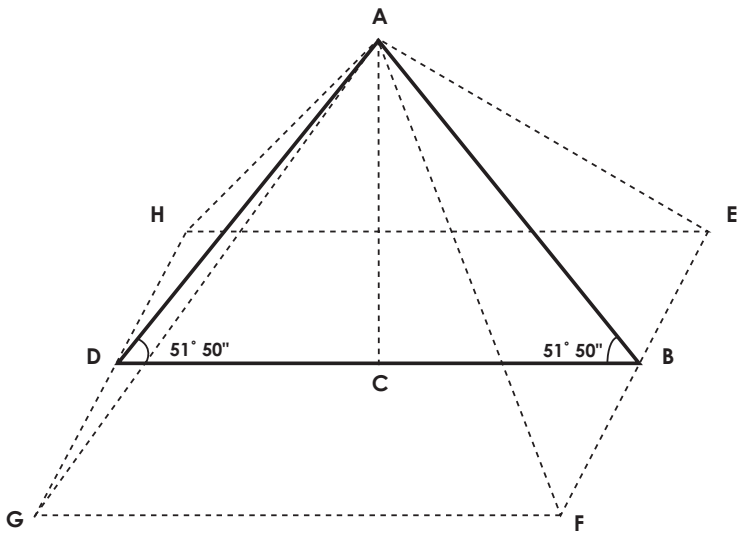

Figure 5. The great pyramid of Giza and Golden Proportion.

An innovative way to explore the Golden Proportion is make models from clay or cardboard of the Great Pyramid of Giza. By creating the same angles as the Great Pyramid, students can manipulate the relationships between the many angles and lengths that exist in the pyramid. The exploration of the relationship between various lengths and in particular the angles of the right angled triangles can lead to a deeper understanding of the ratio that is created when the lengths and angles are compared. Questions such as, “Do other Egyptians pyramids have the same ratio?”, offer further investigations. This initial activity provides scope for students to explore and gather a rich record of mathematical data about various Egyptian structures and the ratios involved in right angle triangles. These activities support other investigations into the relationship between the trigonometric functions of sine and cosine. They offer a different method to the traditional manner that trigonometry is taught usually, and may support the development of the conceptual ideas that for many students can be confusing due to the emphasis on learning algorithmic rules rather than conceptual understanding.

Further investigation can occur with students of reviewing other structures and looking for ratios in famous and not so famous buildings. The Golden Proportion can be found in some of the world’s most famous buildings. A quick Internet search can find pictures of some of these buildings. Ask the students to predict what the ratio of height to the length of the building may be, and search for any other relationships they may see.

Ask whether the building may meet the ratio of the Golden Proportion. Ask students whether other buildings such as the Whitehouse—the seat of USA Government—or the Sydney Harbour Bridge display the same ratio qualities. What about their own homes? These can be excellent questions to engage and provide a series of exploratory activities that are both open ended and inquiry based to engender interest in mathematical learning.

Summary

The investigation of Golden Proportion in a mathematics classroom provides a unique opportunity to explore many aspects of the Australian Curriculum (2014) in mathematics. Exploring the Golden Proportion in the classroom affords students with the opportunity to extend and deepen their learning in many diverse and creative areas. Investigating where the Golden Proportion can be found in everyday life offers a broad and powerful learning experience. These broad learning experiences promotes rigorous thinking and also support the often divergent needs of student learning. It presents educators with a unique opportunity of integrating learning through authentic investigation without watering down learning to a less rigorous level. It facilitates higher order learning while engaging in an interdisciplinary approach where students can be active learners sharing within an Inquiry Based Learning process. Students can engage in a rich historical mathematical journey that covers many areas of the crowded curriculum making proportional reasoning, an area of the mathematics curriculum that is traditionally difficult to teach and learn, both fun and interesting, making the Golden Proportion really golden!

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