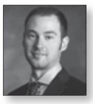
# Zooming in on children's thinking

## How a number line app revealed, concealed, and developed children's number understanding



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An investigation of how an app can influence children's mathematical learning in different ways. Suggestions are made on several aspects teachers may consider when selecting apps for students.

Teachers increasingly use virtual manipulatives and other apps on touch-screen devices (e.g., *iPads*) in an effort to help students understand mathematics concepts. However, students experience these apps and their affordances in different ways. The purpose of this article is to inform teachers' decisions about app implementation in the classroom through discussion of four case studies illustrating ways children interacted with the app *Motion Math: Zoom*, and how these interactions revealed, concealed, and developed children's mathematical understanding. These results suggest that mathematics virtual manipulative apps on touch-screen devices can be useful tools when thoughtfully implemented.

## Mathematics apps and virtual manipulatives

An abundance of mathematics apps are available for the *iPad* and other touch-screen devices (e.g., Highfield & Goodwin, 2013; Larkin, 2014; Tucker et al., 2014). Apps are generally defined as applications used on any computing device, often specifically touch-screen devices such as tablet PCs (Gröger, Silcher, Westkämper, & Mitschang, 2013). Many mathematics apps contain virtual manipulatives (VMs), defined as "an interactive… visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p. 373). Other mathematics apps, such as flash cards or drill games, do not include VMs. These types of apps lack the interactive visual representations of dynamic objects. Apps that contain VMs may provide a variety of affordances, including those linked to positive effects on student performance: simultaneous linking, efficient precision, focused constraint, creative variation, and motivation (Moyer-Packenham & Westenskow, 2013). Research suggests that mathematics VM *iPad* apps use influences student performance (Moyer-Packenham et al., 2015; Riconscente, 2013) and strategy use (Baccaglini-Frank & Maracci, 2015) while certain ways of accessing specific affordances may directly link to performance and efficiency outcomes (Moyer-Packenham et al., in press). However, research also indicates that children access the same affordance in different ways, with some access changing over time (e.g., Paek, 2012; Tucker & Moyer-Packenham, 2014; Tucker, Moyer-Packenham, Westenskow, & Jordan, 2015).

### Choosing the right app

Educators evaluate apps to match children's understandings of a concept and intended learning trajectories, while also considering the apps' technological appropriateness. For example, during app piloting, some children focused on overcoming technological difficulties rather than learning mathematical content (Rick, 2012). Ginsburg, Jamalian, and Creighan (2013) proposed cognitive principles for app design that are applicable to app selection, including mathematically appropriate content, use of appropriate models, and appropriate physical interactions. Sedig and Liang (2006) call the appropriateness 'distance', referring to the gap between how children understand how to act upon the app and how the app requires students to interact with it in order to demonstrate understanding. Children may have difficulty demonstrating their understanding of the mathematics if distance is too great, but children may find tasks too easy if distance is too small. This is akin to applying Vygotsky's Zone of Proximal Development (1978) to interactions with technology (e.g., Murray & Arroyo, 2002). Thus, maintaining appropriate distance is a key component for learning while interacting with technology (Sedig, Klawe, & Westrom, 2001).

This analysis was part of a larger study that included 100 children, 33 of whom were in second grade (ages 7–8 years old) (see also Moyer-Packenham et al., 2016). Each child participated in an individual video-recorded clinical interview that involved interacting with mathematics VM iPad apps chosen for their age group. Researchers focused this analysis on children's five minutes of interactions with the app *Motion Math: Zoom* due to the emergence of distinctive patterns in children's interactions with this app. During this part of the interview, the interviewer offered support for interacting with the app as needed but avoided intervening to guide understanding of mathematical content.

## *Motion Math: Zoom*

*Motion Math: Zoom* is a VM iPad app designed to allow students to demonstrate and develop their understanding of number comparisons, estimation, place value, and magnitude on the number line. In the app, students use touch-based interactions to navigate the number line and pop bubbles to place target numbers in the correct empty spaces (see Figure 1). This interactive representation is a type of "idealised number line" (Kirby, 2013) that was not possible before digital tools (Carpenter, 2013); in this case featuring changeable scales and fluid movement to navigate the number line (Zhang, Trussell, Gallegos, & Asam, 2015). Initial app levels include mathematical and technological content appropriate for second-grade (Year 2) students, and mathematical content becomes progressively more advanced. Within each level, animals of varying sizes separate intervals between numbers and can be simultaneously visible as a reference point.

For positive numbers, the animals face rightward, for negative numbers, the animals face leftward, indicating directionality on the number line, rather than quantity. Users swipe or drag the number line left or right to view numbers along the line. Zooming in or out requires bringing two fingers apart or together ('pinching') horizontally, which decreases or increases the intervals between visible numbers accordingly (e.g., ones, tens, hundreds, etc.). This study focused on learning rather than timed practice and thus did not use the optional needle feature that would pop the bubble if the user completed a task slowly.

## Developing number sense with the number line

The visual representation used in the *Motion Math: Zoom* app is the number line. The number line is a useful model for helping children develop their understandings of number relationships and the number system (Geary, Hoard, Nugent, & Bailey, 2013). By second grade or Year 2, students

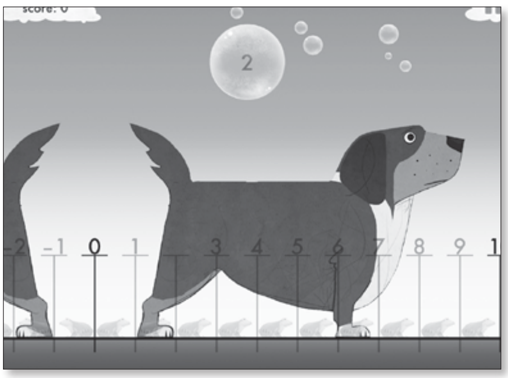


Figure 1. Screen-shot of *Motion Math: Zoom*.

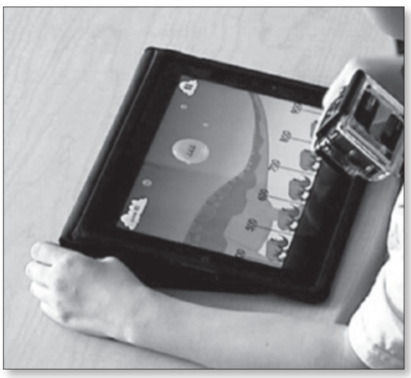

Figure 2a. Finding 444 from intervals of 100.

Figure 2b. Zooming in to tens intervals at 400.

are expected to represent whole numbers and whole-number sums and differences within 100 on a number line diagram (Australian Curriculum, Assessment and Reporting Authority, 2012; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). To meet this expectation, students need to have an understanding of relative magnitude of numerals, comparison and ordering of numerals, and the composition of numbers (Holloway & Ansari, 2009). In *Motion Math: Zoom*, children use these understandings to navigate the number line and determine the target number's position on the number line. To accomplish this, children must consider the target number's relationship to the numbers and/or number ranges on the number line. Children must also use their understandings of magnitude and place value to estimate where to zoom in or out to more efficiently position the target number on the number line.

In *Motion Math: Zoom*, children are given target numbers to place on the number line. Children can zoom out to increase the intervals between visible numbers to efficiently cover greater gaps between starting and target numbers. When zooming out, children estimate the placement of a given number by moving along the number line to find the range wherein the target number is located before zooming in. For example, in figures 2a–2e the child first found 444 from intervals of 100 (Figure 2a), zoomed to tens intervals at 400 (Figure 2b), swiped to increase the numbers along the number line (Figure 2c), zoomed in to intervals of ones at 440–450 (Figure 2d), and popped the bubble to correctly place 444 (Figure 2e). This sequence demonstrates understanding of systematic relationships among numbers.

## Case studies

The following four case studies exemplify trends in how interactions with the app *Motion Math: Zoom* revealed, concealed, and developed students' mathematical understanding. Dave, Nick, Ed and Jacob each received a brief demonstration of basic app interactions and then began *Motion Math: Zoom* on a level involving navigation of one- and two-digit numbers with intervals of one visible on the number line. Although each child used touch-screen devices at home, Nick had the most experience with *iPads* and mathematics apps. During these interactions, Dave and Nick encountered technological distance appropriate for revealing and developing their mathematical understanding. Ed struggled to decrease the technological distance to reveal and develop aspects of his mathematical understanding, but Jacob encountered too much technological distance to effectively reveal and develop much of his mathematical understanding.

## Dave and Nick: revealing and developing understanding with appropriate distance

Dave and Nick began *Motion Math: Zoom* at a level with appropriate mathematical and technological distance. Both children quickly mastered the swiping motion to navigate the number line, at times tapping the screen to stop the number line's motion near the correct space or using fine adjustments for efficient, precise placement of numbers. As gaps between numbers grew, Dave and Nick accessed the app's zoom function. This enabled them to reveal their understanding of the number line by zooming to find specific numbers or ranges. Dave demonstrated proficiency working with intervals of 10 with two-digit numbers, such as when he zoomed in at a range of 70–80 to locate 76 (ACMNA013).

Figure 2c. Swiping to increase along the number line.

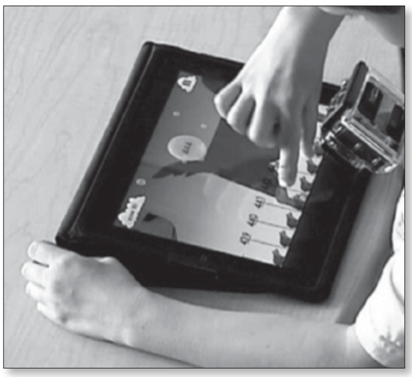

Figure 2d. Zooming in to intervals of one in the range of 440–450.

Figure 2e. Popping the bubble to correctly place 444.

However, Dave was less proficient at working with intervals of 10 for three-digit numbers (ACMNA027), often zooming in at an interval that did not include the target number. For example, to locate 402 from 450, Dave zoomed in at 440–450 and travelled by intervals of 1 until reaching 402. Nick's interactions revealed a more advanced understanding of ranges. For example, Nick zoomed in between 470–480 to find 475, but he zoomed in directly at 480 to find 479. He often zoomed in so the gaps between consecutive numbers were wide enough for precise zooming, accessing the app's affordance of efficient precision. These actions revealed the children's understandings of magnitude and location of numbers on the number line and numbers within a given range.

Throughout the interview, Dave and Nick refined their interactions with the app. Dave gradually improved his zooming efficiency as he oriented the motion to be more horizontal, allowing him to focus more on the mathematical content. As Nick progressed to larger numbers, he increased his efficiency by planning his moves in advance. He often zoomed out to the greatest place value the level permitted (e.g., 10,000: ACMNA052) after placing the previous number but before the next number prompt appeared, accessing the affordance of focused constraint that restricted infinite navigation. This allowed him to quickly cover greater distances between numbers (Figure 3). These planning actions revealed Nick's developing understanding of number magnitude. For Dave and Nick, the technological distance and mathematical distance were different but appropriate, revealing their developing mathematical understandings as they accessed the app's affordances.

## Ed: developing mathematical understanding limited by great technological distance

Ed appeared to enjoy *Motion Math: Zoom* but the technological distance may have limited the development of his mathematical understandings. Ed often popped the bubbles while the number line was in motion, sometimes anticipating the target number's space before it appeared. This revealed his understanding that the numbers' magnitude increased rightward and decreased leftward. After the zoom technique demonstration, Ed attempted to zoom by twisting or pinching vertically or diagonally, instead of pinching horizontally (see Figures 4a–d). For example, when finding 74 starting from 60 with the ranges of ten visible (i.e., 60, 70, 80, 90), Ed tried to zoom in between 70 and 80. However, his ineffective zooming led him to zoom in and out repeatedly before placing the number. His inaccurate motions also caused him to zoom in at ranges other than those he intended. After several similar experiences, Ed avoided zooming altogether. For example, he swiped by ones all the way from 63 to 94 instead of changing intervals. His decision to avoid the zoom input meant that he revealed and developed his understanding of number comparison by ones but he may not have revealed the full extent of his understanding of ranges and he did little to develop this understanding.

Therefore, Ed's difficulty zooming limited the extent to which his understanding of ranges and place value were revealed and developed. Although Ed appeared to understand much of the mathematics content he encountered, his inefficient zooming limited his demonstration of these understandings and kept him from progressing to levels with more advanced mathematics content. However, Ed accessed the affordance of motivation, as he reacted positively to scoring points for his correct responses. The technological distance for Ed was much greater than the distance for Nick or Dave, and this greater technological distance might have concealed some of his mathematical understanding while hindering its development.

Figure 3. Planning by zooming out after placing a number but before the next prompt appears.

Figure 4. Example of Ed's attempt to zoom by twisting.

### Jacob: extent of mathematical understanding concealed by great technological distance

Jacob lacked the technological fluency required by *Motion Math: Zoom* therefore much of his mathematical knowledge may have been concealed and there was little evident development of his mathematical understanding. Jacob was fluent in swiping to navigate the number line, but he did not attempt to zoom until his final task. Much like Ed he swiped to find 60 from 15 by ones or find 200 from 80 by tens. Although Jacob revealed some understanding of magnitude and comparison of numbers by ones and tens on the number line, his lack of zooming concealed any knowledge of larger numbers and number ranges. In an earlier part of the interview, Jacob hinted at an advanced understanding of place value and base ten, stating that tens were tenths of one hundred and ones were tenths of ten (ACMNA079). Yet his interactions with the app did not reveal or develop this understanding.

Due to the great technological distance, Jacob's experience with the app's affordances and mathematical content was very different from the other students. Dave's interactions showed access to the affordance of simultaneously linking representations and actions while revealing nuances in mathematical understandings. However, Jacob did not explore representations that would have accessed the full extent of his mathematical understanding, such as his knowledge of decimals. Whereas Nick used the app's affordances to iterate multiple solution strategies, Jacob repeatedly used the same strategy. Unlike Ed, who was motivated to efficiently complete tasks and earn points, Jacob did not attempt to increase his efficiency.

## Conclusion

The results indicate that an app can influence children's mathematics learning in various ways, including revealing, concealing, and developing mathematical understanding. Therefore, when implementing apps, teachers should consider mathematical and technological distance, which vary due to characteristics of both apps and students (e.g., Tucker, 2015). Prior research implies that teachers already attempt to balance mathematical distance by selecting apps and levels within apps that include mathematical content appropriate for a given student (e.g., Highfield & Goodwin, 2013; Larkin, 2014). However, these cases suggest that teachers should also be aware that technological aspects of an app influence students' mathematical learning experiences, aligning with implications of prior research (e.g., Rick, 2012). Teachers can balance technological distance by assisting students who need help as they learn the technology required to interact with the app. This may include explicitly using scaffolding provided by the app, leading a guided introduction, or reminding students about appropriate interactions after an initial exploration phase (e.g., Aronin & Floyd, 2013). Both the interactions with apps (e.g., Tucker, 2015) and the discourse involved in these experiences (e.g., Anderson-Pence, 2014) can serve as formative assessments, revealing development of mathematical understanding. Facilitating students' discussions of how they experienced the mathematics in these tools may influence technological distance in related situations. Students' experiences with mathematics virtual manipulative apps and their affordances reveal, conceal, and develop mathematical understanding, implying that they can be valuable tools for learning mathematics when thoughtfully implemented.

# References


Anderson-Pence, K. (2014). *Examining the impact of different virtual manipulative types on the nature of students' small-group discussions: An exploratory mixed-methods case study of techno-mathematical discourse* (Doctoral dissertation). Retrieved from http://digitalcommons.usu.edu/etd/2176/

Aronin, S., & Floyd, K. K. (2013). Using an iPad in inclusive preschool classrooms to introduce STEM concepts. *Teaching Exceptional Children, 45*(4), 34–39.

Australian Curriculum, Assessment and Reporting Authority. (2012). *The Australian Curriculum* (3.0 ed.). Retrieved from http://www.australiancurriculum.edu.au/

Baccaglini-Frank, A., & Maracci, M. (2015). Multi-touch technology and preschoolers' development of number-sense. *Digital Experiences in Mathematics Education, 1*(1), 7–27. http://doi.org/10.1007/s40751-015-0002-4

Carpenter, K. K. (2013). *Strategy instruction in early childhood math software: Detecting and teaching single-digit addition strategies* (Doctoral Dissertation). Columbia University. Retrieved from http://academiccommons.columbia.edu/catalog/ac:160522

Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2013). Adolescents' functional numeracy is predicted by their school entry number system knowledge. *PLoS ONE, 8*(1), e54651. http://doi.org/10.1371/journal.pone.0054651

Ginsburg, H. P., Jamalian, A., & Creighan, S. (2013). Cognitive guidelines for the design and evaluation of early mathematics software: The example of MathemAntics. In L. D. English, & J. T. Mulligan (Eds.), *Reconceptualizing early mathematics learning,* 83–120. Springer Netherlands. Retrieved from http://link.springer.com/chapter/10.1007/978-94-007-6440-8_6

Highfield, K., & Goodwin, K. (2013). Apps for mathematics learning: A review of "educational" apps from the iTunes App Store. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today, and tomorrow*, 378–385. Melbourne, Australia: MERGA.

Holloway, I. D., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology, 103*(1), 17–29. http://doi.org/10.1016/j.jecp.2008.04.001

Kirby, K. D. de. (2013). *The development of an idealized number line: Differentiating physical inscription from mathematical object.* Presented at the American Educational Research Association, San Francisco, CA.

Larkin, K. (2014). iPad apps that promote mathematical knowledge?: Yes, they exist! *Australian Primary Mathematics Classroom, 19*(2), 28–32.

Moyer-Packenham, P. S., Bullock, E. P., Shumway, J. F., Tucker, S. I., Watts, C., Westenskow, A., … Jordan, K. E. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulatives mathematics iPad apps. *Mathematics Education Research Journal.* http://doi.org/10.1007/s13394-015-0161-z

Moyer-Packenham, P. S., Shumway, J. F., Bullock, E., Tucker, S. I., Anderson-Pence, K. L., Westenskow, A., … Jordan, K. (2015). Young children's learning performance and efficiency when using virtual manipulative mathematics iPad apps. *Journal of Computers in Mathematics and Science Teaching, 34*(1), 41–69.

Moyer-Packenham, P. S. & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. *International Journal of Virtual and Personal Learning Environments, 4*(3), 35–50.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372–377.

Murray, T., & Arroyo, I. (2002). Toward measuring and maintaining the Zone of Proximal Development in adaptive instructional systems. In S. A. Cerri, G. Gouardères, & F. Paraguaçu (Eds.), *Intelligent tutoring systems* , 749–758. Springer Berlin Heidelberg. Retrieved from http://link.springer.com/chapter/10.1007/3-540-47987-2_75

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Paek, S. (2012). The impact of multimodal virtual manipulatives on young children's mathematics learning (Doctoral dissertation). Retrieved from *ProQuest dissertations & theses full text.* (UMI No. 3554708)

Rick, J. (2012). Proportion: a tablet app for collaborative learning. In *Proceedings of the 11th International conference on interaction design and children,* 316–319. New York, NY, USA: ACM. http://doi.org/10.1145/2307096.2307155

Riconscente, M. M. (2013). Results from a controlled study of the iPad fractions game Motion Math. *Games and Culture, 8*(4), 186–214. http://doi.org/10.1177/1555412013496894

Sedig, K., Klawe, M., & Westrom, M. (2001). Role of interface manipulation style and scaffolding on cognition and concept learning in learnware. *ACM Transactions Computer-Human Interactions, 8*(1), 34–59. http://doi.org/10.1145/371127.371159

Sedig, K., & Liang, H. N. (2006). Interactivity of visual mathematical representations: Factors affecting learning and cognitive processes. *Journal of Interactive Learning Research, 17*(2), 179–212.

Tucker, S. I. (2015). *An exploratory study of attributes, affordances, abilities, and distance in children's use of mathematics virtual manipulative iPad apps* (Doctoral dissertation). Retrieved from http://gradworks.umi.com/37/23/3723089.html

Tucker, S. I., & Moyer-Packenham, P. S. (2014). Virtual manipulatives' affordances influence student learning. In S. Oesterle, C. Nicol, P. Liljedahl, & D. Allan (Eds.), *Proceedings of the joint meeting of PME 38 and PME-NA 36 Vol. 6*, 251. Vancouver, Canada: PME.

Tucker, S. I., Moyer-Packenham, P. S., Boyer-Thurgood, J. M., Anderson, K. L., Shumway, J. F., Westenskow, A.,…The Virtual Manipulatives Research Group at Utah State University. (2014). Literature supporting an investigation of the nexus of mathematics, strategy, and technology in second-graders' interactions with iPad-based virtual manipulatives. In *Proceedings of the 12th Annual Hawaii International Conference on Education (HICE)*, 2338–2346. Honolulu, Hawaii. http://doi.org/10.13140/2.1.3392.4169

Tucker, S. I., Moyer-Packenham, P. S., Westenskow, A., & Jordan, K. E. (2016, in press). The complexity of the affordance-ability relationship when second-grade children interact with mathematics virtual manipulative apps. Technology, Knowledge and Learning. Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Zhang, M., Trussell, R. P., Gallegos, B., & Asam, R. R. (2015). Using math apps for improving student learning: An exploratory study in an inclusive fourth grade classroom. *Tech Trends, 59*(2), 32–39. http://doi.org/10.1007/s11528-015-0837-y