Language, Mathematics and English language learners

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Introduction

Mathematics is sometimes referred to as a 'universal language', implying anybody with mathematical understanding can solve mathematical problems regardless of the language they speak. While arithmetical notations may be mutually understood across some languages—although certainly not all—most mathematical tasks that learners encounter in school are not 'language free'. Moreover, the language required to make sense of those tasks is not the same as the language encountered in other parts of a learner's school day. The mathematics classroom generates its own complex mix of everyday language and discipline specific language and mastery of this is key to success in the mathematics classroom. The shift between everyday and specialist mathematical language is regarded as key to the development of mathematical understandings. This is evident in most mathematics curricula, which focus on everyday language in the junior grades and specialist language in senior grades (ACARA, 2012; Barwell, 2012).

Whilst all learners find the shifts in language use in the mathematics classroom challenging, it is particularly problematic for learners who speak *English as an additional language or dialect* (EAL/D) as they are learning the English language at the same time as they are learning mathematics *through* that language. Given the growing numbers of EAL/D learners in US, UK and Australian classrooms understanding the language challenges that mathematical problem solving presents to EAL/D learners is a skill all mathematics teachers must develop.

This challenge for English language learners is acknowledged in the latest curricula in the USA Common Core Standards and the Australian Curriculum, but ways to manage these challenges are not proffered. The preamble to the Common Core Standards for Maths warns,

> It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives (Common Core Standards State Initiative, 2012).

Similarly the new *Australian Curriculum: Mathematics* begins with the following statement:

> The aims of the 'Australian Curriculum: Mathematics' are ultimately the same for all students. However, EAL/D students are simultaneously learning a new language and the knowledge, understanding and skills of the 'Australian Curriculum: Mathematics' through that new language. They require additional time and support, along with informed teaching that explicitly addresses their language needs, and assessments that take into account their developing language proficiency (ACARA, 2012).

Thus, despite widespread agreement that language is crucial to mathematical achievement, mathematics textbooks and curricula do not make the language demands of their tasks evident to mathematics teachers, and teachers are very often unaware of the linguistic complexity of the mathematical tasks they present to learners (Lucero, 2012).

In this article we provide an overview of the linguistic features of mathematical language, and use this to analyse the language challenges embedded in a typical mathematical task from a Year 10 mathematics textbook. We suggest some strategies that simultaneously address the mathematical language and the mathematical concepts. This will support English language learners in particular but we suggest that the language challenges and solutions we proffer would be useful for any learner who is struggling with the discipline language of mathematics, including native speakers of English.

The language of mathematics

Research indicates that English language learners experience a disadvantage of up to 15% in mathematics because of issues with language (Barton, Chan, King, Neville-Barton, & Sneddon, 2005; Riordain & O'Donoghue, 2009). We also know that failure to achieve in mathematics in schools is linked to poor outcomes in post school lives (Fuchs, Fuchs, & Compton, 2012). If we fail to acknowledge and address the language challenges in mathematics we cannot be certain whether underachievement is a reflection of language challenges or challenges with mathematical concepts, and we are unable to adjust our pedagogical responses accordingly.

In recent years there has been a shift to 'reformed' mathematics in schools, where mathematical reasoning is situated in real life contexts. This shift has occurred partly in order to make mathematics more accessible and relevant to learners. However, paradoxically, it can make mathematics less accessible to those who are learning the language, as the approach results in word problems that increase literacy demands in the mathematics classroom (Zevenbergen, 2001), which, in turn, increases the challenges for English language learners in the mathematics classroom (Slavit & Ernst-Slavit, 2007). Whilst increased contextual information may be beneficial for proficient language users as reported by some research (e.g., Rosales, Vicente, Chamoso, Munez & Orrantia, 2012), Halladay and Neumann (2012) note that mathematical word problems often contain extraneous information in their efforts to depict real world applications for mathematical reasoning. They note that "real-life connections can distract students with 'seductive details' and can hinder their problem-solving efforts" (Halladay & Neumann, 2012). Yet these kinds of problems are increasingly part of standardised testing (Hipwell & Klenowski, 2011).

Turner (2011) notes that with this shift comes a more sophisticated set of competencies for solving mathematical problems. He describes six competencies, two of which are directly concerned with the development of language skills in mathematics: communication, and using symbolic, formal and technical language (Turner, 2011). Importantly he suggests “the more an individual processes these competencies, the more able he or she will be able to make effective use of his or her mathematical knowledge to solve contextualised problems” (Turner, 2011). Students for whom English is an additional language require substantial support to develop these linguistically bound competencies.

Teachers are very often unaware of the challenges mathematical language poses (Gough, 2007), as the language is so familiar to them it is difficult for them to make visible to their learners what has now become invisible to them. Halliday and Martin (1993) describe mathematical language as a specific register of English that has developed over several hundred years of the development of the discipline of mathematics. Mathematicians are insiders in the discipline, with linguistic knowledge that those outside the discipline are not privy to. Gough (2007) equates mathematics teachers to multilinguals who must now consciously pass that linguistic expertise to their students. However, researchers observe a mismatch between the language of instruction in the classroom and the language of mathematics required to make sense of the tasks in the textbook (Gough, 2007; Slavit & Ernst-Slavit, 2007; Warren, Young & de Vries, 2007). In fact, Slavit and Ernst-Slavik (2007) observe “conversation in mathematics classrooms can be a barrier to understanding for English Language Learners” (Slavit & Ernst-Slavit, 2007) and Galvan Carlan (n.d.) concludes “fluency in interpersonal conversation does not equate to fluency in concepts and the discipline-specific language of mathematics”. Rosales et al. (2012) suggest that teachers themselves often revert to a “superficial approach to problem-solving” when working with word problems, failing to make the most of opportunities to teach mathematical language and concepts within the word problem (Rosales et al., 2012). Others suggest that the multiplicity of discourses in mathematics is inevitable and to talk about everyday versus specialist mathematical language in a dichotomous manner is an inaccurate description of the language required to communicate and operate successfully within the discipline of mathematics (Barwell, 2012; Moschkovich, 2008). Whilst we may accept the existence of multiple discourse patterns in mathematical language, we must also accept that these discourses, whether they be the ‘everyday’, the ‘specialist’, or some combination of these, are specific to the mathematical context. As such, the mathematics teacher cannot assume that the language required to access mathematical learning has been developed and nurtured in other curriculum classrooms. Successful reading in the English classroom does not guarantee comprehension of the text book in the mathematics classroom.

A useful way to understand the ways in the language of mathematics differs from ‘everyday’ English language and the language of other discipline areas is to examine language at three levels: text level, sentence level and word level.

Text level

Mathematical word problems are often placed in procedural or narrative contexts to give them real life application. This can introduce new information and contexts that are unfamiliar to EAL/D students and which distract them from the mathematical task embedded within the word problem. References to recipes, sport, transport, entertainment and technology common in word problems, may all be new to EAL/D learners, particularly those from refugee or remote backgrounds (Quinnell & Carter, 2011).

As students move through school, the text types encountered in the Learning Areas are increasingly specific to those learning areas: e.g., physical models with accompanying oral explanations in Science, conducting interviews and producing oral histories, historical narratives and biographies in History, oral procedural texts which explain a mathematical process in Mathematics. These text-types have not been encountered or analysed in subject English, they will only be encountered in the context of their discipline areas and must be taught within these contexts.

Visuals often accompany mathematics problems, often as contextual prompts to the language of the accompanying word problems, but sometimes as adjunct information requisite to the problem. There is a common sense perception that visuals are self-explanatory and neutral, however visual tools may look similar but be read differently across the disciplines. For example, number lines in Mathematics are scaled whereas in History timelines often are not; i.e., in Mathematics the divisions on the timeline are equally spaced, whereas in History they may be broken.

Sentence level

In their studies, Fuchs et al. (2012) have found that for students who struggle with word problems in mathematics the difficulty lay in understanding what they are to do with the numbers which are deeply embedded in a narrative; i.e., to both understand what the salient pieces of information are, and to process the language itself.

Word order or syntax can shift in mathematical word sentences. Usually in English sentences, the beginning of sentences (theme) in sentences, and topic sentences in paragraphs are important as they provide a point of departure for reading the rest of the sentence or paragraph. This provides an important sequential logic to readers of English sentences. However in Mathematics, the reader may read the sentence sequentially from left to right but the order in which they must respond to the sentence is often from right to left. For example, "Draw a circle with a diameter of one-third the sum of 6 + 9 + 15" requires the learner to start the operation from the end of the sentence and move backwards through to the beginning of the sentence. Moreover, algorithms may need to be read vertically (Adams, 2003). Even visuals in mathematics can disrupt normal English reading directionality. For example, negative number lines need to be read right to left from zero.

Sometimes sentences within mathematical texts often have very little redundancy, that is, every word is important to understanding the text. In the Language Arts, unknown words can often be guessed from context, or even skipped when the reader is unfamiliar with them and meaning can still be maintained. However, in Mathematics word problems, if one word is not understood it is probable the entire sentence will be misconstrued. For example: "Which route shows *the greatest change of direction*?" Every word in the italicised phrase is crucial to achieving the right answer.

Sometimes the meaning of the sentence is embedded within mathematical symbols, with a single symbol representing complex linguistic and mathematical concepts. For example, ≥ means greater than or equal to, representing important comparative language which is already somewhat challenging for EAL/D learners. Mathematical symbols are not universal and may be used differently in different languages (Adams, 2003). As a consequence, EAL/D learners may have other expectations of their meanings.

Sentences in mathematics convey complex relationships and abstract ideas. To achieve this they often have linguistically complex sentence structures with dependent clauses. Martiniello's analysis of test items (2008) found these structures are the most difficult for EAL/D learners to comprehend. One important function of sentences in mathematics is to hypothesise or justify. This requires the use of complex sentence structures, including combinations of past, present and future tenses. Linguistically, hypothesis is realised through conditional sentence types where we begin with a dependent conditional clause ("If x is two times y…") before asking the question in an independent clause ("What is the value of x?"). Like all grammatical structures, this syntactical pattern is particular to English and needs to be explained to English Language Learners. Indeed, some languages, including Aboriginal languages, (Roberts, 1998) do not have a specific sentence structure to signal hypothesis, instead relying on the listener to infer hypothesis.

Sentences in the passive voice are common in mathematics texts as they allow the meaning focus to be on process as those responsible for the process are irrelevant to the sentence's message. The passive voice structures are italicised in this mathematics word problem in an Australian high school mathematics textbook: "99 Roman soldiers who fled from battle *were to be punished*. The group *was lined up* and *[was] decimated*. How many *were killed*?" However, whilst the passive voice is a logical choice for the sentences, the structure is a challenging one for students who are learning English as an additional language or dialect.

Word level

Vocabulary is key to success in reading comprehension, and this is as true in mathematical reading as it is in any discipline area—perhaps even more so. In the word problem given above, the word 'decimated' is key to the mathematics of the problem. In this word problem, 'decimated' maintains its original meaning 'one in ten'. However in common parlance 'decimated' has come to mean 'completely wiped out'. Clearly, the two interpretations of the word 'decimated' will each result in a very different answer to this mathematical word problem, but only one will be correct.

Pierce and Fontaine (2003 p. 239) suggest, "the depth and breadth of a child's mathematical vocabulary is more likely than ever to influence a child's success in math". Words are used precisely in mathematical texts. Sometimes these words are discipline specific and not encountered in any other learning area, but often they are words that may be used in more commonsense ways in everyday speech. For example, "*Find* the value of x" does not require the learner to simply look for something, but to complete an algorithm. Table can mean 'times table' or table of values in mathematics, whilst having other meanings outside the classroom and in the other discipline areas: e.g., timetable, water table, table and chair, to table a report. These ambiguities are often overlooked by teachers but can cause students difficulties in word problems (Pierce & Fontaine, 2009).

Words used in specific and technical ways in mathematics are sometimes in contradiction to everyday usage of those words (Gough, 2007). Occasionally precise meanings in mathematical language grow to have a popular but less precise meaning in everyday language. Gough (2007) quotes Apulo's (1988) observation of the everyday use of the word 'fraction', as in 'selling for a fraction of its original price', implying that a fraction is small amount of a whole. This everyday appropriation of a mathematical concept may confuse students' understanding of fractions, which may indeed be larger than a whole. Studies in the US indicate that 50% of middle school students cannot order fractions from smallest to largest (US Department of Education, 2008) with similar levels of confusion with fractions found in an analysis of Australia's national mathematics tests (Perso, 2009).

Vocabulary is key to comprehension and as we progress through the years of schooling, the vocabulary of mathematics becomes increasingly idiosyncratic to the learning area and as the concepts become more abstract so does the vocabulary. Teachers need to ensure vocabulary for new topic areas is taught to these learners, for example in statistics words like cumulative, frequency, histogram, distribution and mean, median, mode. Teachers must also not make assumptions about what vocabulary their learners will already have. EAL/D learners have usually not had the benefit of years of cumulative exposure to the mathematics curriculum, and so do not have a complete mathematical vocabulary.

Additionally, words with specialist meanings in Mathematics may also have different specialist meanings within other disciplines. In Mathematics, 'positive' and 'negative' refer to integer values; in Science, they can refer to electrical charges and, in History, they can refer to attitudes. Another example is 'root' as in square root, or family roots in history, or roots and stems in Science.

Abbreviations are common in mathematics, but may be being encountered for the first time by EAL/D learners. It is important to teach the source words of the abbreviations to support these students' understanding of the concept the abbreviation is representing, for example, cm^2 = centimetres squared. Centimetre can be understood through its morphemes (meaningful parts) 'centi' meaning one hundred and 'metre', meaning measure.

Vocabulary and comprehension is developed by building students' knowledge of morphemes and word origins within words, for example, 'decimated', 'decade', 'decahedron' all contain the Greek origin morpheme 'deca', meaning ten. This knowledge helps an EAL/D learner deduce the meanings of the words, including the important fact that decimated has the mathematical meaning of 'one in ten', rather than the everyday meaning of 'wiped out'.

Understanding word families and word classes helps students understand the links between the concepts each word represents and the mathematical task they require. For example: 'multiple' is an adjective; 'the multiples of 10' is a noun; 'multiply' is a verb; 'multiplication' is a noun, a nominalisation of the process 'multiply'.

Pedagogical principles

Mathematics teachers play an important role in "helping students to use language effectively", and to meet "the need for explicit teaching of language in mathematics" (Meiers, 2010, p. 4). Whilst language may be used differently in mathematics, pedagogical strategies that work well in the English classroom are equally effective in the mathematics classroom (Halladay & Neumann, 2012). When language is unpacked and scaffolded for the learner as a routine in the mathematics classroom, learning outcomes improve (Fuchs et al., 2012). When we scaffold learners in this way we reduce the demands on their working memory, identified as an important blocker for learners for whom the language is new or challenging (Swanson, 2006) and allow them to focus on the mathematical content.

Discussion is crucial to the development of the cognitive competencies required to be successful in mathematics (Gonzalez & DeJarnette, 2012; Gough, 2007; Turner, 2011). This not only improves their ability to comprehend and communicate mathematical understandings, but strengthens the development of their mathematical ideas (Turner, 2011). However, discussion which is unstructured and unmonitored often means more language the EAL/D learner needs to decipher in order to make sense of the mathematical task.

There are multiple expressions in mathematics to express similar algorithmic functions e.g subtract, take away, minus, less, difference (Quinnell & Carter, 2011). Teachers may use all expressions in the same conversation with students in an effort to bridge between the everyday and the technical vocabulary. However for EAL/D learners this may simply cause more confusion if their synonymity is not made explicit.

Drawings and manipulatives, even in advanced classes, are a useful adjunct to teaching specific mathematical knowledge as they provide a concrete artefact upon which to build shared meaning with the learner and to scaffold the discussion.

Looking at the language of mathematics

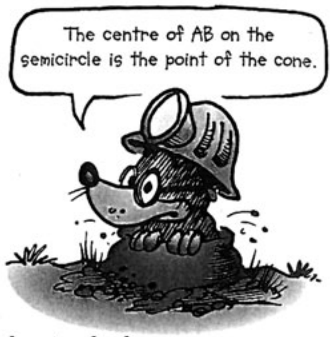
In this section we provide a language analysis of one typical mathematical task from a common mathematical textbook for high school students in Australia (McSeveny, Conway, Hardham & Wilkes, 2010; see Figure 1). The task is concerned with calculating the surface area of a cone. We identify the text, sentence and word level language features as well as the contextual knowledge required in order to make good sense of the mathematical task. We then propose teaching strategies which incorporate a language-focus to the lesson in order to make content knowledge more easily accessible to

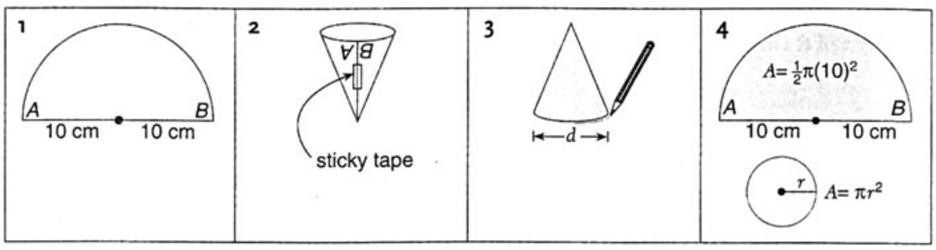
Investigation 6:03 | **The surface area of a cone**

The surface area of a cone comprises two parts: a circle and a curved surface.
The curved surface is formed from a sector of a circle.

- This investigation involves the making of two cones and the calculation of their surface area.

  Step 1
  Draw a semicircle of radius 10 cm.
  Step 2
  Make a cone by joining opposite sides of the semicircle, as shown below.
  Step 3
  Put the cone face down and trace the circular base.
  Measure the diameter of this base.
  Step 4
  Calculate the area of the original semicircle plus the area of the circular base.
  This would be the total surface area of the cone if it were closed.





- Repeat the steps above, making the original sector a quarter circle of radius 10 cm.
  What is the surface area of a closed cone of these dimensions?

Figure 1. Sample textbook task. Source: McSeveny, Conway, Hardham & Wilkes (2010). Reproduced courtesy of Pearson Australia.

EAL/D learners. The structure provided can serve as a model for any mathematical word problem.

Text level

This is a procedural math text, with an identifiable text structure of:
- title: "Investigation 6:03 The surface area of a cone";
- a definition of the topic;
- numbered instructions accompanied by diagrams.

The cohesive devices used to hold this text together include 'referring' words—words that refer back or forward to an action the reader must perform. For example:
- the definite article 'the'—e.g., Repeat *the* steps above;
- pronouns—e.g., This would be the total surface area of the cone if *it* were closed, where 'it' refers to the cone;
- pointing words—e.g., Measure the diameter of *this* base.

Some very important information is contained in cartoon speech bubbles. The mole tells us, "The centre of AB on the semicircle is the point of the cone." In other pages in the book, the cartoon characters are used to make mathematical jokes, so it may not be immediately obvious to the students that important knowledge is attached to this particular cartoon. Students from EAL/D backgrounds may not have any experience with mathematical information being presented in this manner.

Sentence level

The functions of the sentences in this mathematics problem include statements, imperatives (sentences which begin with verbs) and a question. Simple, compound and complex sentences are used as follows (the verbs are italicised):
- Simple sentence (has one verb):

  The surface area of a cone *comprises* two parts: a circle and a curved surface.
- Compound sentence (two simple sentences joined with a conjunction):

  Put the cone face down <u>and</u> *trace* the circular base.
- Complex sentences (one simple sentence, with additional dependent clauses):

  Make a cone by *joining* opposite sides of the semicircle, as *shown* below.

Complex sentences are more difficult to comprehend than simple sentences, and a higher degree of English language proficiency is required. One of the challenges of mathematical texts is the ambiguity which can occur between the clauses in a sentence (Martiniello, 2008). For example, in the complex sentence, "*Repeat* the steps above, *making* the original sector a quarter circle of radius 10 cm", the cause and effect relationship is implied rather than clearly stated. What it is really saying is to 'follow the above steps, but at step 1 draw a quarter circle of radius 10 cm instead of a semicircle'.

Another example of a complex sentence in the maths problem is "This would be the total surface area of the cone, *if it were closed*." To understand this sentence, which ostensibly provides the summary for calculating the area of a cone, the reader must understand:

1. 'This' refers to the calculation achieved from successfully completing the previous four steps.

2. ‘if it were closed’ is a crucial dependent clause which describes the actual structure of a cone
3. ‘it’ refers to the net of the cone

Similarly challenging is the shift between the imperative voice, where the reader is instructed what to do ‘Draw a semicircle of radius 10 cm’ and the use of the passive voice where the action is assumed to be done by an unknown actor ‘The curved surface is formed from a sector of a circle’.

The phrases in each of the sentences in this mathematical task contain no redundancy; all words must be understood for the learner to perform the operation accurately. For example, to comprehend ‘the area of the original semicircle’, the learner must have a complete mathematical understanding of the words area, original, semicircle. Additionally, the word order of some of the phrases do not follow the patterns of everyday English, for example, ‘a semi circle of radius 10 cm’ rather than ‘a semi circle with a 10 cm radius’. Other examples of elaborate noun phrases in this text include ‘the surface area of a cone’, ‘a sector of a circle’, ‘opposite sides of the semicircle’, ‘the diameter of this base’, ‘the total surface area of the cone’, ‘the surface area of a closed cone of these dimensions’.

Word level

The word ‘sector’ is key to this text. It refers to a part of a circle, which may be different from other encounters students may have had with the words, for example, the financial sector or the public sector. Other context specific vocabulary which may be encountered for the first time by EAL/D learners include radius and diameter. Words which may be familiar to learners in an everyday sense, but which have a specific mathematical meaning in this context include: area, surface, cone, base, dimension. Two important processes in this mathematical task are encapsulated in the words ‘investigation’ and ‘calculation’. These two nominalisations replace the process words ‘investigate’ and ‘calculate’ and make the text more abstract and harder to comprehend for English language learners.

Teaching activities

Mathematical word problems are often challenging for EAL/D learners. They often miss crucial information because of their limited linguistic resources, such as not being familiar with contextual knowledge, text type knowledge, grammar and vocabulary. To ensure that mathematical concepts are learned, teachers should incorporate language teaching into their mathematical instruction. Strategies can be used in preparation for reading the mathematical task with the students, as well as during and after reading to consolidate and confirm understanding.

Before reading, ensure there is a shared understanding of the topic by looking at cones, for example deconstructing and reconstructing cardboard conical party hats. Analyse the text to be studied, similar to the analysis provided above, and identify key vocabulary. Build a class glossary of definitions of key words and phrases, accompanied by diagrams where appropriate.

During reading, identify the salient parts of the word problem, those words and phrases that are crucial to the mathematics of the problem. The following lesson procedure could be used for the example mathematics task in this article.

- Give students the title “Investigation 6:03 The surface area of a cone” and four numberless diagrams.
- Ask students to sequence the four diagrams in a correct order. Teachers may ask students with high English proficiency to explain how they sequence the diagrams.
- Ask students label each of the diagrams in their own words.
- Give students cut-out sentence instructions as written in the textbook to match with the diagrams. Also provide the cartoon mole’s statement for students to match with the diagram that represents the statement.
- Ask students to compare their own sentences to the cut-out sentences. Have they said the same thing? In what ways is the text book language different from the student language?
- Complete math activity (make two cones and calculate both cones’ surface area and volume)
- After reading, students practice new vocabulary and new sentence structures
- Ask students to write a sentence or two to compare and contrast those two cones they made (one made by a semicircle, and one made by a quarter of a circle);
- Students write a set of instructions for making a cone and calculating surface area, using different dimensions. Notice the imperative sentence structures in procedural text.
- Students make their own activity in pairs. They may draw geometry shapes, such as a cylinder, give their partners verbal instructions (the partners do not know what the shape looks like), check each other’s drawings and swap the roles.

Conclusion

There is a correlation between language proficiency and achievement in mathematics (Riordain & O’Donoghue, 2009) and this is particularly evident for children who speak English as an additional language or dialect. More effort needs to be made in mathematics classrooms to develop cognitive competencies, including the ability to decode and encode mathematical problems, and use appropriate mathematical language when doing so (Turner, 2011).

Although the mathematics teacher has much content to deliver, mathematics content is delivered through language and so all mathematics teachers are teachers of the language of mathematics. Yet, this is a language that has become invisible and intuitive to the teachers of mathematics but which remains invisible and confounding to their learners. Gough (2007) makes the worrying observation that when teachers are not aware of the language ambiguities and challenges in mathematics they fail in their teaching responsibilities and instead lay the blame at the feet of their students, quoting “‘learning difficulties’, cognitive confusion, attention deficits” (Gough, 2007). The consequence is that learners struggling in the mathematics classroom are misdiagnosed, and presented with ineffective or irrelevant interventions.

When students are left to struggle, with their challenges misunderstood, their achievement levels in mathematics continue to drop along with their opportunities for positive post-school outcomes (Fuchs et al., 2012). If the national curricula and Core Standards in countries around the world are to make good on their claims to promote excellence and equity in education then mathematics teachers must take up the challenge and teach both the content and the language that is specific to Mathematics.

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