Using technology effectively to teach about fractions

Karina K.R. Hensberry
University of South Florida,
St. Petersburg USA.
<khensberry@usfsp.edu>

Emily B. Moore
University of Colorado, Boulder, USA.
<Emily.Moore@colorado.edu>

Katherine Perkins
University of Colorado, Boulder, USA.
<katherine.perkins@colorado.edu>

This article describes how well designed fraction simulation software carefully introduced in classrooms can assist students to understand fractional concepts through problem solving and reasoning. How to implement such an approach in the classroom is examined.

Introduction

"This is pretty cool," announces Alera, a 6th grade student, as she explores the PhET *Build a Fraction* interactive simulation (all names are pseudonyms). Third grader Jenny responds similarly to PhET's *Fractions Intro Simulation,* giggling and making sound effects as she investigates fractions. You read that correctly: students, excited about learning fractions! In this article, we describe classroom use of technology that successfully engaged fourth grade students (typically aged 9–10) in the United States in learning about fractions. The activities involved the use of an interactive simulation designed to support student learning of fractions, and whole-class discussion where students were prompted to reflect on their learning with the simulation. We found this discourse-rich environment coupled with simulation use to be motivating to students and supportive of their growing understanding of fraction ideas.

What are PhET simulations?

Computer technology is most effective when combined with standards-based instruction (Li & Ma, 2010) and constructive practices (Vahey et al., 2010; Wenglensky, 2005). One example of computer technology that is designed based on constructivist principles and intended to support standards-based instruction comes from the PhET Interactive Simulations Project at the University of Colorado Boulder in the United States. The PhET project has designed a suite of free interactive simulations, or "sims", for teaching mathematics and science. The sims, which cover a range of topics for all grade levels and are available at http://phet.colorado.edu, can be used to teach many of the standards described in the *Australian Curriculum: Mathematics.*

Interactive simulations are dynamic virtual environments that model a system and allow users to interact with that system, as well as receive feedback during their interactions (Hensberry, Paul, Moore, Podolefsky, & Perkins, 2013). Similar to virtual manipulatives, these sims provide students with a "visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Boylard, & Spikell, 2002, p. 373). Both types of tools support conceptual understanding, are designed to be used interactively, encourage and support the active process of knowledge construction, provide immediate feedback, and support students to make connections between multiple representations (Moyer et al., 2002; Reimer & Moyer, 2005). Additionally, PhET sims build upon the capabilities of virtual manipulatives by including implicit scaffolding—guidance and feedback provided without the use of explicit on-screen text instructions—to support students in developing informal rules and relationships (Paul, Podolefsky, & Perkins, 2012; Podolefsky, Moore,

& Perkins, 2014). This implicit scaffolding allows for students to use effectively the sims with minimal written instructions, making them flexible for teachers as well as useful tools for struggling readers or second language learners. Finally, something we do not typically see with virtual manipulatives but is a common element of PhET sims is the inclusion of content-focused games in which students can test their understanding and revise and refine their ideas.

While technology can have inherent benefits for both teachers and students, how that technology is implemented in the classroom also impacts learning (Drijvers, 2012). Here, we highlight one sim and offer a classroom example of its use to explain the ways teachers can use sims and provide students with opportunities to actively explore and discuss mathematical ideas to support them in building conceptual understanding.

An example PhET simulation

PhET's *Fractions Intro* sim (http://phet.colorado.edu/en/simulation/fractions-intro) has four tabs (Figure 1), and focuses on early fraction concepts as part of the *Australian Curriculum: Mathematics* for Years 1–5. Notice that there are few words and no directions. Each PhET sim is designed to be intuitive and to implicitly guide—rather than use on-screen text directions to explicitly guide—student interactions.

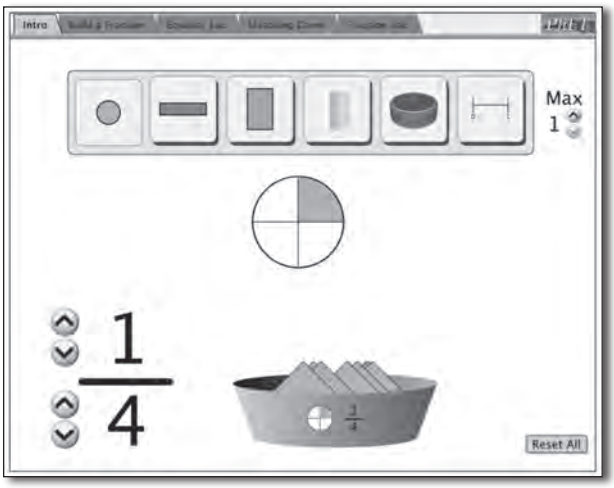


Figure 1. PhET's Fractions Intro simulation addresses learning goals related to developing an understanding of the definition of a fraction, and fraction equivalence, as well as make a connection across representations. Ms Brown displayed the sim at the front of the room for all students to see. This helped to focus their whole-class discussions.

The first tab, Intro, functions like a virtual manipulative lab with the goal of allowing students to explore the concept and symbolic notation of fractions. Specifically, students can build fractions by either dragging fraction pieces into the central play area or by modifying the symbolic notation of a fraction by clicking the yellow arrows. These actions enable students to learn to model and represent unit fractions and their multiples to complete a whole, through exploring, for example, how $\frac{1}{4}$ is the amount obtained by taking the whole, dividing it into four equal parts, and taking one of those parts. Translation among representations is also supported by the sim's ability to model for the user how a change in one representation results in an immediate and corresponding change in another representation. For example, as a student changes the numerical value of the denominator, the number of partitions in the shape and the size of the fraction pieces change as well. In the Intro tab, students can also explore realistic representations (e.g., cakes or graduated cylinders), and what the same fraction looks like in different pictorial representations (e.g., circles, rectangles, or a number line). The sim begins with one whole for simplicity, which students can change using the "max" buttons to increase from one to six wholes.

The second tab, Build a Fraction, challenges students to create specific target fractions (see Figure 2). Goals of this tab are to enable students to build an understanding of fractions as partitioning and iteration (see Siebert & Gaskin, 2006), to support representational fluency, and to provide scaffolding toward building an understanding of equivalence. Students are presented with several fractions (represented pictorially) that they must construct—or 'build'—using symbolic (i.e., numerical) cards. Other challenges present students with symbolic fractions, allowing students to construct these fractions using a pictorial representation. As students progress through each series of challenges, the type and number of build pieces available are constrained in order to implicitly guide students to build fractions in more than one way (e.g., building $\frac{5}{6}$ with five sixth-sized pieces, or building $\frac{1}{3}$ with two sixth-sized pieces when there are no more one-third sized pieces available). Students must build all of the target fractions correctly to complete a given level,

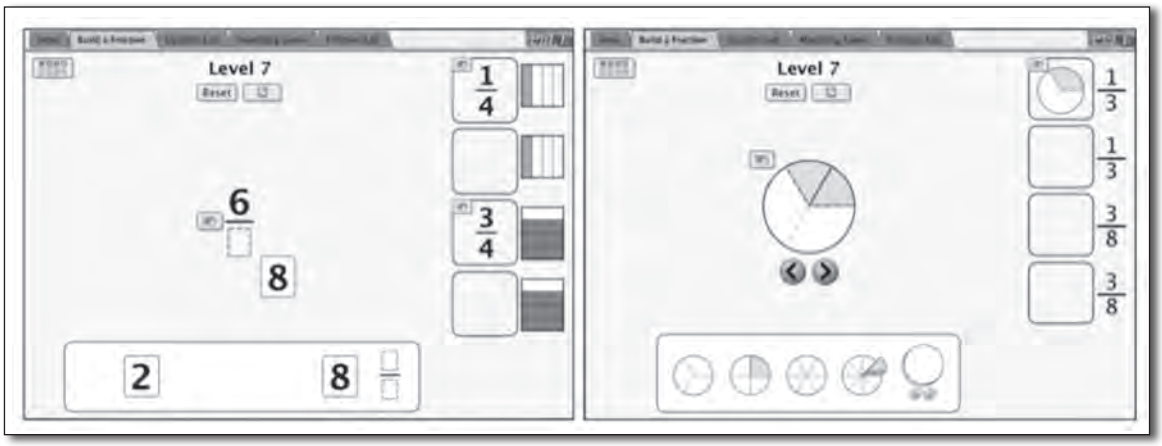


Figure 2. Challenges in the Build a Fraction tab implicitly support students to develop representational fluency and an understanding of fraction equivalence.

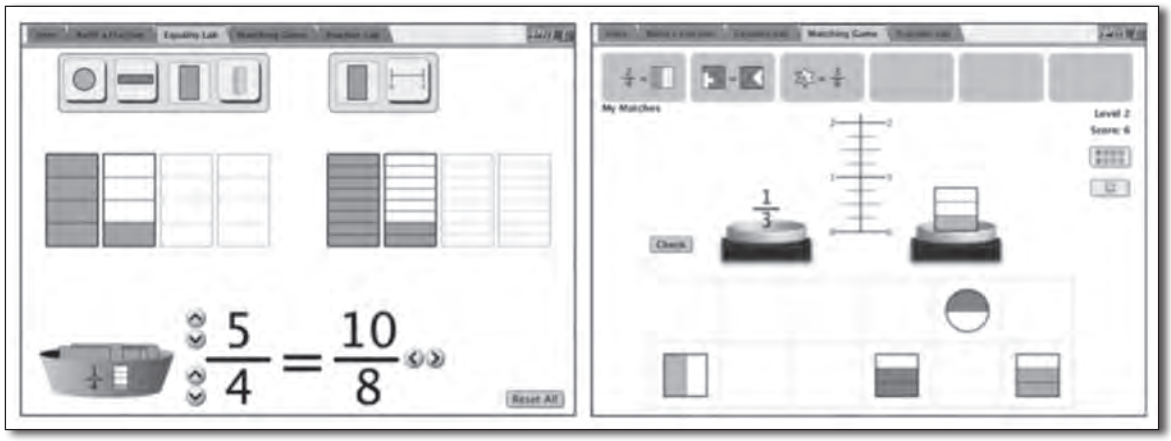


Figure 3. The Equality Lab and Matching Game tabs of PhET's Fractions Intro simulation. These tabs address content goals related to equivalence.

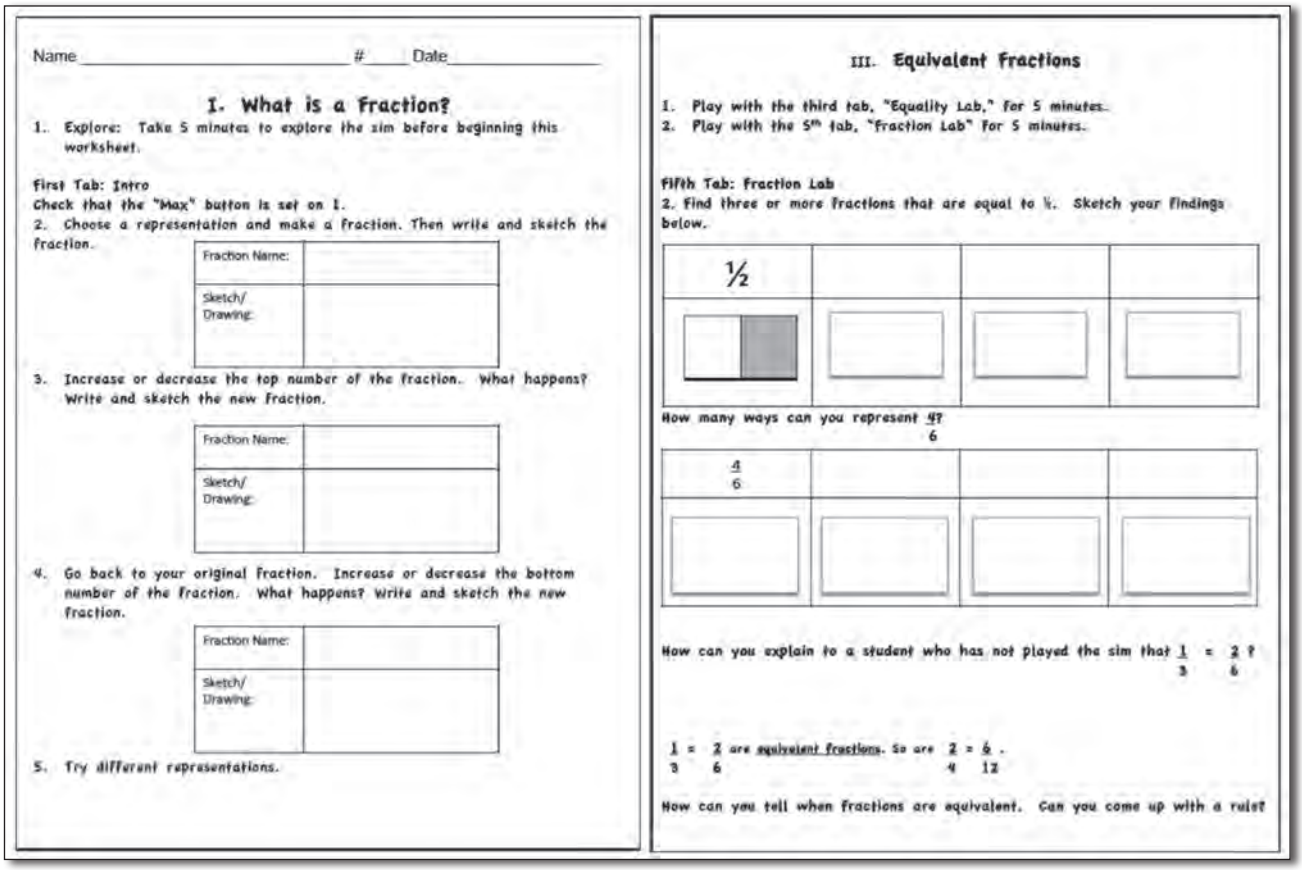
but they can do so at their own pace and in any sequence with no penalties for building a target fraction incorrectly.

The third tab, Equality Lab, is similar to the Intro tab in that students can make and modify fractions, though Equality Lab allows students to explore equivalent fractions rather than a single fraction. Multiple representations of the fractions (i.e., pictorial, symbolic, number line) are displayed (see Figure 3).

The final tab, Matching Game, features a game in which students must identify equivalent fractions. Once students select a pair of pictorial or symbolic fractions that they believe to be equivalent, the computer displays both fractions on a number line—to indicate whether the solution is correct or to provide visual feedback if students' selections are incorrect. Students accumulate points for each correct answer and are provided with opportunities to revise incorrect solutions.

Teaching with PhET simulations

How might you make use of sim technology in order to support your students' knowledge of fractions? We have found that the sim, effective teacher facilitation, and a well-designed activity sheet can work together to create an effective learning environment for students. Each component supports how students approach the lesson, the mathematics content, and their interactions with the technology. PhET sims are intentionally designed to be intuitive, flexible tools and can be integrated into instruction in many ways: as part of a student-centered lesson; as a pre-class activity; as a post-instruction assignment (e.g., homework); or in lieu of or in conjunction with traditional concrete manipulatives, to name a few. This section first describes effective teaching strategies enacted by one teacher, who used *Fractions Intro* and engaged students in discourse around the ideas they explored in the sim.



Name _____ # ____ Date _____

I. What is a Fraction?

1. Explore: Take 5 minutes to explore the sim before beginning this worksheet.

First Tab: Intro
Check that the "Max" button is set on 1.

2. Choose a representation and make a fraction. Then write and sketch the fraction.

Fraction Name:	
Sketch/ Drawing:	

3. Increase or decrease the top number of the fraction. What happens? Write and sketch the new fraction.

Fraction Name:	
Sketch/ Drawing:	

4. Go back to your original fraction. Increase or decrease the bottom number of the fraction. What happens? Write and sketch the new fraction.

Fraction Name:	
Sketch/ Drawing:	

5. Try different representations.

III. Equivalent Fractions

1. Play with the third tab, "Equality Lab," for 5 minutes.
2. Play with the 5th tab, "Fraction Lab" for 5 minutes.

Fifth Tab: Fraction Lab

2. Find three or more fractions that are equal to ½. Sketch your findings below.

½			

How many ways can you represent $\frac{4}{6}$?

$\frac{4}{6}$			

How can you explain to a student who has not played the sim that $\frac{1}{3} = \frac{2}{6}$?

$\frac{1}{3} = \frac{2}{6}$ are equivalent fractions. So are $\frac{2}{4} = \frac{6}{12}$.

How can you tell when fractions are equivalent. Can you come up with a rule?

Figure 4. Examples of activity sheets used in Ms. Brown's class. After completing the explorations guided by the sheet on the left, students were asked to describe the role of the top and bottom numbers in a fraction. The sheet on the right includes the questions that were the culminating activity for the week, focusing students on developing their own rules for determining equivalence.

Next, we highlight the use of PhET sims with student-centered, 'hands-on-sim' activities and provide guidelines for designing and evaluating sim-based activities.

Teacher facilitation in the classroom

Interactive simulations provide students with a shared experience upon which to reflect, creating unique opportunities for whole-class discussions. The following vignette from a four-day sim-based unit using the *Fractions Intro* sim, taught by fourth grade teacher, Ms Brown, highlights strategies teachers can use to effectively engage students in sim-based lessons. This vignette offers an example of an authentic classroom episode useful for highlighting specific strategies teachers can use to support learning with this technology. Simulations offer many more opportunities beyond what is represented in this episode. For instance, Ms. Brown could also have taken advantage of affordances of the sim to support thinking about fractions as partitioning and iteration.

We use italics throughout this section to explicitly highlight the effective strategies used by Ms. Brown that can support learning mathematics with sim technology. The unit addressed US fraction content standards equivalent to those outlined in the *Australian Curriculum: Mathematics* for Years 3 and 4. Each day, students explored the sim without instruction for five minutes, and then worked through an activity sheet that guided their exploration with the sim for 15—20 minutes (see Figure 4 for an example). Throughout the lesson, Ms Brown engaged students in reflection and discussion about the mathematics.

This vignette occurred toward the end of the first day's lesson, after students had utilised the sim and completed the first page of the activity sheet. Students had already established that pieces in any given fraction must be equal in size, and were working on understanding fraction notation. Ms. Brown invited students to sit at the front of the room for a class discussion about the components of a fraction. She projected the first tab of the sim onto the interactive white board and displayed the fraction $\frac{1}{4}$ (see Figure 1) as students shared out what they had discovered

through their interactions with the sim and small group discussions.

Ms Brown: How would you describe the top number in a fraction? … Derek, what do you think?
Derek: Me and Randy said that that the top number was one of the add-ins.
Ms Brown: What do you mean by that?
Derek: The top number is how many is in, or is not in, the bottom number.
Ms Brown: Okay, so Derek is saying the top number, is how many are "in" the bottom number. What do you mean by that? So, in this example? [Points to the screen.]
Derek: In this example, one is how many … pieces there are in the four different slots.
Ms Brown: Okay, does anybody have anything they want to add on to Derek's thinking? Jessica?
Jessica: Tereza and Taka decided that it's like how much you shaded in the fraction.

Here, Ms Brown pressed Derek to communicate precisely, and pointed him toward a specific example in the sim to help facilitate his explanation. Ms Brown also encouraged students to build ideas together when she asked Jessica to add on to Derek's thinking. This co-construction of ideas is an important part of building understanding in a discourse-rich mathematics classroom.

Next, M. Brown revoiced student ideas by restating Derek and Jessica's contributions (and later, Carolina's) in a more precise manner:

Ms Brown: Do you guys agree? Derek and [Jessica's] groups had a similar understanding. They said the top number was kinda (sic) how much you shaded in. [Students agree in unison.]
Ms Brown: Okay, so what would happen if I increased that top number right now? Carolina, what would happen?
Carolina: If you put it at [two-fourths], then two of the shapes would be shaded in.
Ms Brown: Okay, do you agree with Carolina? So she said, if we increase the top number to two, then two pieces would be shaded in? [Students agree in unison.]

Ms Brown prompted Carolina to make a prediction when she asked, "What would happen if…?" She frequently used prediction questions during classroom discussions and while circulating as students worked. This type of practice can support students to engage in mathematical practices such as making conjectures. The sim is a useful tool for supporting problem solving, perseverance, and reasoning skills, as students can check their predictions and receive immediate feedback regarding the accuracy of their predictions. Many of the students in Ms. Brown's class had independently taken on this important behavior of asking questions and making predictions in their small groups as well.

Ms Brown: Okay, lets move to the bottom number. Take a look at the second question [on the back of your worksheet]. How would you describe the bottom number of a fraction? We'll start with that. So what's that bottom number, Rick?
Rick: How many that there are in all, like, there's (sic) four squares.
Ms. Brown: Can someone paraphrase what Rick is saying? [Long pause.] Rick, say it one more time, and then if you have an idea of what Rick is saying you can say it in a different way.
Rick: It's how many spaces there are in the circle or whatever you are using.
Ms. Brown: Okay, Dave, what is Rick thinking?
Dave: He, like, the bottom is how many are in all, and then the top number is how many is shaded in.
Ms Brown: Okay, so the bottom number is how many what in all? A lot of you are saying, "how many in all". How many? Hudson?
Hudson: Spaces.
Ms Brown: Spaces. How many?
Carolina: Pieces!
Ms Brown: How many pieces, and what do you notice about the pieces?…If I increased the bottom number, what would happen? [Pause. Points at projected sim on board.] So right now there's (sic) four pieces. If I click up and the bottom number increases, what happens? [Several students speaking at once.]
Ms Brown: It changes the amount of what?
Student: It will make the pieces smaller.
Ms Brown: The pieces will get smaller, yes.

Similar to revoicing, Ms Brown has asked Dave to rephrase what Rick has said. Asking students to rephrase what others have said encourages them to listen to each other, which is an important part of class discussions (Cazden, 2001) because it can allow students to learn from one another (Hintz, 2014). Furthermore, paraphrasing allows an unclear explanation to be made clearer to other students, and may support students in developing their argumentation skills. Ms Brown's regard for precise and clear explanations is also evident when she asks Hudson and Carolina to more accurately describe how many of what the bottom number represents.

Finally, notice that in this discussion, neither Ms Brown nor the students ever use the terms "numerator" or "denominator". Formal language can overwhelm or distract students if it is introduced too early; Ms Brown chose to introduce formal language the following day, after students had explored fraction ideas on their own. The sim allowed students the opportunity to build conceptual understanding through their exploration before being introduced to formal vocabulary.

Activity design for classroom use

In addition to the sim and teacher facilitation, an activity sheet like the one used by Ms Brown can be an important part of a successful sim-based lesson. An effective activity sheet will keep students' primary focus on engaging with and learning from the sim—as opposed to demanding their focus be on the activity sheet itself—while also gently guiding student engagement toward the learning goals, and providing a space for students to record ideas and findings. The authors have outlined strategies for designing activity sheets elsewhere (Hensberry et al., 2013; PhET Interactive Simulations, 2012). A particularly effective strategy for these activity sheets is seizing opportunities within sims to challenge students by asking open-ended questions such as: "What is the largest fraction you can make?", "How many ways can you represent $\frac{4}{6}$?", and "Develop a strategy for identifying equivalent fractions."

Whether you write your own activities or use or adapt activities from the PhET activity database (http://phet.colorado.edu/en/for-teachers/browse-activities), the following list of questions can help you evaluate how the sim-based activity aligns with PhET's recommended strategies:

- Does the activity begin with 5–10 minutes of free exploration time? This allows students an opportunity to explore the sim and begin thinking about the concepts before continuing.
- Are explicit directions on how to manipulate the sim minimised or removed? Providing explicit directions can trigger direction following as opposed to reasoning and sense-making. With their intuitive controls, sims allow teachers to ask open-ended questions without providing explicit instruction on sim use.
- Is the activity sheet short and does it contain minimal wording? In this way, the activity sheet acts as tool for guiding and organising progress. Making use of 'concept tables' and providing sim representations like those in Figure 4 can reduce the need for lengthy directions. Together, these approaches allow teachers the time to solicit and flexibly respond to student-generated ideas through facilitated discussion.
- Does the activity solicit student ideas? Does it provide opportunities for students topractice mathematical reasoning? Inviting students—in paired or whole-class discussions—to share ideas, demonstrate and justify their findings, and engage in problem posing and sense making through the use of the sim provides opportunities to direct attention, give feedback, and increase student ownership over their learning.

We found that the discourse-rich, sim-centred unit described above led to learning gains on assessments that measured students' procedural and conceptual understanding of fraction ideas related to numerator, denominator, and equivalence. Learning gains were observed with all students, including English-language learners and special education students (Hensberry, Moore, & Perkins, [in press]). These findings indicate that use of PhET interactive simulations, coupled with teacher facilitation and activity sheets that support students' engagement and discussion of mathematical ideas, can result in effective lessons incorporating technology.

Conclusion

PhET interactive simulations are a new tool for teaching conceptual mathematics and mathematical practices. New research and feedback from teachers suggest that PhET sims engage students and support student learning of mathematics. As we saw in Ms Brown's classroom, sims can provide students with shared experiences, serving as a context to focus whole-class discussions. Bringing the class together after students have had time to explore the sim and develop some concepts on their own allowed for rich conversations that supported student learning of fraction ideas. We encourage educators to explore the range of PhET simulations available and browse or contribute to the activities database.

Note to readers

Share the ways you are using PhET sims for teaching mathematics, and send us your suggestions for new sim topics by emailing phethelp@colorado.edu.

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