Measurement
Five considerations to add even more impact to your program

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The potential of using Measurement as a way of "tuning students into mathematics" is demonstrated. Five ideas that can form the basis of focusing on measurement to access other strands of the mathematics curriculum are examined.

In this article I would like to look at some key considerations which have proven to be very handy in the teaching of measurement in the primary classroom. Developing an understanding of the following considerations and keeping them up-front in my teaching has given my measurement lessons much more focus than they once had, and has allowed me to deliver a more comprehensive and hopefully engaging program of work in this important strand.

One of the refrains you can hear coming from the mathematics classroom is: "When will I ever use this?" This is not an accusation that should be legitimately used about the measurement strand. The ubiquitous nature of measurement in dealing with our societal, professional and personal lives means that anyone who does not have the capacity to be efficient and effective with measurement could not really call themselves numerate (Serow, Callingham & Muir, 2014). On any given day we require measurement to operate in the world. Measurement is a rich place to think about mathematics and can be used as a way of giving students a contextual need for learning and using other mathematical strands, particularly number and place value, but also geometry and probability (Van de Walle, Karp & Bay-Williams, 2013).

Measurement is also a strand of mathematics which can be shown to naturally link with many other parts of the school curriculum. Examples of measurement can be readily found in areas such as art, science, history, geography and music. There is also a plethora of literature which is predicated on measurement or requires an understanding of measurement in order to be properly understood. Further, as Reys et al. (2012) wrote:

> Another reason measurement is an important part of the mathematics curriculum is not so much the mathematical but the pedagogical. Measurement is an effective way to engage many types of student, some of whom would be less motivated to learn other topics. They often see the usefulness of the tasks as it relates to them personally (p. 404).

This engagement stems from the fact that if taught properly, measurement requires action, socialisation and reflection. A measurement lesson should not be a 'passive' lesson.

If you teach in a school in which mathematics seems to cause anxiety and is not well received, then it may just be worth considering centring the mathematics program on measurement for some of the school year. Rather than making the emphasis of the mathematics program come from a focus on the number strand, some very deliberate and purposeful teaching and learning of number authentically rising from the need to complete measurement tasks may be a way in which to proceed.

I would like to share five ideas that I have found useful to be aware of regarding the teaching of the measurement strand.

There is a teaching sequence

There is a well-researched and well-established sequence to teaching measurement (Booker, Bond, Sparrow & Swan, 2010; Serow, Callingham & Muir, 2014; Van de Walle, Karp & Bay-Williams, 2013). This sequence works across all attributes of measurement.

Identify the attribute to be measured

The first step in the measurement process is to ask the question about what is being measured. For example, if a box is placed in the middle of the room and the question of how big it is is asked, the first response from the students should be about what needs to be measured: is it the weight of the box, the height of the box, the width of the box, the volume of it, the capacity of it, or even the surface area?

Compare and order

A key and arguably, *the* key to any measurement is the idea of *comparison* (Booker et al., 2010; Drake, 2014). Comparison is part of human nature. On an individual level we seem to be constantly comparing ourselves to those around us. Early on, the comparison is often quite general in nature; questions such as "Who is taller? What is heavier? Which is longer?" [If you have any doubts about how inquisitive we are about these sorts things, just consider the sales for Guinness World Records (formerly The Guinness Book of Records) which between 1955 and 2013 had sold 132 002 542) copies. These sorts of questions then give way to a desire to be more specific and this is when the need for quantification starts to become apparent.

Use non-standard units

It may be surprising to some people that the majority of understandings that will be eventually developed to work with standard units are the same understandings which are developed with non-standard (or informal) units. Understandings such as:

- The unit of measurement chosen is chosen through convenience and appropriateness. It is more appropriate to measure the length of a netball court with strides than hand-spans or with metres than centimetres.

- Once the unit has been chosen it must stay the same. It is not appropriate to start a measurement in hand-spans and then change to hand-lengths. This is an understanding which many young students find challenging (Clements & Sarama, 2007).
- The largest number of the same unit represents the greatest amount of the attribute being measured, that is, (under the constriction that the units are the same 'size') four cubes is 'more' than three cubes.
- Estimation should always precede measurement.
- Understand the degree of accuracy appropriate in a given situation and work within an acceptable level of tolerance of error.
- There can be no gaps or over-laps when lining up the units.
- Measurement relies on an inverse relationship, that is, the larger the unit, the less of that unit is required to measure an object. For example, five heavy washers may be the same mass as 12 light washers.
- A smaller unit gives a more exact measurement (Booker, et al., 2012; Reys, et al., 2012; Serow, Callingham & Muir, 2014).

A second consideration which may prove challenging is that standard units are actually no more accurate than non-standard units. That is, if something is 12 paper clips long, this is just as accurate as saying something is 24 centimetres long. The issue with non-standard units is not with accuracy but more about when we try to transport or communicate these measurements to others. Can they replicate the length of my 24 paper clips exactly, or will the length of one of their paper clips be different from the ones I am employing and therefore the over-all length be different?

Use standard units

A quick scan of the understandings which can be developed through the use of non-standard units reveals that these are the same understandings that are required to work efficiently and effectively with standard units. In Australia the standard units are derived from the metric system, a very uniform system which is predicated on a base-10 under-standing and therefore has a strong link between measurement and number. Students first need to develop a familiarity with the standard unit,

the basic size of the units, which units are appropriate to which situations for a required level of precision, and what the relationships are between commonly used standard units (Siemon et al., 2011; Van de Walle, Karp & Bay-Williams, 2013).

Standard units are important when measurements need to be recorded for later use, or need to be communicated over time or distance, or are to be used in a calculation. As standard units are an agreed measure, a person can be sure that if they order a piece of piping to be shipped from Europe to Australia that is to be three metres long and has a diameter of 1.5 centimetres, then that is what will arrive. Asking someone in Europe for a piece of pipe which is two and a bit arm spans long and that my index finger will fit snugly into may not provide you with exactly what is required!

Application of measurement (formulas)

Once students are proficient with using standard units they can be provided with opportunities to use formulas and apply their measurement skills and knowledge. The formulas for area, perimeter, volume and surface area are usually developed in the later years of primary school. Whilst formulas are very necessary, they should be a product of carefully developed processes and understandings, not a replacement of them.

There are several different attributes of measurement that need attention

In discussing the teaching sequence, the attributes of measurement have often been mentioned. There are seven common attributes of measurement taught in primary schools and these are: length, area, volume, capacity, mass, time and temperature. An eighth attribute is angle but mention of this is not made in the *Australian Curriculum* in the measurement strand but rather the geometry strand (ACARA, 2014). However most mathematics educators consider it to be a measurement attribute (e.g. Reys et al., 2012; Serow, Callingham & Muir, 2014; Van de Walle, Karp & Bay-Williams, 2013).

It may be surprising, but many mathematics educators also include money as a ninth attribute (Booker, et al., 2010; Serow, Callingham & Muir,

2014; Van de Walle, Karp & Bay-Williams, 2013) as they regard money as a measure of value. There are also three less well explored attributes of measurement in the primary school setting: speed, density and probability (Reys et al., 2012).

When working with all attributes of measurement there seems to be a natural hierarchy of questions.

- Which is biggest (longest, widest, tallest, heaviest, etc.)?
- How big (long, wide, tall, heavy, etc.) is it?
- How much bigger (longer, wider, taller, heavier ,etc.) is it than the other one?

Each of these questions shows an increasing level of mathematical sophistication. The first question asks for a perceptual comparison; quite simply, if the items start at a common baseline (for example the table top), which is taller? This then can be developed from a direct comparison of objects to an indirect comparison of objects. If I cannot move two objects together to make a comparison, I can (for example) cut a piece of string the length of the first object, and place it next to the second object make the comparison.

Question two asks students to quantify the attribute. Whether the quantification is using standard or non-standard units, the notion is to determine how many units 'big' something is, for example, 24 paper clips long, eight marbles heavy, 13 centimetres tall. It should be noted that there is quite a pronounced difference between using non-standard units and additively determining the size of an object, and reading a scale from an instrument to determine its size.

As in question two, question three is about quantifying, but in this case, establishing the difference between two or more objects or shapes. This usually means measuring all of the objects which are being compared and then performing a mathematical operation (usually, but not always, a subtraction) to calculate the difference. It is at this point when a 'problem' can become apparent, and this will be discussed in the following 'idea'.

Many measurement lessons are not measurement lessons

The 'problem' that can become apparent is that once calculations start to become necessary in a

measurement lesson, it can quite easily become the point where students stop engaging in some of the necessary skills and understandings of measurement, and the lesson essentially becomes a number lesson. It can happen that entire measurement lessons can be conducted without the students necessarily engaging in any actual measurement. They are completing calculations using measurement as the context, but not actually engaging in measurement. That is not to suggest that these calculation lessons are not important and worthwhile, but they should not be categorised as measurement lessons. It is always worth remembering, that in every measurement lesson, an act of measurement should occur.

Estimation is important

I would like to propose that in every measurement lesson (and in fact in every lesson) the students are required to use and develop their powers of estimation, and therefore that as teachers we develop the notion that that there is not measurement without estimation. Estimation is an important and, I believe, a much under-utilised skill. Estimation is important because:

- without it, the focus of the mathematics lesson can be on number rather than measurement (Booker, Bond, Sparrow & Swan, 2010);
- it reinforces the size of units and the relationship among units (Reys, et al., 2012; Van de Walle, Karp & Bay-Williams, 2013);
- it has practical implications (Reys, et al. 2012) as it is an essential skill for anyone involved in trade to determine the amount of materials that may be required to complete a job;
- it helps focus on the attribute being measured and the measuring process (Van de Walle, Karp & Bay-Williams, 2013);
- it provides intrinsic motivation for measurement as it is interesting to see how close your estimation is (Van de Walle, Karp & Bay-Williams, 2013.)

The significance of estimation as an everyday and natural aspect of measurement needs to be conveyed to students, as many students tend to view estimation as difficult, where success is judged by how close the estimation is to that of

the estimation of the teacher (Muir, 2005), rather than an expected and necessary part of the measuring process.

Understandings developed well in one attribute can be transferred to other attributes

As is illustrated in the previous four ideas, if students deeply understand measurement ideas about one kind of attribute, they can transfer their knowledge to other attributes (Booker et al., 2010). This is good news for busy teachers. What it means is that the teaching and learning experiences for the attribute of length (length is the attribute that most students encounter first (Reys, et al., 2012)) should be systematic, well-considered and aiming for depth of understanding. By doing so, when the next attribute is being taught and learned, the teaching sequence, how students think about the attributes and the need for estimation, along with sound pedagogical practices have already been established. Investing time and energy into teaching the attribute of length well, will have many positive benefits.

The measurement strand is mathematically powerful and has rich pedagogical possibilities. It is a part of the mathematics curriculum which is replete with possibilities to engage students in meaningful, rigorous and rewarding mathematics. Many of the understandings needed to be efficient and effective with measurement are relatively straightforward and readily applicable, something which allows access into the world of mathematics success for some students. The potential of the measurement strand for tuning students into mathematics is great and should not be underestimated.

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