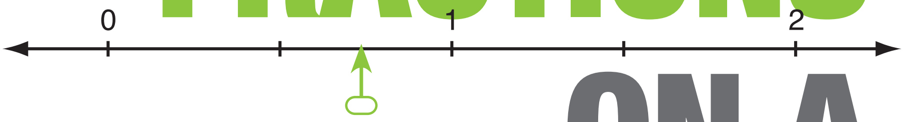
LOCATING FRACTIONS ON A NUMBER LINE



Monica Wong
Australian Catholic University
<monica.wong@acu.edu.au>

Monica Wong continues her discussion of students' understandings and misunderstandings of fractions. In this article, students' strategies for locating fractions on a number line are presented.

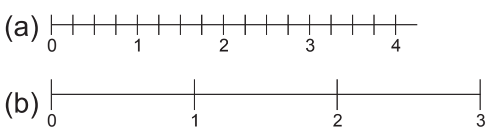
Understanding fractions remains problematic for many students. Wright (2013) suggests that part of the reason is the over-use and dominance of the part–whole model in students' thinking. He advises that students should be exposed to other concepts of fractions such as measurement, whereby a fraction represents a quantity. The use of the number line aids in this understanding, but requires students to recognise that a fraction represents the distance from zero to a dot or arrow marked on a number line which is a linear scale. Hence, without this understanding, students may encounter difficulties achieving the knowledge requirement within the Australian Curriculum to "[c]ount by quarters, halves and thirds… Locate and represent these fractions on a number line (ACMNA078)" (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2013).

This paper continues the discussion from *Identifying Fractions on a Number Line* (Wong, 2013) in which students' understanding of the key features of a number line (proportionality, scale, location of zero) and fractions as a quantity were explored. In this paper, two questions which highlight students' strategies for locating fractions on a number line are presented, along with common student responses. The data were collected from three primary schools in the Sydney region. Students from Years 3 to 6 completed a pencil and paper assessment, the *Assessment of Fraction Understanding v2* (Wong, 2009), which measured their knowledge and understanding of fractions and equivalence. Based on their responses to the written assessment, the tasks were re-administered to students during one-on-one interviews.

Representing a fraction on a number line

To advance students' learning, teachers must gain insight into their students' thinking. By identifying students' incorrect strategies and conceptualisations, teachers can adjust their instruction by focussing tasks and lessons on particular areas of concern. The two questions shown in Figure 1 enable teachers to identify some of the strategies students employ when locating fractions on a number line. Question 11 required students to locate $\frac{1}{2}$ on two number lines, both starting from zero with whole numbers marked; additionally 11(a) was marked in quarters. As part of ACMNA078, students should become competent in "converting mixed numbers to improper fractions and vice versa" (ACARA, 2013, p. 41), thus question 20 required students to locate $\frac{3}{2}$ on the number line.

11). Put a cross (X) where the number $\frac{1}{2}$ would be on the number lines below.



20). Put a cross (X) where the number $\frac{3}{2}$ would be on the number line below.

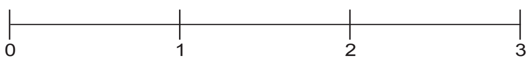


Figure 1. Locating fractions on a number line tasks.

A two-way table (see Appendix 1) was constructed to examine the written responses for question 11(a) and (b), which was completed by 349 students from Years 3 to 6. Three response categories were identified:

- 15.5% (n = 54) of students correctly located half on the number line for both (a) and (b). An additional two per cent of students correctly located half for either (a) or (b).
- 24.4% (n = 90) of students placed a cross halfway along the length of the entire number line for both number lines. For 11(a), half was marked at 2, and for 11(b), half was marked at $1\frac{1}{2}$.
- 27.2% (n = 95) of students marked half midway between 1 and 2, at $1\frac{1}{2}$ on both number lines.
- The many remaining response categories comprised less than 4.7% of respondents as shown in the two-way table in Appendix 1.

Of the 163 students in Years 5 and 6, who completed question 20 in the pencil and paper assessment, 10.3% (n = 36) placed a cross between 2 and $2\frac{1}{2}$, 10.6% (n = 37) placed at exactly $2\frac{1}{2}$. Only 6.3% (n = 22) of the students were able to locate $\frac{3}{2}$ on the number line correctly. The errors exhibited by students for questions 11 and 20 suggest the use of two strategies: 'fraction implies an action' and 'cue strategy'. A number of students were re-administered questions 11 and 20 during interviews to examine their thinking and to validate the existence of these strategies.

Fraction implies an action

Mitchell and Horne (2008) and Pearn and Stephens (2007) found that locating $\frac{1}{2}$ evokes 'half of' something, whether it is a number or half the entire number line. Similarly, locating $\frac{2}{3}$ results in placing a dot or an arrow two-thirds of the way along the length of entire number line. Sam, a Year 5 student, completed question 11 during his interview. He read aloud: "Put a cross where the number half would be on the number line". He placed a cross, halfway between 2 and 3, as shown in Figure 2. When asked how he worked it out, Sam replied, "Half of it. Well zero's not a number, so it's between these [indicating location of the 1 and 4 on the number-line]… This was the middle [counting in from both sides]." Sam ignored zero as he did not consider it a number, thus halfway between 1 and 4 was located at $2\frac{1}{2}$. Sam's response during the interview and the responses of 24.4% of students who completed the question in the pencil and paper assessment support the notion that half implies an action rather than a quantity or distance from zero.

10 (a). Put a cross (X) where the number $\frac{1}{2}$ would be on the number line.

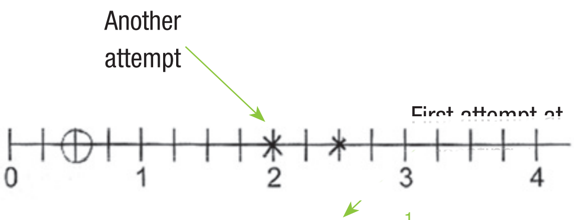


Figure 2. Sam's (Year 5) attempts to locate $\frac{1}{2}$ on a number line.

Michael, a Year 4 student, was asked to draw a number line (see Figure 3) during his interview. When he was asked, "Where would a half be on the number line?" he replied, "Half of 18, here 9 [labelling the location with an arrow]." Although Michael used half as an action, the language "Where would *a half* be on the number line" could have been confused with "half of". Gould (1996) talks about mathematical English, how word order is important and changes in word order can result in different mathematical structures (Gould, 1996, p. 22): (a) What number is half of six? (b) What number is six half of? Words in English, 'to', 'of', and 'by' are crucial in deciphering mathematical English and the mathematical structure embedded within the statement. Indeed, when Michael was asked to locate the quantities "half", "one and a half" and "thirteen and a half" he completed the task by drawing arrows at the correct location and labelling it accordingly (see Figure 3).

The cue strategy

The responses provided by students suggested that students rely on a *cue* strategy, locating the fraction somewhere between the whole number locations of the numerator and denominator on the number line. Questions 11 and 20 were completed by Anna, a Year 6 student, during her interview. She placed a cross halfway between 1 and 2 for question 11 (see Figure 4). When asked to identify the cross placed by the researcher on the same number line between 2 and 3, Anna wrote the fraction $\frac{2}{3}$ underneath it. An arrow was then marked on the number line by the researcher and Anna identified the arrow as 1/0. She used the whole numbers as cues to locate the fraction quantity somewhere within the interval. Similarly, to identify a fraction quantity, the symbolic representation was constructed using the closest whole numbers.

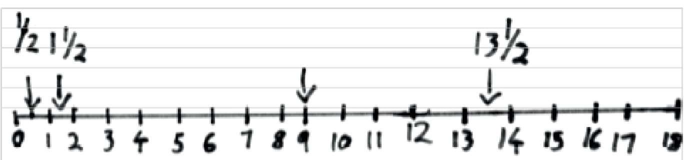


Figure 3. The number-line drawn by Michael during his interview.

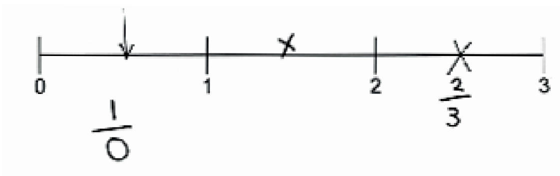


Figure 4. Anna's attempts at locating and identifying fractions on a number line.

Variations to the cue strategy were also observed during interviews. After locating $\frac{1}{2}$ halfway between 1 and 2, Rachel, a Year 5 student, was asked to locate the fraction $\frac{2}{3}$ on the number line. She explained (see Figure 5):

> I'll look at the top number [points to the 2 in $\frac{2}{3}$]. See how it has the word two, so I look… [moves her finger along the number-line until she reaches 2], found 2 and look at the 3 [locates whole number 3 on the number-line] as well. So I have to think, alright, if half is here [points to half way between 2 and 3] so the cross should be a little more higher [to the right of $2\frac{1}{2}$].

Rachel appeared to show some understanding of fraction quantities as she

Figure 5. Rachel locates $\frac{1}{2}$ (between 1 and 2) and $\frac{2}{3}$ on a number line.

acknowledged $\frac{2}{3}$ was greater than $\frac{1}{2}$. When Rachel was then asked to locate $\frac{3}{2}$ on another number line, she vocalised $\frac{3}{2}$ as "two thirds." When asked if was she sure, Rachel then said, "Three twos," and continued:

> If I want to do it, it is going to be somewhere out here [pointing to the right of 3 on the number line in Figure 6]. So I think two thirds is just here [motioning to right of three]. Here should be three ones [where the author placed a vertical line and label $\frac{3}{1}$ to indicate the location identified by Rachel]. Here should be three twos further to right of three ones [Rachel marks the location with a cross and the author writes $\frac{3}{2}$ above it].

Rachel's interpretation of fraction notation, $\frac{3}{2}$ was inconsistent, and her uncertainty and vocalisation of "three twos" guided her actions.

Figure 6. Rachel places "three twos" on a number line.

Interpreting fraction notation

As shown in previous examples, students interpret fraction notation and vocalise it in many ways. Halford (1993) and Kieren (1999) have found that students intertwine vocalisation with their thoughts and these should be considered an externalisation of their understanding. Thus incorrect vocalisations may represent incorrect conceptualisations.

Different vocalisations for the fraction $\frac{1}{2}$ were observed when students read aloud their interview tasks. Gillian, a Year 3 student, read $\frac{1}{2}$ as "one and a half" and explained, "One [pointing to the numerator] and that's like a sign of half [running her finger along the horizontal line or fraction bar], one and a half."

Anton, another Year 3 student said, "One over two," and Christina, also in Year 3, said, "One in a half." Not only did vocalisations differ between students, a number of different vocalisations for $\frac{1}{2}$ were used by Tina (Year 3): "one half of two", "one and a half" and "half". She also vocalised $\frac{3}{2}$ as "three half of two." Rachel (Year 5) interpreted of $\frac{3}{2}$ as "three twos", which was preceded by "three ones". These examples show that students have difficulties with the language of fractions.

David (Year 5) who correctly located "one half" and recognised one-quarter and other fractions on a number line, was able to state that $\frac{2}{2}$ represents one whole. He was unsure of the meaning of $\frac{3}{2}$, which he vocalised $\frac{3}{2}$ as "three over two," then changed it to "three two" before placing a cross just before 3 on the number-line. He then stated: "I'm confused 'cause it's three out of two". Thus $\frac{3}{2}$ or improper fractions are more difficult for students to conceptualise than proper fractions.

Fraction language

Fraction language can vary depending on the context. For example, "one out of two", "five out of eight" or *x out of y* makes sense when you are talking about a whole. If the understanding of fractions is confined to the part–whole model, fractions are often regarded as a pair of whole numbers and promote whole number thinking. David found that improper fractions (e.g., "three out of two") have little meaning in this context.

When considering fractions as a quantity, the language used is different. Fraction language in the form, "five-eighths", "three-quarters", etc., is more helpful. For example, the eighth (denominator) tells us the size of the section or partition and how many sections are needed to recreate the unit, thus identifying its size—which is important in measurement. The five (numerator) informs us of how many eighths we have. Creating a ruler using informal units as described in Wong (2013), enables the systematic measuring and counting of fractions (e.g., $\frac{1}{2}$ one-half, $\frac{2}{2}$ two-halves, $\frac{3}{2}$ three-halves), with the one, two, three tells us how many halves there are, thus linking fractional language to fraction quantities depicted on a number line. Hence appropriate fraction models need to be used to convey the various fraction conceptions. The use of the number line and appropriate language enables students to make sense of improper fractions.

Conclusion

Questions 11 and 20 provide a method for teachers to identify their students' knowledge of fractions and the number line model. Examining students' written and verbal responses to these questions can provide teachers with clues to their students' thinking and understanding. The questions enable the identification of incorrect strategies for locating fractions. The strategies observed from the pencil and paper assessment and one-on-one interviews included: (a) fraction implies an *action*; (b) considering the numerator and denominator as a *cue* to the location of the fraction, and (c) incorrect vocalisation of fraction notation. Strategies were sometimes combined, incorporating reasoning based on both fraction and whole number knowledge.

In a measurement context, a fraction is the symbolic representation of a quantity or distance from zero to a mark or dot on a number line. Without this foundational knowledge, inappropriate strategies and incorrect interpretation of fraction notation can occur. Activities such as creating a number line or ruler (see Wong, 2013), which enables students to explore the attributes of the number line (scale, proportionality and location of zero), fraction notation and fraction language, provides opportunities for students to acquire these skills.

References

Australian Curriculum, Assessment and Reporting Authority [ACARA]. (2013). *The Australian Curriculum: Mathematics v4.1[dated Monday, 20 May, 2013]*. Retrieved from http://www.australiancurriculum.edu.au/Download.

Gould, P. (1996). Assessing mathematical language — More or less. *Reflections, 21*(1), 22–26.

Halford, G. S. (1993). *Children's understanding: The development of mental models.* Hillsdale: Lawrence Erlbaum Associates.

Kieren, T. (1999). Language use in embodied action and interaction in knowing fractions. *Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Pyschology of Mathematics Education.* Cuernavaca, Mexico. (ERIC Document Reproduction Service No. ED433998)

Mitchell, A. & Horne, M. (2008). Fraction number line tasks and the additivity concept of length measurement. In M. Goos, R. Brown & K. Makar (Eds), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Brisbane, pp. 353–360). Brisbane: MERGA.

Pearn, C. & Stephens, M. (2007). Whole number knowledge and number lines help to develop fraction concepts. In J. Watson & K. Beswick (Eds), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 601–610). Adelaide: MERGA.

Wong, M. (2013). Identifying fractions on a number line. *Australian Primary Mathematics Classroom 18*(3), 13–18.

Wong, M. (2009). *Assessing students' knowledge and conceptual understanding of fraction equivalence.* Unpublished doctoral thesis, University of Sydney, NSW.

Wright, V. (2013). In search of the prototypical fraction. *Australian Primary Mathematics Classroom, 18*(2), 27–32.

Appendix 1

Responses for Items 11(a) and 11(b) (percentage of total responses, $N = 349$)

	Item 11(b)[1]	$\frac{1}{2}$	$1\frac{1}{2}$	$1 - 1\frac{1}{2}$	Other	Total
Item[2] 11(a)	$\frac{1}{2}$	**15.5**	0.3	0.3	0.3	17.8
	1	0.0	0.0	0.0	0.0	1.1
	$1\frac{1}{2}$	0.6	27.2	2.6	0.0	32.4
	2	0.6	$\boxed{24.4}$	0.3	0.0	25.8
	$2\frac{1}{2}$	0.0	1.7	0.0	0.3	2.9
	$1 - 1\frac{1}{2}$	0.0	0.9	4.6	0.3	6.6
	$1\frac{1}{2} - 2$	0.0	0.3	0.3	0.0	1.4
	Other	0.0	0.3	0.0	2.0	3.2
	Total	17.5	57.3	8.3	2.9	100.0

NOTES

1 Nine categories for item 11(b) were omitted from the table, where students placed the cross as follows: '1' (on the number-line) with 5 responses, '2' with 3 responses, '2' with $2\frac{1}{2}$ responses '<$\frac{1}{2}$' with 9 responses, '2-$2\frac{1}{2}$' with 7 responses, '$1\frac{1}{2}$-2' with 4 responses, '$\frac{1}{2}$-1' with 3 responses, '0' with 2 responses and 14 no*n-attempts*.

2 Five categories for item 11(a) were omitted from the table, category '<$\frac{1}{2}$' (on the number-line) with 7 responses, '2-$2\frac{1}{2}$' with 9 responses, '$\frac{1}{2}$-1' with 3 responses, '0' with 4 responses and 8 non-*attempts*.