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How Rwandan Grade 6 Mathematics Teachers Teach – A First Investigation

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We would like to acknowledge that this paper reports the part of the study on teaching and learning in Rwandan Grade 6 classrooms and that it replicated a similar study conducted in two of the South Africa's provinces to enable comparisons. The project is self-funded

Abstract

This paper offers a tentative snapshot of Grade 6 mathematics classroom practices in Rwanda based on twenty video recorded lessons. It has an objective of investigating through evaluation, the teaching strategies used by Rwandan Grade 6 Mathematics teachers in their classrooms. In the absence of sufficient instruments to measure Pedagogical Content Knowledge (PCK) in practice, a coding scheme was developed and applied to our video recorded lesson data, which helped us to answer our question of knowing the PCK levels of Rwandan Grade 6 Mathematics teachers. Choosing Rwanda as research site is mainly based on our will to know Grade 6 Mathematics practices in developing countries and particularly in Rwanda context where there has not been any study of this kind done before, which could add to researchers' existing understanding of practical PCK. The findings suggest that there are differences in the extent to which practical PCK is engaged as some teachers seem more likely to use teaching strategies that research suggests are effective like being able to unpack the content in their teaching. Even if the overall impression is that it is fair to assume that teaching in Rwanda is not in a calamity, some teaching practices need to be improved. These include making connections and linking the contents. The paper ends with a discussion of methodological issues.
Keywords: classroom practices, Rwanda, teachers' practices, procedural PCK, teaching strategies.

Introduction

Teachers are considered the most important input element to schooling (Makuwa, 2011), apart from those which are out of control of the education system such as the learners' abilities and home situation (Hattie, 2003). If others interested in the knowledge and practices of Mathematics teachers, it is easier to collect data on teachers' qualifications, experience, or training, but it is harder to get a reasonably precise idea of their command of relevant declarative knowledge, and hardest to get insights into their behaviour in the classroom (cf. Gabrielle, 2009). Nonetheless, as it is the actual practice that is most likely to affect learning, this is a crucial piece in the puzzle of understanding teaching, including unearthing links between declarative and procedural knowledge. It is for this reason that we were interested in what we have deemed 'practical PCK– knowledge of how to facilitate learning of the particular content as it is manifested in practice. Measuring practical PCK has been seen as a difficult task by a number of researchers. Rohaan et al.(2009) put forward three main difficulties related to it namely: the fact that teachers' PCK is often implicit; that it cannot be determined entirely from behavior; and difficulties related to making judgments about teachers' practical PCK. For this purpose, an instrument was developed based, as far as possible, on existing research. This will be presented in more detail below.

At the time of writing, there was no available literature on classroom based teaching in Rwanda. This work added then to scholars understanding of practical PCK in developing countries and Rwanda in particular. Such studies have been conducted elsewhere. The ORACLE study in UK is considered to be the first classroom based study (Amidon & Hough, 1967) in which the researchers focused on classroom verbal behaviors. Many other studies have followed across the world trying to ascertain teachers' knowledge as it manifests in the classroom actions or in talking about teaching (Baumert et al., 2010; Boaler, 2002; Cobb, 1991; Douek, 2005; Kersting et al., 2012; Ramdhany, 2010; Rowan et al., 2001; Sfard, 2007; Sfard et al., 1999; Stigler et al., 2000). Within the literature, there are a range of methods and analytical categories used in trying to identify teachers' actions in

classroom situation. For example, Baumert et al.(2004) considered important the selection of tasks and the way teachers assign those tasks to learners during classroom teaching accompanied with suitable reaction on learners' answers and assignment of homework, although this aspect may not be considered by others.

Research evidence on PCK

Different conceptions on what to take as the most sensitive parts of PCK have opened debates, and have influenced different authors to come up with diverging ideas on PCK sub-categories.Loughran et al.(2004) confirms that PCK is a complex notion and continues to be a seductive theoretical construct but not an easily identifiable aspect of practice. The above observation inspired us in this research to find out a way of identifying PCK in classroom practices as our major aim. On one hand, as previously noted, authors who researched teachers' PCK (e.g., Ball et al., 2008; Neubrand, 2006; Stump, 2001) were mainly using tasks either given to teachers or to learners by teachers for them to have sense on how teachers put their knowledge into practice. A different approach of asking teachers to comment on video recordings and then coding their responses have also been used (Kersting et al.2012).

The issue of what to take as PCK sub-categories or aspects continues to constitute a challenge. Some authors focused on the role played by the content even if their perception of PCK also included pedagogy (Sorto et al., 2009). A different perception of PCK in mathematics education in particular came with the work of Ball et al.(2008) in which they considered Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC) as PCK sub-categories. Their main emphasis was on KCS which itself is a subset of the larger construct of what they called Mathematical Knowledge for Teaching (MKT) (Adler & Zain, 2006; Ball, Hill, & Bass, 2005). Based on their analysis of the mathematical demands of teaching, (Ball, Thames, & Phelps 2008) hypothesized that Shulman's categories of Content Knowledge (CK) and PCK (Grossman, 1990; Shulman, 1987) can be subdivided into Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK) on one hand, and Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) on the other hand.

For Ball, et al.(2008b), KCS, KCT and KCC are the ones which can be considered as PCK sub-categories. Other aspects like CCK, Horizon Content Knowledge (HCK) and SCK have been considered as subparts of CK which form MKT when added to PCK. We used these sub-categories of knowledge specific to mathematics teaching to provide an overview of leading authors' conceptions of teachers' knowledge and what it includes (Table 1) in which p stands for present and MfT stands for Mathematics for Teaching. However, as argued by Karstein (2014), there is no sufficient agreement even when it comes to the distinction between PCK and CK, so different authors may have different conceptions of the categories below even when there appears to be agreement.

Table 1: Authors' view of PCK subcategories.

Scholars	Teachers' job equipments							
	CK (Subject Matter Knowledge)			PCK				
	CCK	HCK	SCK	KCT	KCS	KCC		
	Common content of math	Knowledge of math horizontally	MfT Knowledge of effective teaching practices in a given content area	General PK	Knowledge of Learners and their characteristics	Curriculum Knowledge	Knowledge of education ends	Knowledge of educational context
			Specialized math knowledge					
Shulman (1987)	P		P	P	P	P	P	P
Grossman (1990)	P		P	P		P		
Brian, Steven (2001)	P		P					
Ball, Hill, Schilling (2005)	P		P					
Adler (2006)			P					
Hill, Ball, Schilling (2008)	P	P	P	P	P	P		
Baumert (2010)	P	P	P					
Hurrel (2013)	P	P	P	P	P	P		

In the present research we have worked with sub-categories of PCK (Deapepe et al., 2014, p14). Those are KCS, KCT and KCC which have been analyzed using our developed analysis tool based on the existing researches which tried to inform on classroom teaching like the work by Fennema & Romberg (1999; Hurrell (2013; Ramdhan (2010); Reid (1995; Stigler et al. (2000), among others.

Looking into the components of PCK more specifically, one can ask to what extent different researchers agree on the dimensions? As can be seen in Table 2 below that we developed based on the reviews by Soonhy (2008) and Depaepe (2013) providing an overview of the components of PCK, we also find here substantial differences in perceptions.

Table 2: Authors' view of PCK components [Developed on the basis of the reviews in Soonhy (2008) and Depaepe (2013)]

Scholars	Student understanding	Teaching Purposes for a subject matter	Curriculum	Instructional strategies and representation	Assessment	Context	Pedagogy	Subject matter	Math tasks & Cognitive demand
Shulman (1987)	p			p	p				
Tamir (1988)	p		p	p					
Smith and Naele (1989)	p	p		p					
Grossman (1990)	p	p	p						
Cochran et al. (1993)	p					p	p	p	
Geddis et al. (1993)	p		p	p					
Even (1993)	p			p					
Fernandez-Balboa and Stiehl (1995)	p	p		p		p		p	
Magnusson et al. (1999)	p	p	p	p	p				
Brian, Steven (2001)	p		P				p		
Hasweh (2005)	p	p	p	p	p	p	p	p	
Ball, Hill, Schilling (2005)	p								p
Loughran et al. (2006)	p	p		p		p	p	p	
Adler (2006)	p								p
Ball et al. (2008)	p		p	p					
Krauss et al. (2008)	p			p					p
Hill, Ball, Schilling (2008)	p			p					p
Baumert et al. (2010)	p			p					p
Nilssen (2010)	p					p	p		
Watson and Nathan (2010)	p			p					p

Nonetheless, there are some components which have generally been considered crucial, namely student understanding, instructional strategies, and representations which appeared in conceptions of almost all the scholars (cf. Table 2). Mathematics tasks and cognitive demand is also seen as an important element to consider while measuring teachers' practical PCK (Stigler et al., 2000). However Taylor (2008) argues that some teachers still exhibit weak understanding of assessment and lack of feedback on students' responses. This might be reflected from any teacher as experienced teachers are not necessarily expert teachers (Hattie, 2003; Tobin & Garnett, 1988). On this basis, we decided to construct our own research informed instrument for categorization of teachers' PCK in as much as descriptive and non-normative way as we possibly could.

The analytical instrument

We decided to work with the following sub-categories of PCK: KCT, KCS and KCC, because we judged them to reflect most of teachers' practices in classroom. Within the notion of KCT, we worked with the dimensions like content connections to create new knowledge and unpacking the method/concept to make the content more accessible for learners. Likewise, under KCS a survey of literature on effective teaching suggested to us further sub-divisions, here referred to as *component* of teaching which also act as *indicators* of that particular PCK sub-category. These were, for example, the effort that teachers use to concretize the lesson by illustrative examples and teaching aids (representation) and the way they assess learners' prior knowledge. Under KCC we were looking for example, at the way progression of the lesson and linkages to other sessions were sequenced (see appendix). Within the table, the first column indicates the PCK subcategory we were targeting, the second column indicates the component of teaching to be observed as teacher action in order for us to get sense of what a particular teacher was doing related to the targeted subcategory.

Within each component of teaching, we considered different *approaches* or *options*. For instance, under KCT, one component of teaching was the connections the teacher makes between different 'parts' of content knowledge. Using the distinctions made by Mhlolo at al.(2012) derived from Businskas (2008), we then worked with six options.

(a) No connections were made between parts of content.
(b) Different representations were used in engaging the same concept or process.
(c) Content was connected through logical implications such as proofs or semi-proofs.
(d) Connections were made between procedures and concepts.
(e) Explicit connections to previously taught content were made.
(f) Part-whole relationships were evoked.

As these categories may suggest, it was possible to evoke more than one type of connections in a lesson. As was shown in the Appendix, we ended with a coding system of 10 separate components, with 4-6 options for each.

Method

The data used to generate this paper were collected in 2013.The tools we used included a teacher questionnaire, a teacher test on content knowledge and PCK, a learner test given at the beginning of the school year and repeated

towards the end of school year, a learner questionnaire and one video recorded lesson from each teacher who consented to participate in this research. However, for the purpose of this paper, we only used the data from our recorded video lessons.

Sample

To select our sample, we used random stratified sampling. Selecting seven districts from three provinces from the five constituting Rwanda. We ended up with twenty primary schools as our final sample. We note that our sampling took into account the socio-economic state of the schools and that it included both public and private schools.

Nature of the video data

The lessons were recorded when convenient and therefore not selected to be topic based. Any of the teachers could teach a topic which was different from the one by his/her colleague even if the topics were from the same mathematics sub branch like algebra, geometry etc. Teachers were teaching in English. This is mainly based on the fact that Rwanda changed the language of instruction in schools from French to English (Gahigi, 2008) five years ago. As the teachers have been educated in French and have Kinyarwanda as their mother tongue, this may mean that they were less comfortable teaching in English possibly resulting in them exhibiting less practical PCK.

Ethics

Ethical clearance was obtained from the University of KwaZulu-Natal in South Africa, and permission to conduct the research was obtained from the Ministry of Education in Rwanda. Research participants are protected through absolute anonymity; each teacher, school and learner has been assigning a code, and all data have been processed using these codes, not actual names.

Analysis

Video recorded lessons were analyzed in five minute intervals. After each five minute period, the video was stopped, and notes were made of any component of teaching and any option within this component which had been used in that time frame. This means that short instances of an option could weigh as much in our data analysis as longer instances, except that the latter were more likely to extend over more than one five minute intervals.

Upon coding all the videos, we removed the option categories which had not been used by any teacher in the sample. This, however, does not mean that the removed options are not useful in classroom practices measurement, and indeed could be relevant in a different data set. For instance, no teachers in the sample worked from concrete to abstract when linking new to previously taught material. Few teachers determined learners' prior knowledge and those that did so did not use it in any detectable way to inform their teaching. To simplify the data set in order to investigate any differences, we also opted to exclude any option categories used by only one or two teachers for 20% of the time or less. This included the occurrence of implication connections and the comparison of different methods/ways to unpack the concept. Although these categories have not been considered in their own right, they were correlated with others in our tool which suggests to us that we have not lost the image of teachers' actions in their classroom teaching. Furthermore, we excluded the category of 'no evidence' (the third column in Appendix) from our analysis, as their non occurrences within a particular time implied the occurrence of other different PCK sub-categories at that particular time. Finally, to reduce the data set, we collapsed similar categories. We then looked for patterns inductively and in particular for similarities across teachers.

Results

Differences between teachers

We realised that there were teachers who showed little evidence of any form of practical PCK (see Figure 1 for an example reflecting the classroom actions of a teacher with code number Tr35[1]). Those teachers were tending to engage the whole class or to involve learners to work individually by giving them a lot of tasks or product feedback (Bitchener, Young, & Cameron, 2005; Hattie & Timperley, 2007), but little else.

[1] An example of a code number given to teachers to mask their names

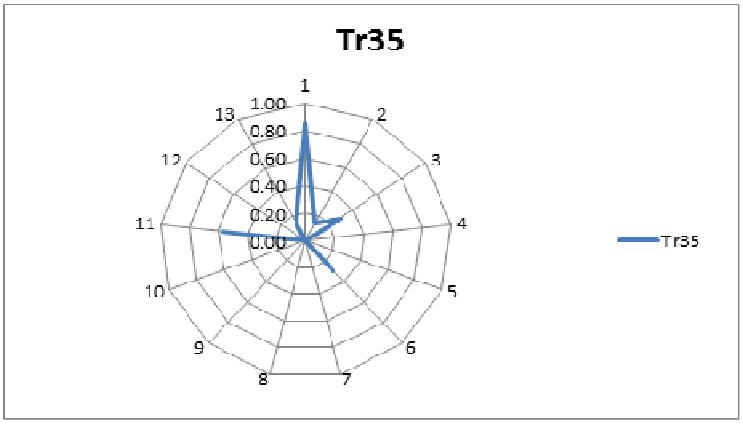

Figure 1. An example of a teacher with little practical PCK evidence in the observed lesson.

In the Figure1, Category 1 is engaging the whole class or letting them work individually on practice tasks, while category 11 is giving task or product feedback. There were other teachers who demonstrated traditional use of tasks and one additional substantial PCK category. Those teachers chose to give more process oriented feedback (Hattie & Timperley, 2007) or to engage more in representations (Cuoco, 2001) as shown on figure 2.

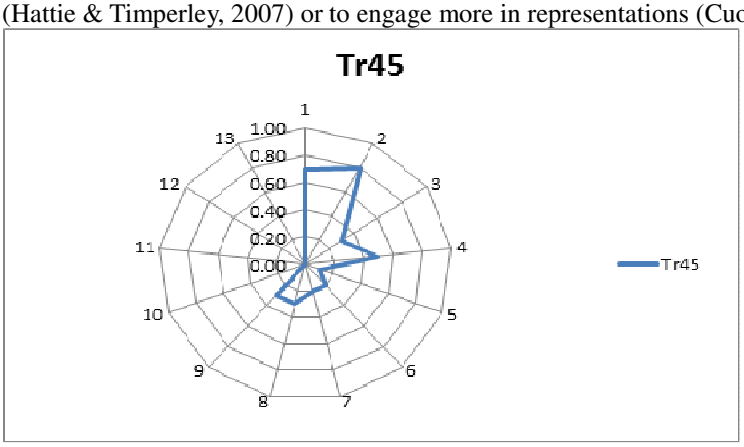

Figure 2: Example of a teacher with traditional use of tasks and one substantial PCK category. Category 2 is the use of representations such as manipulatives or drawings.

A third category consisted of teachers with more than one category of substantial practical PCK used frequently (Figure 3).

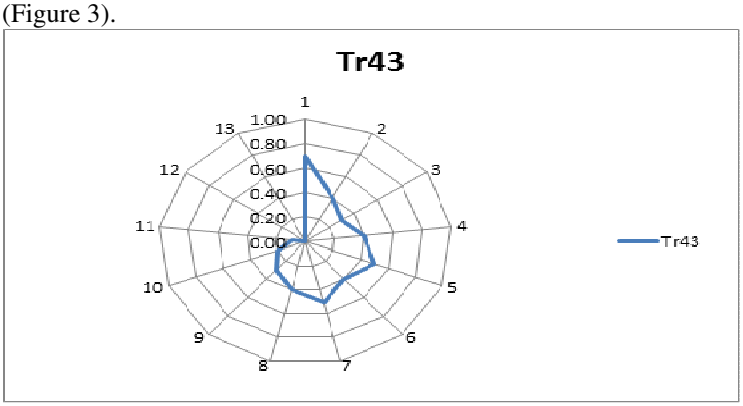

Figure 3: Example of a teacher with more than one category of substantial practical PCK.

Category 7 is engaging in more conceptual unpacking or showing learners more than one method, and category 5 is engaging learners in mathematical communication.

The majority of the teachers in the sample were using tasks traditionally, but some PCK indicators could be observed such as the usage of representations, engaging learners' errors/misconceptions, work with more connections in the content, unpacking procedures, engaging learners in mathematical content constructions (cf. Businskas, 2008; Crowley, 1987; Cuoco, 2001; Mhlolo, et al., 2012; Ramdhany, 2010) without forgetting to give learners more process feedback and engaging them more in content constructions (cf. de Villiers, 2004; Hattie & Timperley, 2007). Finally, some teachers demonstrated more extended use of practical PCK (see example in Figure 4).

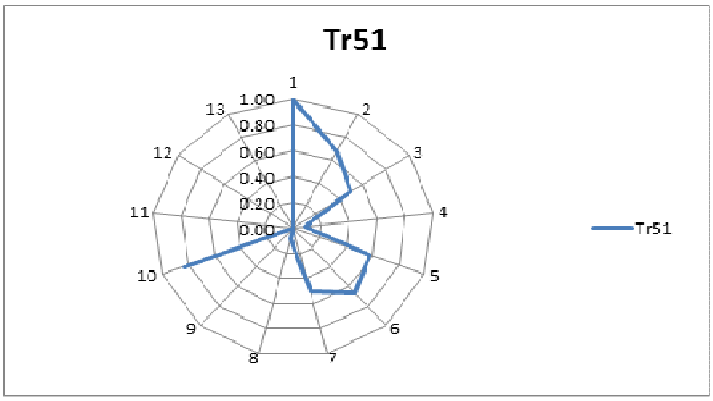

Figure 4: Example of a teacher with more extended use of PCK.

This teacher whose actions are shown on Figure 4 was the only teacher to use content connections a lot of the time (category 10) in her teachings (cf. Mhlolo et al., 2012). Yet she shared with other teachers in giving more process feedback, showing different methods to unpack the content, engaging learners' errors, using representations, engaging in construction of mathematical content, and linking the content (Hattie & Timperley, 2007; Hill et al., 2008).

Types and frequency of practical PCK

It must be acknowledged that the grouping of teachers presented above is only the result of a first exploration, and that a different grouping of categories may have led to a slightly different impression. Nonetheless, the data suggests that there are substantial varieties in the extent to which Rwandan Grade 6 teachers use practical PCK. If we add up the frequencies with which any of the considered categories of practical PCK occurred, we get the picture shown in Figure 5.

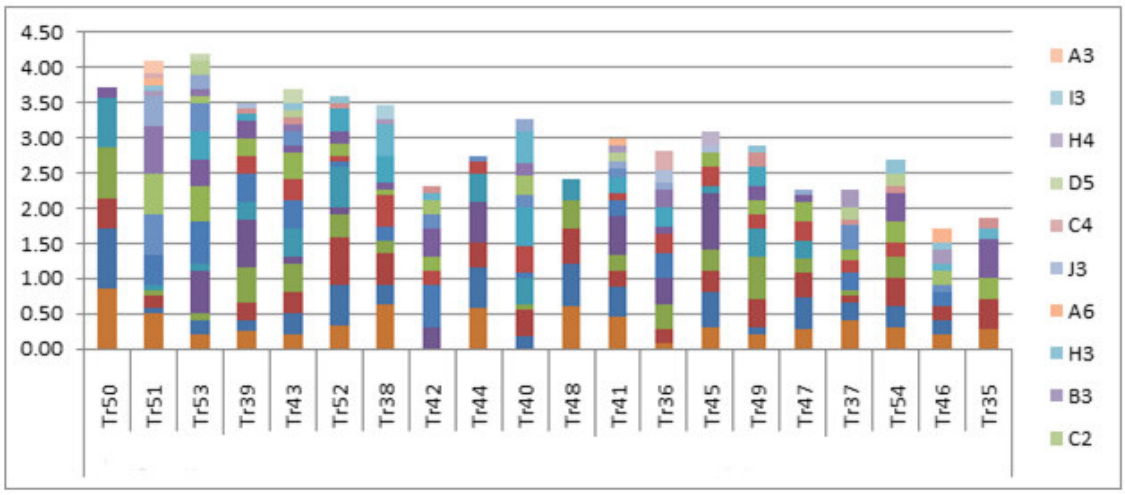

Figure 5.The distribution of ten categories of practical PCK on the twenty teachers. For each category, the height of the bar indicates the fraction of the lesson this option was observed.

Thus, all the teachers in the sample exhibit several aspects of practical PCK, though the literature suggest some are more effective than others, and thus it is fair to assume that teaching in Rwanda is not in a crisis. Nonetheless, it seems relevant to distinguish between strategies recommended in the literature and those less favored. In our future research the relationship between them and strategies recommended in Rwandan Math Curriculum policy or syllabus for Grade 6 will be considered.

There was only one strategy used by all the teachers, and that was J2. Teachers identified learners' prior knowledge but it was not used to inform the next topic. In addition to the negative picture teachers, giving of 'self feedback'[2] which may affect learners' self-esteem (G5) was a common strategy. The practice of giving indirect feedback can have both strengths and weaknesses. It was used by only eight of the teachers, but varying from infrequent to frequent. Of the more constructive strategies, the most common were sharing of seatwork (I4, see Figure 6) and process feedback (G3, see Figure 7).

[2]Feedback on learners' personality, general aptitude or general behavior rather than on particular method or result.

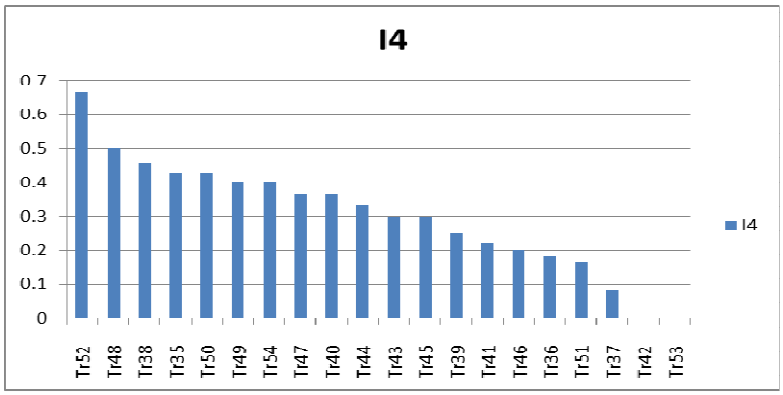

Figure 6.The frequency of sharing of seatwork for the different lessons.

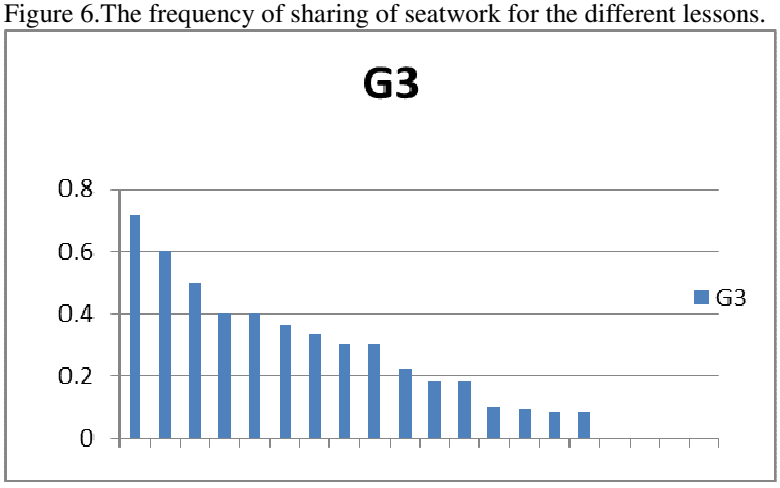

Figure 7.The frequency of process feedback for the different lessons.

Overall, this gives the impression that some Grade six teachers could improve on their use of more effective teaching strategies.

Validity issues

We again remind that our practical PCK investigation was not based on topic specific perspective. Due to that fact, it might be possible for a teacher to not show/use a certain PCK dimension not because s/he was not able to do so but may be because his/her topic of the day was not requiring that specific type of PCK. Alternatively, their PCK may be content specific as suggested by Hill et al. (2008). For example it was easier for teachers who were teaching topics related to geometry to engage their learners in tasks with manipulative and drawing teaching aids than teachers whose topics fell within algebra. As we alluded to in the beginning of this paper, there is no given way to determine the practical PCK of teachers, whether done in a single lesson with its obvious limitations, over longer time, or in discussion of video recordings. It is therefore pertinent that we reflect on the quality of our instrument in its current form, on living up to our expectations in this respect.

Firstly, we note that our desire to create an instrument which would be descriptive and non-normative lead to difficulties in interpreting the results. When it is noted that self feedback is used, for instance, it may appear that the teacher is using a useful component of practical PCK when indeed it is a highly problematic one. Thus, our main realization upon using this instrument on empirical data is that the data have to be more carefully interrogated. However, this also means that once this type of analysis is conducted on a sufficiently large data set, it may be possible to investigate the extent to which the predictions of which teaching strategies are more effective may hold true.

Secondly, we note that collapsing categories for ease of analysis may distort the picture, no matter which process is used. If, for instance, the frequencies for categories are simply added, the new collapsed category may be given an unfairly high frequency compared to categories which could have been sub-divided but were not. If, on the other hand, collapsed categories are counted only as present or non-present, a completely different picture emerges. This means that more consideration should probably be given to the nature of the different options we have listed, so they are of compatible weighting. We have not yet reached a solution to this problem.

Conclusion

While the data set is rather limited, it still has given us a snapshot of Grade 6 mathematics teaching across Rwanda. It is evident that conventional teaching approaches with whole class interaction and practice tasks are still widespread. Nonetheless, there appears to be substantial variations amongst teachers, with some using more different strategies than others, and some using what the literature suggests are more effective strategies. So while teaching may not be in a crisis, there is still room for substantial improvement.

We also conclude that the instrument may need further development, and the results will always depend on the nature of the instrument used, in particular the number of sub-categories, how occurrences are measured, and whether or not all categories are given equal weighting. The next part of our study will interrogate the results of the analysis of the video recorded lessons in relation to the learner gains and the teachers' performance on the test.

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Appendix

PCK sub domains in this study	Criteria	Option one	Option two	Option three	Option four	Option five	Option six
K.C.T (Knowledge of Content and Teaching.)	Content connections to create new knowledge within the lesson are observed.	No kind of connections showed.	Different representations are used (equivalent or alternate).	Implication connections are used.	Procedure connections are used.	Prerequisites connections are observed.	Part-whole relationships are observed.
K.C.C (Knowledge of Content and Curriculum.)	Progression of the lesson and linkage to other sessions, for learners to assimilate the concept are sequenced.	No linkage observed.	The linkage with other sessions is shown from simple to complex.	The linkage with other sessions is shown from particular to general or vice verse.	The linkage with other sessions moved from theory to practical.	The linkage with other sessions is shown from concrete to abstract.	The linkage with other sessions is shown from every day to specialize.
K.C.T/ (S.C.K: Special Content Knowledge.)	Mathematical content construction through practices/variations is observed.	Any kind of mathematical content construction through practices/variations is observed.	Investigation by observation of the object/image through continuous variation/contrast is observed.	Mathematical terms are used by learners to explain why the conjecture is true or false through discussions/separation.	Verifications are done to clarify areas in which learners exhibit doubts by expressing themselves within their math vocabulary.	Generalization of the concept by leaving or adding properties from complex tasks under organized is observed.	Learners are encouraged to communicate mathematically while performing a task.
K.C.S (Knowledge of Content and Students.)/ K.C.T	Effort for using illustrative examples and teaching aids for lesson concretization / representation is shown.	No examples and teaching aids used both verbally and practically.	Examples and teaching aids for lesson concretization / representation are verbally cited.	Drawn teaching aids /representations are used.	Manipulative teaching aids /representations are used.	A combination of drawn and manipulative teaching aids / representation is observed.	
K.C.S/S.C.K	Teacher recognizes errors and misconceptions and addresses them.	Errors and misconceptions are not observable.	Errors and misconceptions are present but not recognized.	Errors and misconceptions are recognized but ignored and incorrect answers are simply interpreted/corrected.	Incorrect answers from risen misconception/ errors have been individually challenged.	Errors and misconceptions are shared and discussed with learners.	
K.C.T	The teacher is giving the How feedback to learners.	No How feedback is observed.	Direct feedback is given.	Inexplicit feedback is given.	Cognitive conflict type of feedback is given.	The feedback through class-debate is observed.	
K.C.S/K.C.T	The teacher is giving the	No What feedback is	The given feedback is	The feedback given is about	The given feedback is	The personal feedback (self)	

	What feedback to learners.	observed.	about task or product.	process to create product.	on self regulation level.	is given.	
K.C.T	Teacher is unpacking the methods / concept to make the content more approachable for learners.	Any attempt to unpacking the methods / concept is observed.	Only rules/procedural descriptions are used to unpack the methods.	More than one methods/ways are shown to unpack the methods /concepts but not followed by their comparison/analysis.	More than one methods/ways are shown to unpack the methods/concepts and besides their Comparison/analysis is observed.	Only definitions/conceptual are used to unpack the concepts.	
K.C.T	Teacher is putting into place problems to clarify the concept and alternative strategies are observed.	Problems to clarify the concept and alternative strategies have not been in place.	Posed problems have been worked on by teacher-learner direct interaction.	Problems given have been worked on, checked as seatwork individually or in individual group but not shared.	Posed problems as seatwork individually or in individual group have been worked on, checked and shared.		
K.C.S/ K.C.T	Learners' prior knowledge is assessed.	Prior knowledge has not been engaged.	The captured learners' prior knowledge has not been used as foundation of the new topic to learn.	The captured learners' prior knowledge has been used as foundation of the new topic to learn.			