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Structured Arrangement Supporting The Development of Splitting Level in Doing Multiplication By Number Up To 20

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Abstract

In guiding students to construct a mathematical concept themselves, learning process should be started by a context which is suit with the concept. In this research, we focused on structured arrangement which was believed to be able to support students ages 8 – 9 years old developing splitting strategy in doing multiplication. This study was a design research underlined by tenets of Realistic Mathematics Education (RME). The result show that giving structured objects, recognizing number relation in multiplication, and maintaining flexible calculation have important roles in supporting the development of mental calculation achieving splitting level.

Key words: design research, mental calculation, splitting level, structured arrangement

Abstrak

Penelitian ini telah dilaksanakan di MIN 2 Palembang, melibatkan 28 siswa dan seorang guru di kelas tiga. Tujuan dari penelitian ini adalah merancang rangkaian aktivitas pembelajaran yang mendukung siswa untuk mengembangkan mental berhitung mereka hingga tahap pemisahan (splitting level) dalam perkalian hingga bilangan pengali 20. Semua aktivitas dirancang dengan mempertimbangkan prinsip - prinsip dari Realistic Mathematics Education (RME). Dalam merancang aktivitas, kita memperhatikan: proses berpikir siswa; bagaimana aktivitas yang dirancang mampu membantu mereka dalam mengembangkan mental berhitung mereka dalam perkalian hingga tahap splitting; bagaimana diskusi kelas membantu siswa berkemampuan rendah untuk belajar; dan bagaimana peran guru dalam mendukung siswa untuk belajar. Pemberian objek – objek yang terstruktur, kemampuan mengenal hubungan antar bilangan dalam perkalian, dan pembiasaan berhitung secara fleksibel mempunyai peran penting dalam mendukung perkembangan mental berhitung siswa hingga tahap splitting.

Kata kunci: design research, mental berhitung, splitting level, objek - objek terstruktur

Introduction

Multiplication is a fundamental concept that supports concepts in mathematics like: division, fraction, percentage, etc. The knowledge of multiplication is also required in

estimation and mental computation in (Heerge, 1985). Issac (Braddock, 2010) asked, "How can students use 80 x 40 to estimate 84 x 41 if they do not know 8 x 4?"

In Indonesia, students have learned multiplication since grade 2 and 3. However, most of the time, they are required to memorize multiplication table. Meanwhile, memorizing multiplication does not help students to solve extended multiplication because they do not know the meaning of it (Armanto, 2002).

Freudenthal (1991) argued that students need experience mathematics and achieve concept of mathematics including multiplication through sequence of learning activities since mathematics itself is as 'a human activity'. Gravemeijer (Foxman, and Beishuizen, 2002) suggested that learning process in multiplication should be started by informal strategies which are promoted both by realistic contextual problems and by mental calculation. Realistic context problems are not always connected with real world but is related to the emphasis that it puts on offering the students problem situations in which they can imagine (Van den Heuvel-Panhuizen, 2000). Using context problems help students to start constructing their knowledge because they can use their own experiences related to the problem. On the other hand, mental calculation is a way of approaching numbers and numerical information in which numbers are dealt with a handy and flexible way. It is not strictly related to a certain number area or operation (Van den Heuvel-Panhuizen, 2001). Thus, students also can learn multiplication using their own way based on their mental calculation without forcing them to memorize.

Considering the need to develop students' mental calculation on multiplication and the need to connect mathematics to reality, we design instructional sequence of activities starting from structure context which is conjectured to be able to lead students to achieve splitting level on multiplication. Therefore, we formulate a research question:

"How can structures support the development of splitting strategy in doing multiplication by numbers up to 20?"

Theoretical Framework

1. **Realistic Mathematics Education**

   Realistic Mathematics Education is a theory of teaching and learning in mathematics education that was firstly introduced and developed by Freudenthal

Institute in Netherland. One of central principles of Realistic Mathematics Education which is mostly determined by Freudenthal's idea in mathematics education is that mathematics is as 'a human activity'. It means that mathematics should be taught in the way where students can experience and attain concepts. Therefore, this study design and develop a learning sequence in multiplication in which students can develop their mental calculation achieving splitting level on multiplication by numbers up to 20 through experiencing a sequence of learning activities rather than memorizing multiplication table. However, the sequence of learning activities in this study is only a part of longer series of learning trajectory in developing students' mental calculation on multiplication.

The sequence of learning activities is designed based on five tenets of Realistic Mathematics Education which are stated by Treffers (Bakker, 2004). We describe the five tenets of RME as following:

1. *Phenomenological exploration.*

   Contextual situation is used as a starting point in the first instructional activity. Beginning with contextual situation, students can use their informal knowledge since they experience the situation. *Kondangan*, a ceremony event in Indonesia that is held because of marriage or circumcision one or more sons, is a context that will be used in learning process about multiplication. It contains structured objects that can be used to lead student to achieve splitting level of mental calculation. Contexts such as spoon boxes arrangement and arrangement of guess chairs are considered as a starting point to stimulate student's informal multiplication strategies directing to splitting strategy.

2. *Using models and symbols for progressive mathematization.*

   The development from informal to formal mathematical concepts is a gradual process of progressive mathematization. From contextual situation, students start to mathematize by making model of situation and continue to generalize by making model for calculation. Rectangle models can be used to support their development of mental calculation achieving splitting level in multipication. These instruments are meaningful for the students in representing situation and have the potential for generalization and abstraction.

3. *Using students' own constructions and productions.*

   It is assumed that what students make on their own is meaningful for them. Hence, using students' constructions and productions is promoted as an essential part of instruction. Students use their own way in representing situation. They also work using their own strategies although we direct them to achieve splitting level n doing multiplication.

4. *Interactivity.*

   Students' own contributions can be used to compare their works. Students can learn from each other in small groups or in whole-class discussions. Through discussion, they can evaluate and improve the way they achieve splitting level in multiplication. Discussion also can help low achieving students to learn.

5. *Intertwinement.*

   It is important to consider an instructional sequence in its relation to other domains. In learning multiplication, students also can intertwint with another concepts like addition and subtraction in basic forms of mental calculations.

2. Splitting Level on Multiplication

Splitting strategy involves "breaking down" numbers where multiplicand or multiplier is no longer seen as a whole but it is separated into another numbers – it is usually split into tens and ones in multiplication with larger numbers. In the area of basic multiplication, this is characterized as structuring based multiplication where a problem, for instance, a 6 x 8 structure, is no longer solved by step-by-step counting but is reached with only one intermediate step using the known table product of 5 x 8. According to Ambrose (2003), splitting is acquainted alongside when they have extended repeated addition or doubling which is categorized as stringing strategies. Using multiplication situations based on different structure will lead to the splitting strategy becoming independent of the concrete examples. In the area of multiplication, three kinds of context situation like situation with a line, group, and rectangular structure support the insight that 6 x 8 is 5 x 8 plus 1 x 8 (Van den Heuvel, 2002)

Ambrose (Buijs, 2008) mentioned strategies in multi-digit multiplication problems - which are used by students that have not had any formal instruction on the subject - vary from elementary solution strategies such as repeated addition and various form of doubling to more sophisticated strategies based on decimal splitting. It is a process for students to develop their strategy in multiplication where the basic strategies can

be involved into more advanced strategies. In this research, we want to support students to develop their strategy until splitting level through designing a learning trajectory for the domain of multiplication based on the realistic approach of mathematics education. We try with basic multiplication because students have to be advanced with multiplication table before they acquaint with splitting level in multiplication with larger numbers.

3. Structures on Multiplication

Freudental (1991) suggested that context situation should be used to promote students to learn mathematics because they can use their experience to construct new knowledge. However, the chosen context should be appropriate with attained mathematical idea. Using structured arrangement are believed to be able to make students aware of splitting strategy on multiplication. In this study, structured arrangement mean objects that are arranged in certain condition such that students recognize parts of arrangement and can use multiplication in each part.

Freudenthal (1991) believed that structuring is a means of organizing physical and mathematical phenomena, and even mathematics as a whole. In this study, structures which are used are structured arrangement in group situation and rectangular situation. There are group situation such spoon boxes arrangement and rectangular situation such guest chair arrangement which are used at the beginning of instructional sequence of learning activities. In group situation, some spoon boxes are arranged into some parts purposing students can calculate them in the partition. Guest chair arrangement in rectangular situation provides chair arrangement based on rows and columns.

4. Learning Multiplication

In Indonesia, multiplication generally has been taught in the second and the third grade. They have learned it as repeated addition in the first time (Armanto, 2002). However, most of the time, they are required to memorize it. Whereas, Issac (Braddrock, 2010) argue that memorizing can cause anxiety in students which can lead to a lack of motivation and a bad attitude toward mathematics. Otherwise, children with deep conceptual understanding of multiplication will have an advantage when they have faced with a forgotten multiplication fact. Besides that, Hergee (1985) believes that knowledge of basic multiplication support flexible mental calculation. Since they only know multiplication as repeated addition, Clark and Kamil

(Braddrock, 2010) argue it requires higher order multiplicative thinking which the child develop out of addition.

In learning basic multiplication, Issac & Carrols (Braddrock, 2010) argue that basic multiplication facts should begin with real world multiplication situation that students can model with manipulative and count all the objects. It is in line with some tenets of RME that instruction activity is started with contextual situation and let students to make representation of situation. Drawing pictures such as equal groups and arrays should be a part of the early instruction of basic facts. Using multiplication situation like group or rectangular situation will lead them to recognize multiplication structure. Because of structure context, it promotes students to use splitting strategy.

Sherin and Foson (Braddrock, 2010) believe that students will learn some basic facts on their own like multiplication by 5 and multiplication by 10 since they have ability to count by 5 and 10, but other higher factors like 6, 7, 8, and 9 require explicit instruction. However, Angileri argues that they can use lower factors (from 1 until 5) to solve problems with higher factors.

Research Methodology

In this study, design research is used as a research methodology to achive research goals and discover the answer of research question. According to Gravemeijer (2006), there are three phases of design research: preparation for the experiment, experiment in the classroom, and restropective analyses. These phases are descibed as following:

1. Preparation for the experiment

This phase contains preparations before experiment is conducted. It includes designing hypothetical learning trajectory, determining research subjetcs, and doing pre assessment. A hypotetical learning trajectory which is designed consists of mathematical goal, teaching and learning activities, and conjecture of student's thinking. The designed hypothetical learning trajectory will be implemented in a experiment phase. Before any instruction begin, we also do observation in school where we will conduct the research. Observation includes contact with teacher, condition of school, and students. It is also a phase to choose a number of students which are involved as research subjects. However, pre assessment is also useful to be done. The purpose is to investigate students' pre – knowledge about the way they learn multiplication and how far they have learned it. It is used as starting point before

we do insructional activities. It includes interview with teacher and written test for students in whole class. In written test, students had to answer some open questions. For further need, it is possible to do interview for some students to know more specific information. After hypothetical learning trajectory have been designed, research subjects have been chosen, and pre assessment have been done; designed hypotetical learning trajectory is conducted in classroom experiment.

2. Experiment in the classrrom

In this phase, we do experiment in classroom and test the conjecture of student's thinking which is designed in preparation phase. The aim of this phase is to improve learning trajectory in hopes of helping students to aquaire multiplication basic facts through developing their mental calculation. The experiment is conducted in 6 lesson. Each lesson emphasizes mathematical idea and students' learning process. Before doing each lesson, researcher and teacher discuss about upcoming lesson and make reflection after each lesson is done. During experiment, all data including classroom observations, interviews, poster, and students' worksheet are collected. These gathered data will be analyzed in restropective analyses phase.

3. Restropective analyses

After data are gathered, we analyze all data and get evaluation for hypotetical learning trajectory. According to restropective analysis result, the hypotetical learning trajectory is developed and is tried out in the next cycle. At the end of the last cycle, we discover the answer of research question about what support that we can give to students to develop their mental calculation in acquairing basic multiplication facts based on restropective analysis result.

Research Subjects

This research was conducted in MIN 2 Palembang which has become school partner of PMRI since 2006. Twenty eight students of 3A, one group which consist of 5 students in 3B, and a teacher of grade 3 were involved in this research. The students were about 8 to 9 years old and they had learned about multiplication as repeated addition in grade 2.

Hypothetical Learning Trajectory

According Simon (2004), An HLT consist of the goal for the student's learning, the teaching and learning activity, and hypoteses about the process of the student's

learning. The mathematical goals which want to be reached are as directions for teaching and learning process and hypotesis of student's learning. The selection of learning tasks and the hypoteses of student's learning are interdependent where the tasks are selected based on hypothesis about the learning process and the hypothesis of the learning process is based on the tasks involved. Grameveijer (2004) added that the activity of designing instructional activities is guided by a conjectured local instruction theory, which is developed in advance, and which is refined and adjusted in the process.

Goal	Activity	Mathematical idea	Conjecture
Students can solve multiplication problem by repeated addition	**Calculating *Kondangan* Warmer Square Pans**	Multiplication as a repeated addition	- They make drawing like rectangles, lines or numbers to represent pans. - They use repeated addition in their calculation.
Students are able to use number relation to solve the problem	**Calculating Spoons**	Number Relation	Students may use repeated addition in determining the number of spoons students use number relation when they know that number of spoons in 8 boxes can be known when they add number of spoons in 3 boxes and number of spoons in 5 boxes.
students are able to make structure	**Arranging Guest Chairs**	Multiplication Structure	they may make drawing like the actual chairs, rectangles, circles, or lines as representation of chair arrangements. In determining the number of chairs, they may calculate how many chairs in the left side and in the right side then they sum up to get the total. They may use repeated addition in calculating the number of chairs in each side.
Students can solve multiplication by ten problem.	**Chair for Guests**	Multiplication by ten	Some students solve by repeated adding or doubling, to compute all chairs. When they are able to see their

			drawing in different side they can calculate in different way, they exchange the numbers.
Students are able to use subtraction from the result of multiplication by tens to determine the result of multiplication by nine	**Attending Guests**	Multiplication by nine	Some students may still use repeated addition. The others may use subtraction from the result of multiplication by ten.
Students can solve multiplication problem with larger number than 10 by using number relation between tens and ones.	**Solving Multiplication with Larger Number by Number Relation.**	Number Relation in Multiplication with larger numbers	In solving multiplication larger than ten, they may solve via multiplication by ten plus multiplication by ones.

Result and Analysis

According to result of pretest, most of students are able to do repeated addition in finding the number of objects which are arranged in group situation and rectangle situation. Based on interview and the result of pretest, multiplication by 6, 7, 8, and 9 are categorized as uneasy multiplications for students to learn. Some students admit that they have known multiplication by 1 until 5 and multiplication by 10 but they are struggled in multiplication by 6, 7, 8, and 9.

Based on observation in pilot experiment, we make conclusion as follow:

In activity 1, students used repeated addition in finding result of multiplication. The difficulty for students is that they were not good enough in translating problem into multiplication. To help them, teacher asked students to make drawing and pose some questions which purpose in order students know the meaning of multiplication related to situation in the problem.

In activity 2, students are able to use number relation where they can involve the result of one problem in finding the solution of the other problem. However, they have difficulty in connecting the results that they get with its multiplications. To help them, teacher also pose question which purpose in order they remain and formulate multiplication based on situation in problem.

Students have struggled in determining multiplication based on row and column in chair arrangement that they made. This happened because they did not really understand about array – the role of rows and columns. To help students to use structure, we are planning to use group situation like activity 2 rather than use rectangular situation. Since the purpose of this activity is that students are able to use structure and this purpose can be covered in activity 2, this activity might be not to be used in the second cycle.

There were many ways that students used in explaining situation in activity 4. They are able to find the result of multiplicand by nine by relating to the result of multiplicand by 10. The difficulty is that they were not able to formulate the connection of those multiplications. To help students, teacher can pose some questions in order they see their connection.

To solve multiplication with larger number than 10, students are able to split multiplicand into tens and ones. However, they also have problem in connecting whole multiplication with part multiplication. Similar with previous activity, teacher can help them by posing some questions in order they can see their connection.

Considering students to be able to do flexible calculation in solving multiplication by splitting strategy, there might be needed additional activities where students work in more formal ways. Those activities are purposed to maintain students to involve multiplication that they have already known in solving multiplication problem using splitting strategy.

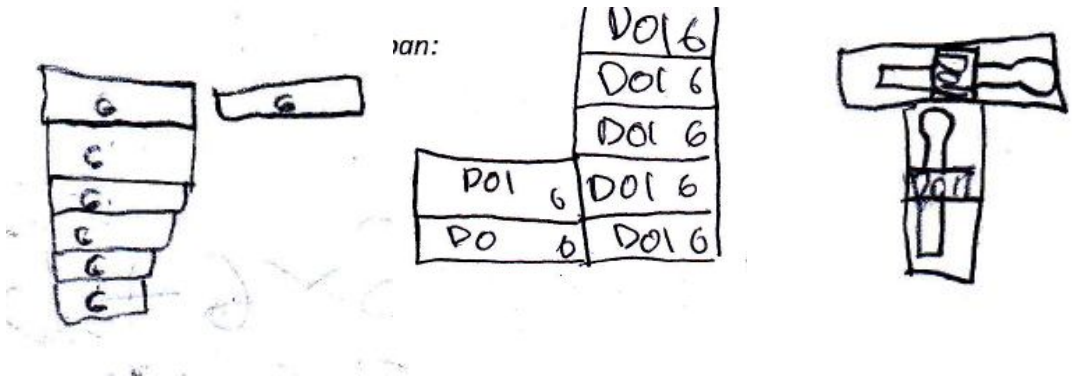
Revised Activities for Second Cycle

At the beginning, there were 5 activities which were designed. After we tried out the designed activities in the first cycle and analyzed them, there are some improvements in some activities. According to our observation and analysis, there is activity that needs to be done outside the second cycle since this activity maintains the basic condition of students before they acquaint with splitting level. There are also some addition activities that we think they need to be included. Therefore, we do revision which is purposed to improve HLT in order we can reach research goal in better way.

No.	Activity	Goal	Expected result
1.	**Using two multiplications for One Multiplication**	Students are able to use the result of two multiplications in determining the result of a	When student have a multiplication, For instance, 8 x 6, they can split it into 5 x 6

		multiplication.	plus 3 x 6
2.	**Knowing Relation between multiplication by 10 and by 9**	Students can use the result of multiplication by 10 in determining the result of multiplication by 9	Students can find the result of 9 x 7 by subtracting 7 from the result of 10 x 7.
3.	**Working with model for**	Students are able to draw rectangle model as a model for representing situation	Students can draw rectangle to describe a pile of boxes of chair arrangements.
4.	**Determining One larger number and one smaller number Multiplications**	Students can determine multiplication "before" and multiplication "after" of a multiplication	When they have a multiplication, e.g. 5 x 8, the can determine the result of 4 x 8 and 6 x 8.
5.	**Using known multiplication for another multiplication**	Students are able to use multiplication that they have known in determining the result of a multiplication	In finding the result of 6 x 8, they use multiplication that they know, e.g. 5 x 8 and add more 8 into it.
6.	**Completing Multiplication table**	Students can complete multiplication table, multiplication by 11 and by 12 by using number relation.	In solving 11 x 7, students split it into 10 x 7 plus 1 x 7

This figures below show how students drew situation of spoon boxes after they were asked to arrange the boxes.



(i) (ii) (iii)

Figure 1. Ribi's drawing (i), Dwi's drawing (ii), and Abdulrahman's drawing (iii)

Furthermore, students were asked to calculate number of spoons in the first pile and the second pile. In determining the number of spoons, they saw number of boxes in one pile. When they knew them and realized that there were 6 spoons in one box, they used multiplication to find the number of spoons. For example, they arranged two

piles of boxes with 2 boxes in one pile and 5 boxes in the other pile. They know that one box contain 6 spoons and they had 2 boxes. Therefore, they know the number of spoons in two boxes was equals to the result of 2 x 6. At the beginning, some student might saw the first pile consisted of 5 boxes therefore they use multiplication 5 x 6 to find the number of spoons. The other students saw that there were 2 boxes. Therefore, the number of spoons was equal to the result of 2 x 6. It happened because they had different decision in determining which one the first pile and the second pile. After discussion, they agreed to decide one pile with two boxes as the first pile and the other as the second pile.

In determining the number of spoons altogether, some students multiplied 7 x 6 since they saw the boxes altogether. They knew that there were 7 boxes with 6 spoons in each box. Therefore they calculated them by multiplying 7 x 6. Some other students saw that the number of all spoons was equal to the number of spoons in the first pile which was added with the number of spoons in the second pile.

In her presentation, Mutia told about number of spoons in each pile of boxes which were arranged by her group. Her group arranged 6 boxes in the first pile and 1 box in the second pile. However, she calculated all spoons by multiplying 7 x 6 because there were 7 boxes altogether with 6 spoons in each box. Ribi did not agree with Mutia's reason since she had two piles of boxes where each pile had different number of boxes but she calculated number of all spoon by multiplying 7 x 6 – the way when she look at the number of boxes altogether.

Through this activity, we could see that students use both whole multiplication and part multiplications to know how many all spoons. Some of them saw all boxes and use whole multiplication to find the number of spoons altogether. However, some students also could see the number of spoons in each pile and added them with the other number of spoons in the other pile. By calculating the number of spoons in each pile and adding them, they could find the number of spoons altogether. The only difficulty that they had is when they tried to formulate and connect between whole multiplication and part of multiplications based on box arrangement that they made. For them who were not able to see the use of two multiplications in finding the number of all spoons, they were helped by illustrating with the real boxes that they arranged. By asking them to see boxes in each pile and formulating its multiplication,

they see that they could use those multiplications by adding them in determining the number of spoons altogether.

In activity 2, most students used multiplication as a handy way. However, some students found the result by connecting two multiplications. They involved previous knowledge where they could use two multiplications in finding the result of a multiplication. Some of them added two multiplications and the others were able to use subtraction. The difficulty that they had is when they formulate form of multiplication for situation that they had. To help them, teacher asked students who were able to formulate the multiplications to explain and to discuss with the other students.

Most students used rectangle model to describe situation in activity 3. This happened since teacher introduced this model at the first time. Although teacher did it, some students still described situation by their ways using rectangle model. They drew rectangles to represent both group situation like a pile of boxes and rectangle situation like chair arrangement. The only unexpected thing happened was that some students made each drawing in each number of questions. This happened because they want to give answer for each question. Knowing this condition, teacher remained students about instruction in the question that asked them to draw in same figure that they made in previous question.

All students started from multiplication that they knew and used it to find the result of other multiplication. Most of time, they started from multiplication that had already stated in the cards and their worksheet. However, there were also some students started from the smallest multiplication. For students who had struggled, teacher helped them by relating their problem to context situation. Therefore, students could think through situation.

We got that most students used multiplication by 5 and multiplication by 10 for multiplications more than ten since they have already known these multiplications. However, they also recognized multiplication that they know after they look at multiplication before or multiplication after. They could start from that multiplication if they know its result.

This following is an example of student's work in solving 16 x 6.

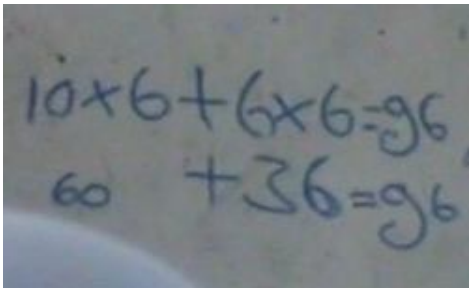


Figure 2. Ribi's solves 16 x 6

In his presentation, Ribi argued that 16 x 6 can be split into 10 x 6 and 6 x 6. It was done after he recognized that he could separate 16 based on tens and ones. Therefore, he split 16 x 6 into 10 x 6 and 6 x 6. The other students also considered that they could split it into 10 x 6 and 6 x 6 because they know the result of multiplication by 10.

We got that most students used multiplication that they have already known in determining result of a multiplication. They could start from multiplication before, multiplication by 5, or multiplication by 10. It happened since they could see number relation of multiplicand which can be used. By separating a whole multiplication into some parts of multiplications that they know, it could help them to find result of the whole multiplication. The only difficulty that they had is when they determine multiplication by ones after they spilt a whole multiplication into multiplication by tens and multiplication by ones. It happened since students sometimes forgot the result.

Conclusion

Following the sequence of learning activities starting from structured context could give opportunity for students to achieve splitting level on basic multiplication. Students could see that unknown multiplication can split into some multiplications to find the result of the unknown multiplication.

In this study, group situation which was used is spoon boxes arrangement and the rectangular situation which was used is guests chair arrangement. Using structured objects like spoon boxes arrangement could lead students to see some parts of arrangement. Thus, they calculate objects by splitting them into the parts. Through discussion, it could promote students to see that a multiplication can be separated into some multiplications because the result of the whole multiplication is same with addition of those multiplications.

In rectangular situation such guest chair arrangement, objects are arranged based on rows and columns. Through arrangement of guest chairs, students could see relation between used objects and unused objects. Because of this condition, they determined the number of used objects by subtracting the number of all objects with the number of unused objects. Therefore, we conclude that giving structured objects, recognizing number relation in multiplication, and maintaining flexible calculation have important roles in supporting the development of mental calculation achieving splitting level.

Students can apply splitting strategy in determining result of multiplication up to 20 since they perceive that they can involve multiplication that they have already known to determine result of forgotten multiplication in multiplication table. In solving multiplication up to 20, most of them agree that multiplication by 10 was an easy multiplication that they can involve to find result of the multiplication. By splitting the multiplication into multiplication by 10 and multiplication by the rest of numbers, they could determine result of multiplication by number up to 20.

This study is only a small part of the development of mental calculation on multiplication. However, because of this small part, it can affect their mental calculation on multiplication by number up to 20. Perhaps, it also applies for multi-digit multiplications. Therefore, it raises new questions such as: how can splitting strategy be applied on multiplication with larger numbers? To answer the question, further research is needed.

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