

## A Mentoring Program for Inquiry-Based Teaching in a College Geometry Class

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### Abstract

This paper describes a mentoring program designed to prepare novice instructors to teach a college geometry class using inquiry-based methods. The mentoring program was used in a medium-sized public university with approximately 12,000 undergraduate students and 1,500 graduate students. The authors worked together to implement a mentoring program for the first time. One author was an associate professor and experienced using inquiry-based learning. The other author was a graduate student in mathematics education. During the course of the year the graduate student first observed and then taught a college level inquiry-based geometry course for pre-service teachers. This article describes the details of this mentoring program and our reflections on how the program went.

**Key words:** Inquiry-based teaching, Mentoring, College geometry, Mathematics education.

### Introduction

Many instructors find it difficult to implement inquiry-based teaching methods for the first time. As discussed by Hodge (Hodge, 2006), the difficulties they encounter may include a sense of loss of control over the classroom, increased workload, resistance on the part of students and issues of “coverage.” In an attempt to ease the transition for novice instructors trying to learn how to successfully teach using an inquiry-based style, an intensive mentoring program has been established at the University of Northern Colorado (UNC), supported by funding from the Educational Advancement Foundation. During the 2007–2008 academic year, the authors worked together to implement this program for the first time. One of us (Nathaniel Miller), an associate professor, was an experienced inquiry-based learning instructor while the other (Nathan Wakefield) was a graduate student in mathematics education. During the course of the year Mr. Wakefield first observed and then taught a college level inquiry-based geometry course for pre-service teachers. This article describes the details of this mentoring program and our reflections on how the program went. The program was quite successful and we would encourage others to adopt a similar model.

UNC is a medium-sized public university located in northern Colorado. The university has approximately 12,000 undergraduate students and 1,500 graduate students. The university was founded in 1889 as the State Normal School for training educators, and although the university has evolved over time, a large emphasis is still placed on teacher preparation. The university offers a secondary teaching emphasis B. S. degree for pre-service secondary math teachers, and also offers a mathematics concentration within its elementary education degree for pre-service elementary teachers.

At UNC, both the mathematics secondary education majors and the elementary education majors with a math concentration are required to take a course called Modern Geometry I. This course is the main focus of the mentoring program. Mathematics secondary education majors are also required to take a second course called Modern Geometry II. Both of these courses are taught in an inquiry-based style. The material for these courses is broken into a series of 9 to 12 small projects which students complete in randomly chosen groups of 2 to 4 students. In addition to time spent in these groups working on the projects, a significant portion of class time is spent on whole class discussions about the projects. Detailed course notes (including all of the projects) for both of these courses are available from the *Journal of Inquiry-Based Learning in Mathematics* (Miller, 2010a,

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2010b). These notes loosely follow the first several chapters of the textbook *Experiencing Geometry* by Henderson & Taimina (Henderson & Taimina, 2005), and cover a mixture of Euclidean and non-Euclidean geometry.

All of the projects that students work on are submitted as written papers. Some papers are submitted individually, while others are submitted in groups. All of these papers go through an intensive process of revisions. Some assignments are designated as formal assignments, which are graded on both their correctness and their clarity of explanation. Students submit a rough draft of these assignments, on which the instructor makes detailed comments, and then submit a final draft, which receives a letter grade. Other assignments are designated as informal assignments. These are graded primarily on the completeness and correctness of their explanations, and students are allowed to revise and resubmit them as many times as they want to. For further details, please see the course notes (Miller, 2010a, 2010b).

The rest of this paper describes the mentoring process that we implemented during the 2007–2008 academic year. We alternate sections to give both of our perspectives on the mentoring process.

## The First Semester of Mentoring

### Observing an experienced mentor

The mentoring process took place over a two-semester academic year. During the fall semester I observed both Modern Geometry I and Modern Geometry II. In addition to observing the courses, I held office hours for students in both courses, which gave me the opportunity to grow more familiar with the material. While observing the classes, I often walked around the classroom, helping Dr. Miller in answering any questions the students had. As the fall semester progressed, I gradually began grading student papers. Initially, I just looked over assignments that Dr. Miller had already graded, focusing on the types of comments that Dr. Miller had made and what his expectations were. As the semester progressed, I moved on to grading the assignments myself and making my own comments. By the end of the semester, I was doing most of the grading for the course.

Dr. Miller does not use a textbook in Modern Geometry I, and students are not allowed to refer to a textbook for the course. The emphasis is placed on the students' own discovery of mathematical ideas. Students are allowed to talk to each other about ideas but no outside resources are allowed. Students typically receive a question to work on in their groups. The questions are designed to have multiple entry points with the goal of developing in the students a deep mathematical understanding of the topic. During the class period, the instructor facilitates group discussion and ensures that groups stay on track. If a group is completely stumped, the instructor may give a hint to trigger their further progress. The hint, however, may often be presented as "another group has found that. . . ." In this way the instructor is not seen as the giver of hints, and instead, students begin to rely on peer-to-peer communication for their ideas.

I felt a lot of stress grading the assignments when I first started. Although I had already read over comments that Dr. Miller made on previous assignments, I still felt that I lacked the intuition to correctly assess student thinking. One item that I struggled with in particular was the variety of student responses: what if a student was thinking about a problem in a novel way, and I simply lacked the intuition to understand it? As the semester progressed I gained confidence in my grading. I began to realize an important notion, which was that if I didn't understand something, the item needed clarification. A key part of the course was training students to communicate mathematical ideas. Prior to this realization, I assumed that my inability to understand students' arguments was often my fault; their arguments were surely correct—I simply didn't understand them. It took me a while to realize that whether or not an argument was correct was irrelevant if it wasn't explained clearly enough that I could understand it. By the time I was teaching in the spring, I felt confident telling a student they needed to better formulate an argument. I spent much less time trying to understand students' arguments and more time helping students clarify their answers.

During the last several weeks of the semester, I acted as a student and joined a group to work on the final project. Joining as a student gave me the opportunity to inject my own ideas and have my work graded. I assumed that after a semester of reading other student papers I would have no trouble explaining my own ideas. I was wrong; I still struggled. I was able to experience the frustration of getting what I thought was a complete paper returned with comments and a request to resubmit. Having my own work graded and commented on helped me to become better at writing comments on student papers and at asking students for better

explanations.

### **The first semester: a gradual immersion in inquiry-based teaching**

One of the greatest challenges in learning to teach in an inquiry-based style is the vast amount of pedagogical content knowledge that the instructor needs. Pedagogical content knowledge (PCK) is knowledge about what is likely to happen in a classroom as the result of particular instructional choices. In an inquiry-based classroom, the instructor is constantly making instructional decisions in responding to what their students have just done or said. For example, what do you say to a group that is stuck on a particular part of a particular problem? When is the appropriate time to interrupt group work for a whole class discussion? How do you direct such a discussion to get to the end point you want when it has started off in a certain way? What comments on a particular mistake made in the write-up of a particular problem are most likely to lead a student to discovering the mistake on their own? The more experience that an instructor has teaching using a particular set of materials, the more likely they are to have answers to these kinds of questions, which can be overwhelming for a novice instructor. This is why mentoring is such a valuable tool in learning to teach using inquiry-based materials. It allows a prospective user of such materials to begin formulating such questions, and it gives them the means to begin answering them before they find themselves in the high-stakes, sink-or-swim situation of teaching a class using the materials.

Our mentoring program was set up to allow Nathan to become more immersed in the pedagogical content knowledge required to teach this course gradually, over the course of a whole semester. At the beginning of the semester, he mainly observed me teaching, and read my comments on student papers. As the semester progressed, he became more involved in teaching the class. First, he was given the responsibility of grading and commenting on rewrites of papers after having seen all of my comments on the first drafts. Then, he slowly took over most of the grading for the class. At first, I would carefully reread papers he had graded and make additional comments and changes as necessary; later on, as we both became more confident in his ability to grade papers on his own, I would only occasionally reread papers he had graded. As time went on, he also had more and better interactions with groups as they worked.

About halfway through the semester, I asked Nathan to completely take over the class while they worked on one project. I observed him teach, and we had long conversations about what worked well and what he might want to change. I think that this was extremely helpful, not only because it gave him a feeling for what it felt like to take over the class, but also because it raised lots of questions for him about instructional decisions, and changed the way that he observed me for the rest of the semester. In the beginning of the semester, he was watching me teach, but he wasn't necessarily aware of many of the instructional decisions that I was making, because he wasn't yet aware of the questions they were addressing. In this regard, learning to teach an inquiry-based class is a lot like being a student in one: you can't appreciate the answer to a question until you have grappled with it for a while and it is a meaningful question for you.

At the end of this class, I always assign one big final group project about geometry on the cone, and I try to assign students to homogenous groups in which all the students are working at roughly the same level. In the particular class that Nathan was observing, there was one student who was much stronger mathematically than everyone else in the class. (He has since graduated and entered a Ph.D. program in mathematics.) It occurred to me that he and Nathan would be roughly evenly matched as partners, and so I asked them to form a group together. This gave Nathan the experience of acting as a student in the class, and also gave the other student the chance to work in a group where he wasn't the strongest member. I think that this experience of being a student in the class informed Nathan's teaching in a different way from his observations of my teaching, and helped prepare him to teach his own section of the course the following semester.

### **The second semester of mentoring**

#### **Teaching an inquiry-based course**

Over the winter break, I began compiling all of the assignments from the previous semester in preparation for teaching during the spring semester. During this semester, I taught a single section of Modern Geometry I. Dr. Miller attended most of the classes as an observer, and met with me at least once a week to discuss the class. The course was taught using the same assignments and grading system from the previous semester. Using the same assignments was probably the single most important part of my experience. Instead of having to try to piece assignments together in a way that made sense to students but still covered the required material, I was able to use assignments that already met these criteria. Had I tried creating my own assignments, I probably

would have spent a considerable amount of time beyond a typical course load. I would strongly encourage anyone new to inquiry-based methods to first try using material developed by someone else.

Inquiry-based learning was a fundamental aspect of the course. Lectures were not used in a traditional sense, but instead, students were given questions to discuss with fellow group members, and students were expected turn in a written answer to each question. I facilitated class discussions, but the emphasis was on students' consideration of other students' ideas. To better facilitate a student-centered class discussion, I often asked students to come forward and present their ideas to the entire class. When a student finished, I would ask, "does anybody want to agree or disagree?" The first few times I asked for students to agree or disagree, I quickly moved on when no one responded. During one of my meetings with Dr. Miller, he challenged me to wait well beyond the time of awkward silence. The first time I waited seemed like a very long time, but eventually a student had a question. After a couple of classes of waiting past the moment of awkward silence, students began to realize they were expected to speak up in class. The students realized that if there was no response I would simply wait until eventually the situation became awkward and someone responded. Toward the end of the semester I noticed that this was no longer necessary and students would usually ask questions or state their agreement without any prodding.

The course met two days a week for an hour and 15 minutes. I also held regular office hours and office hours by appointment. During the class, students quickly learned that I would not give them answers, but instead expected them to find them on their own. I did notice, however, that many students, especially in the first few weeks, came to my office hours to try to get answers to the questions I would not answer during class. I dealt with this simply by insisting that they come up with their own answers. By the end of the semester, students no longer came asking for answers, but instead were asking me to just listen to their ideas.

### **Observing Nathan's class**

It is surprisingly hard to just sit in the back of a class that someone else is teaching using your materials—you find yourself trying to telepathically tell them what you think they should be doing all the time. Because we met after almost every class to discuss how things went, I was able discuss most of these things with Nathan shortly after they happened, and he would often take time at the beginning of one class to address some issues that I had noticed coming up during the previous class. Of course, it would be better to be able to address the issues immediately, but, just as students need the opportunity to make mistakes at first as they learn new skills, new instructors need the opportunity to make some mistakes and see how they influence how the class goes.

The issue of waiting long enough for student responses that Nathan discusses above is a good example. I'm sure that we had discussed waiting for students to respond before the semester began, and Nathan thought he was, in fact, waiting for students to respond at the beginning of the semester. However, he observed that they weren't responding in the way that he wanted, the way he had observed them responding in the class that I taught the previous semester. After this had happened a few times, we discussed the situation, and I told him that I thought he wasn't waiting long enough. He adjusted his teaching accordingly, even though it made him uncomfortable at first, and the students responded to the change.

I think this also speaks to how hard it is to gain pedagogical content knowledge by watching someone else teach if you haven't already tried teaching the material, and don't have particular pedagogical questions in mind. Once Nathan noticed that students weren't responding to his requests for discussion the way he expected, then he wanted to know what I did differently in order to get the students to talk more freely. He had seen me teach the class the preceding semester, but until this was an active question for him, he wasn't in a good position to look and try to see what it was I was doing differently. I was willing to wait longer than he was to get a response, but I probably only had to wait a bit longer and probably only at the very beginning of the semester, since I was actively working to create a classroom culture in which people felt free to speak from the first day of class. Furthermore, many of the things that I was doing to promote this culture probably would not have been visible except to someone who already knew exactly what to look for. For example, before our first big class discussion, I made sure that I knew exactly how far each group had gotten with the problem they were working on; I carefully picked someone who hadn't gotten very far with a problem but who was unafraid to talk about what she had done to call on first to start the discussion; and I made sure that every group got a chance to contribute to the first discussion. Even though he watched me do this, it would have been quite hard for Nathan to really notice the intention behind what I did, just as the students didn't notice how carefully I was managing their discussion. One of the hallmarks of a successful inquiry-based class is that the students don't notice most

of the ways that you are trying to manage what they are doing, and just remember that you posed a problem, and they solved it.

It was clear as the semester progressed that Nathan's questions about my teaching had changed quite a bit, and so just as he had taken over my class a few times the preceding semester, so too did I take over his class a few times. This allowed him to observe me teaching with his students, and I think that it was quite a different experience than watching me teaching my own students during the previous semester, because his pedagogical questions were much more developed.

## **Results**

### **Things that worked well**

My experiences observing classes during the Fall semester proved to be very valuable when I was teaching in the Spring. Of particular benefit was my observation of Modern Geometry II. Observing the second semester course helped me to identify what items were important for students' success in later courses, and this helped to ease some of my "coverage" issues. Observing the second semester helped me to identify specific areas in which students struggled, and allowed me to focus on those areas.

Another key component in the success of the program was the grading mentorship used in the Fall semester. Teaching one course using inquiry-based methods took me, as a new teacher, as much time as teaching two courses traditionally. I am told that this is not the case as one becomes more experienced. Much of the increase in workload can be credited to the increased time spent grading assignments. Participating in the fall semester grading allowed me to absorb a lot of the difficulties in the grading scheme during the semester that I was not teaching. The mentorship also helped me to see what to look for in student work. In retrospect, the program probably would not have been a success had I not been allowed to practice grading the semester before.

Teaching any course for the first time can be a challenge to a novice instructor, and although this course was a challenge, the mid-term feedback from students seemed to indicate that not only were students learning the material, but students were learning material they had not expected. Midway through the semester in which I taught I handed out some course evaluation questions. One student stated that "I admit that this is a class that after hearing from friends I never wanted to take.[sic] However, now that I am taking it, I am truly enjoying the material being taught. Modern Geometry is clearly not a typical math class [and] makes me think about things that I never would have thought about had I not taken this class." I suspect that the class seemed so unusual to the student because he or she had not previously set out to understand material in other classes for him or herself. Another student wrote, "I feel like my views of the world are being changed to be less restrictive." A third student wrote, "So far in this class, I have been extremely proud of my work ... I have learned a lot and am [better] able to express myself and ideas on paper." These comments of surprise at what they were able to learn in the class were typical of the rest of the student comments.

The student comments reflect on an important, non-trivial, aspect of the mentoring process. When I first signed up to be part of the mentoring process I thought to myself, "I had better study geometry—it has been a long time since I took geometry and I need to make sure I know the material." What I failed to realize was that equally important was the idea, "I need to think about how my students will think about this material, and what I can do to get them to think about the material independently." Prior to the mentoring process the second thought had not weighed heavily on my planning process. An important aspect of the mentoring process was how it has moved my perspective on teaching into the realm of helping students learn to think.

### **Ideas for improvement**

I tried to take detailed notes during the fall course. The problem, however, was that I had not taught before using inquiry-based methods and therefore the notes were not focused on the pieces that I later had questions about. My notes were mostly about how Dr. Miller broke the class period into sections and what he assigned on each day. I wish I had taken notes on the questions students seemed to struggle with and how Dr. Miller gave them just enough help to move them past their difficulties. In a future course, it might be useful for both the mentor and graduate student to take notes. Tape recordings might be useful, but capturing the students' questions might be difficult.

One particular question that I had was “why were my notes such a failure?” Dr. Miller and I discussed this and the conclusion we came to may provide some insight into what it takes to teach an inquiry-based course. In a traditional course one of the single most important items in teaching the course is how to introduce material and transition to the next set of material. I was an instructor who had only lectured prior to the mentorship and had been raised in a lecture environment for the preceding 16+ years, so it is not surprising that I chose to record in my notes how Dr. Miller introduced each topic and how he transitioned to a new topic. In retrospect I can see why this was a problem.

In an inquiry-based course, how to introduce a topic or transition to another one is not done via lecture. The development of good questions is key for achieving smooth transitions. If the questions are well prepared, the students transition automatically, and the teacher only acknowledges the transition to make sure all of the students have caught the change. Inquiry-based learning requires a different expertise. One must be ready for any question that might come up and be prepared to ask questions that will guide a student to a deeper understanding. In retrospect, I would like to have taken notes on what questions students asked and how Dr. Miller answered each of these questions.

### **How the experience changed my teaching**

In the time since the mentoring experience, I have left UNC in order to complete a Ph.D. program in pure mathematics at the University of Colorado, where I have had a couple of further opportunities to teach math classes. Prior to the mentoring experience, my teaching style was primarily lecture-based. Now, after the mentoring experience, I typically lecture for a very short time and then proceed with some form of group work. In a course without any pre-set syllabus, I would be willing to attempt a completely inquiry-based approach. In my own personal grading, I find I spend far less time trying to figure out if a student has the right idea and much more time helping a student to explain his or her own ideas. Through the mentoring process, I learned how much students struggle with communication.

Some of the changes in my teaching style have also been passed on to others through my work as the Calculus I assistant coordinator at the University of Colorado. A typical assignment in Modern Geometry was submitted several times. Students would receive comments after each submission and resubmit to get a better grade. I implemented a system at the University of Colorado in which a weekly written assignment from Calculus I was submitted to a TA for grading. The TA made comments and returned the assignment; students then had to respond to the TA comments and resubmit the assignment. This process of submission and resubmission was successfully implemented with more than 500 students. In my own calculus courses I now use group work on a regular basis. A key component of the group work I use is class discussion of the material, it is not uncommon for me to ask a student to attempt a problem in front of the class. I will generally look for a student who does not have a complete solution and ask that student to present as much as they can.

Prior to this experience I facilitated group work by giving groups a “hint” or helping them get past something they were stuck on. After the mentoring experience I have changed my style to ask groups directed questions. These questions might be common questions similar to “well what exactly is the problem asking you to do?” or “what information are you given in the problem?” However, sometimes it is helpful to get students to talk about their own thinking and so questions like “what thought process did you go through to write...” seem to facilitate more learning. Using questions to facilitate group work instead of giving hints has been a major change in my style.

### **Conclusion**

All in all, the mentoring program was a great success. The average student course evaluation rating for the class that Nathan taught was 4.8 out of 5, which is astonishingly high when you consider that the course was being taught by a novice instructor using teaching methods that were radically different than what the students were previously accustomed to. I think that this shows that both the instructor and the students bought fully into the teaching method. I don't think this would have been possible without the intensive mentoring program that allowed Nathan to slowly accumulate the pedagogical content knowledge necessary for success over the course of a whole year.

It is equally clear that the students in Nathan's class learned a lot. On their final course evaluations, four students “agreed” and all of the other fourteen students “strongly agreed” with the statement “I learned a lot in

this course.” Likewise, two students “agreed” and the remaining sixteen students “Strongly agreed” with the statement “I was challenged to think in the this course.” Eight of the eighteen students in Nathan’s class continued on into the second semester course that I taught the following fall, and it was clear that they were well prepared. In fact, the final GPA in that class of these eight students was slightly higher than that of the students who were in the other first-semester class that was taught at the same time as Nathan’s class by an experienced regular faculty member.

Since the year I spent mentoring Nathan Wakefield, the mentoring program has continued with very little change, and it continues to be successful. Four other graduate students, Frieda Parker, Lee Roberson, Sarah Rozner Haley, and Rebecca Dibbs, have each completed the mentoring program in subsequent years.

I’m not surprised that intensive mentoring has been a successful means of preparing an instructor to teach in an inquiry-based style, since that is essentially how I learned to teach in this style myself. As a graduate student at Cornell, I acted as a TA for several different professors who taught in an inquiry-based style, including David Henderson and Avery Solomon, and then looked for opportunities to teach this way myself. An instructor who wants to learn how to teach in an inquiry-based style for the first time and doesn’t have access to mentoring from an experienced practitioner can only acquire the pedagogical content knowledge they need to be successful by trying things and making mistakes with their first class. If things don’t go that well, the instructor is likely to conclude that inquiry-based teaching doesn’t work well or doesn’t work well for them, and may not try again. Mentoring gives the novice teacher the tools and the pedagogical content knowledge needed to be successful their first time teaching an inquiry-based class. I would encourage anyone considering teaching an inquiry-based class for the first time to seek out a mentor, preferably one who is experienced in using the materials that you want to use with the audience that you intend to use them with. Not all mentors and mentees will have the time and resources to engage in such an intensive mentoring program over a whole year, but when it is possible, I would say that such a program is highly desirable.

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