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THE IMPORTANCE OF ACKNOWLEDGING THE CULTURAL DIMENSION IN MATHEMATICS TEACHING AND LEARNING RESEARCH

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Abstract. In this paper, which is in four parts, I make a plea to those involved in research into mathematics teaching and learning of the need to acknowledge, however their work is framed, that it will be located in a culture, not always visible to a reader, that should be made explicit. In the first part I examine three key models of culture and their significance for education. In the second I further highlight the impact of culture on what children are expected by critiquing various models of curriculum. The third part examines how culture informs the particularities of four European mathematics curricula, while the fourth part explores culturally located differences in mathematics teaching. In so doing a plea to researchers is framed: Culture permeates all aspects of educational endeavour and should be acknowledged more explicitly than it is.

Keywords: Comparative education research, comparative curriculum analysis, comparative mathematics teaching, culture, mathematics education research.

Introduction

For nearly twenty years I have had the good fortune to have been funded to visit and observe, videotape and analyse, mathematics classrooms in a number of European countries. In so doing I have been drawn to the conclusion, as was the survey of mathematics and science opportunities team (Schmidt et al., 1996), that mathematics teachers in any country behave in ways that align them more closely with their compatriots than with teachers elsewhere. I have also concluded that aspects of the cultures in which teachers operate may have as much, if not more, influence on student achievement than the ways in which they conceptualise and present mathematics. For example, successive iterations of the programme of international student assessment (PISA) (OECD, 2001, 2004, 2007) have highlighted Finnish students' linguistic and mathematical competence. However, a number of internal commentators have indicated that such successes may be more a consequence of cultural factors than pedagogic excellence. Of these cultural factors, cultural homogeneity (Välijärvi et al, 2002) appears to play a significant role not least because it has made it “comparatively easy in Finland to reach mutual understanding on national education policy and the means for developing the education system”. (Välijärvi et al., 2002, p. 45). Moreover, Finnish cultural homogeneity, born of frequently violent struggles between the Finnish people and oppressors from both east and west, has created a collective mindset not dissimilar to those of Japan and Korea (Simola, 2005). In particular, “there is something archaic, something authoritarian, possibly even something eastern, in the Finnish culture and mentality” (Simola, 2005, p. 458). This sense of the authoritarian collective finds resonance in the tradition that participation in Finnish cultural life has, since post-reformation times, been dependent on a public demonstration of reading competence, which was a precondition not only for receiving the sacraments but also for contracting a Christian marriage (Linnakylä, 2002). Consequently, as Mason (2007, p. 167) notes, “given what we now know of the relationship between

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levels of parental education and the educational achievements of their children, it does not take a social Darwinian perspective ...to realize the effect over centuries of a cultural practice that has meant that almost all children in Finland have been raised in families where both parents are literate". In short, there is more than a little evidence that culture, at least in the context of Finland, may play as great a part as pedagogy in determining the educational achievements of that country; a possibility that should be acknowledged if research into Finnish education is to have any meaning. Consequently, the purpose of this paper is to invite researchers in mathematics education to acknowledge, in the theorising, conducting and reporting of their work, the manner in which culture pervades all aspects of education in general and mathematics education in particular.

Of course, such ambitions may be more difficult to achieve than might be expected as culture is not only "one of the two or three most complicated words in the English language" (Williams, 1976, p. 76) but also a very difficult construct to define. That said, there seems to be a consensus that culture is, essentially, a collective manifestation of psychological conditioning and can be construed as a societal analogy to individual memory (Triandis & Suh, 2002). Culture embodies the "implicitly or explicitly shared abstract ideas about what is good, right, and desirable in a society" (Schwartz, 1999, p. 25). Culture includes those beliefs, artefacts and practices that history has shown to be effective for the maintenance of a society and its future generations (Hofstede, 1980; Triandis & Suh, 2002). In summary, cultures are social, historical and behavioural constructions that reflect the "collective mental programming" of their people (Hofstede, 1980, p. 43). Through the educational transmission of their embedded traditions, values, beliefs, knowledge and skills, they ensure their continuing replication. However, despite a minority perspective that cultures are static, the prevailing view is that they are dynamic systems (Kitayama, 2002) in which people actively seek to coordinate their behaviours with those expected of the particular societal norms. Hence, people's psychological processes "are likely to be configured in different ways across different socio-cultural groups" (Erez & Gati, 2004, p. 568). In the following I consider several educationally-salient features of culture and show, in various ways, how they impact on the opportunities to learn students receive. These are various models of culture and how they shape educational systems, models of curricula and their culturally derived expectations and a selection from the mathematics education literature highlighting the extent to which culture influences classroom practices.

Culture and its various models

A number of researchers have attempted to categorise the ways in which cultures differ, and it is to them that I turn first. Hofstede's well known study initially identified four dimensions of culture. The first, *power distance*, concerns the extent to which followers accept being led. "A society's power distance level is bred in its families through the extent to which its children are socialized toward obedience or toward initiative" (Hofstede & McCrae, 2004, p. 62). The second, *uncertainty avoidance*, relates to the extent to which "a culture programs its members to feel either uncomfortable or comfortable in unstructured situations". The third is *individualism*, or the "degree to which individuals are integrated into groups". Lastly there are *masculine* as opposed to *feminine* cultures. *Masculine* cultures "strive for maximal distinction between what men are expected to do and what women are expected to do". His analyses showed that cultural differences influence how businesses in general and employer-employee relations in particular are conducted (Hofstede 1980). In other words, Hofstede's dimensions may help explain not only how educational practices evolved in different countries but also differences between countries. To this end he has proposed four areas in which cultures differ with respect to schooling; differences in the social positions of teachers and students, curriculum relevance, cognitive expectations, and differences in the patterns of participant interactions (Hofstede 1986). Others, have exploited Hofstede's dimensions. For example, in a study of 43 countries that confounds received wisdom concerning class size, Cheung & Chan (2008) found that cultural norms associated with power distance and collectivism had more significant impact on decisions concerning class size than economic factors.

Triandis (2001) has proposed an eleven dimensional model of culture, which, by way of brevity, is presented without discussion. Cultures differ in their *complexity*; for example, cities are more complex than villages. Cultures differ in their tightness; *loose* societies tend to be more tolerant of diversity and non-conformist than *tight* cultures. Cultures differ in their perspectives on the individual and collective. For example, tight and simple cultures tend to the collectivist. They differ in their *verticality* and *horizontality*, where the former are more accepting of hierarchical differences than the latter. There are *active* and *passive* cultures. In the former people try to mould their environment to suit them and tend to be more competitive. *Universalist* cultures differ from *particularist* cultures in that any suitably qualified person is eligible for a job. In *diffuse* cultures people are judged holistically while in *specific* cultures they are not. *Ascriptive* cultures judge according to an individual's attributes while *achievement* cultures judge them on their achievement. In *instrumentalist* cultures social activity is subordinated to work, while the opposite is found in *expressive* cultures. Cultures may be *emotionally expression* or *emotionally suppressive*. Lastly, the extent to which norms concerning *physical contact* varies from one culture to another. As with Hofstede's work, and there is clearly much resonance between his and Triandis' categorisations, such characterisations help us understand the role of culture in framing humans' day-to-day actions, decision making in general and the processes of education in particular.

In this latter respect, Schwartz (1999), examined elementary teachers' and undergraduate students' education-related values and, drawing on data from more than 40 countries, identified seven dimensions of culture including conservatism (embeddedness), hierarchy, mastery, autonomy, egalitarian commitment (egalitarianism), and harmony. Thus, with respect to culture and education, for example, a conservative culture which emphasises the "maintenance of the status quo, propriety, and restraint of actions or inclinations that might disrupt the solidary group or the traditional order" would structure educational opportunities very differently from an autonomous culture in which an individual finds "meaning in his or her own uniqueness, who seeks to express his or her own internal attributes (preferences, traits, feelings, motives) and is encouraged to do so" (Schwartz, 1999, p. 24). Importantly, not only for justifying his choice of research subjects but also for highlighting the significance of culture in this discussion, Schwartz (1999, p. 34) writes that teachers were chosen for his study because they "play an explicit role in value socialisation", are likely to be "key carriers of culture, and... reflect the mid-range of prevailing value priorities in most societies". All such models help us to understand that wherever we are located, forces, possibly beyond our consciousness, act to shape what happens in classrooms. They determine what is to be taught, to whom it is taught, how it is to be taught and where it is taught. In the following I consider how such matters find manifestation.

Culture and curricula

The curriculum, according to the second international mathematics study, comprises intended, implemented and attained forms. However, what is frequently missing from such discussions is the extent to which historical and cultural forces have shaped the development of the curricula of different countries. One example of an alternative perspective on this can be found in the work of Holmes and McLean (1989), who summarise four curricular traditions. The first, Essentialism, is linked primarily to English education and refers to the liberal arts curriculum of the late mediaeval English public[1] school, its replication in early models of public education, and its rejection of science and engineering as subjects studied by gentlemen. Cummings (1999, p. 423) describes the public school as "a boarding school with many rules, mandatory chapel, an emphasis on mind and body, which included daily athletics... a strong classical thrust in the curriculum, and so forth". The second, Encyclopaedism, is generally linked to post-revolutionary France and draws on Enlightenment principles that education

[1] The use of the adjective public in the English context is misleading as the English public school is not the state-funded school that most children attend, but an elite, usually old, independent school that charges very high fees. Public schools typically employ competitive entrance examinations and are frequently so popular among the wealthy classes that parents may have to register their desire for their child to attend shortly after the child's birth. They are steeped in tradition and continue, as they have for many centuries, to furnish the higher ranks of the English civil service, the clergy, the judiciary, around half of all students at the leading universities of Cambridge and Oxford, and, of course, a disproportionately high number of members of parliament. Despite the name, they should not be confused with the public school systems of other countries.

should include all human knowledge, be accessible to all and free from superstition (Cummings, 1999). The third, Polytechnicalism, is linked to the Soviet Union and incorporates notions of vocationalism into an essentially encyclopaedic model of knowledge located in a socialist moral philosophy (Cummings, 1999). Lastly, Pragmatism, is linked to the United States and addresses the acquisition of the knowledge and skills necessary for tackling real world problems of a participative democracy. Inevitably, despite their crudity and blurred edges, such models help explain why, for example, French teachers present an intellectually sophisticated mathematics in comparison to the intellectually simplified mathematics presented by their English colleagues (Jennings & Dunne, 1996).

A similar perspective can be found in the work of Kamens et al. (1996). They write that upper secondary curricula tend to fall into four categories. *Classical curricula* address the maintenance of the natural social order through the training of a political and social elite by means of an "intellectually demanding and character-enhancing" experience that produces "well-rounded generalists rather than highly trained specialists" (Kamens et al., 1996, p. 119). *Comprehensive curricula*, reflecting egalitarian principles, typify cultures privileging popular democracy and aim not only to allow all children, irrespective of background, similar opportunities to learn and achieve but also "to produce competent citizens and productive workers rather than technical specialists" (Kamens et al., 1996, p. 120). *Mathematics and science curricula* are, essentially, reactions to increasing industrialisation reflecting both technocratic and egalitarian orientations and economic needs for a technologically competent work-force. Lastly, *arts, humanities, and modern languages curricula,* reflecting a modernisation of the classical European curriculum, focus on the maintenance of an elite high culture through emphases on, for example, particular representations of history and language.

From a different standpoint, Cummings (1999) writes that all curricula draw on cultural constructions of the ideal person, which, in the light of the discussions above, vary considerably from one culture to another. His analyses led him to conclude that most modern educational systems are derivatives of the traditions of six *core* curricula – the Prussian, Russian, French, English, Japanese and United States. He offers a compelling account of not only the development of these six traditions but also the ways in which they have influenced curricula world-wide. In respect of three of these six core nations, Pepin (1999) has argued that the English tradition, with an emphasis on personal morality, has an anti-rational humanist core privileging intuitive knowledge over the systematic construction of knowledge. With regard to the French tradition she writes that the post-revolutionary principles of *égalité* and *laïcité* facilitated the removal of social inequalities through a common broad curriculum and the expectation, unlike in England, that pastoral and moral issues will be addressed by the family. Lastly, the German humanistic tradition, drawing on notions of *Bildung*[2], incorporates both encyclopaedic rationalism as well as moralism in its unified promotion of academic and moral education (Prange, 2004). Such views lead to the equal valuing of academic and practical knowledge as found in the tripartite structure of German schools and the fact that higher education remains an option for all students, irrespective of the secondary school attended.

Of course, such characterisations are inevitably crude and may not reflect the idiosyncrasies of individual states or nations. For example, social scientists have tended to "conceptualize individualism as the opposite of collectivism... especially when contrasting European American and East Asian cultural frames (Oyserman et al., 2002, p. 3). However, such conceptions are, in some ways, confounded by analyses of the European context. Weber (1930) observed that Protestant cultures, focused on the promotion of self-reliance, tended to the individualist while Catholic, due to emphases on the maintenance of established hierarchical relationships, tended to the collectivist. Such cultural underpinnings may help to explain, for example, differences between the English, French and German curricular traditions discussed above – the individualism of the English, the collectivism of the French and the juxtaposition of the two of the German. That is, models such as those presented above highlight well how curricula vary substantially in both principle and manifestation, although few modern curricula are so tightly located in a single cultural context that they remain untouched by developments elsewhere (Kamens et al., 1996), particularly in the light of recent international tests of

[2] According to Prange, Bildung is a broad concept that does not translate easily, but is "something noble and undeniably good... *Bildung* is much better than mere education, or *Erziehung*, to give the German word. It is associated with liberty and human dignity, whereas education is associated with teaching skills and morals" (Prange 2004: 502).

student achievement, especially TIMSS and PISA.

Culture and mathematics curricula

Of course, the extent to which the underlying principles and traditions discussed above find manifestation in the written, or intended, presentation of school mathematics may vary from one country to another. Moreover, by way of facilitating discussion, Mason (2007, p. 172) notes that a curriculum is "both material artefact", the domain of the cultural anthropologist, and "symbolic system", the domain of the cultural sociologist. That is, the curriculum reflects both a way of life, including the "shared values and meanings common to members of the group" and the practices by which meaning is constructed and shared within the group (Mason, 2007: 172). In the following four European perspectives on the teaching of linear equations at the lower secondary level are presented before being discussed in relation to Hofstede's dimensions of culture. In so doing, it is becomes clear how curricula, as anthropological and sociological entities, are constructed. Choice, in respect of the countries under scrutiny, was constrained by the availability of curricula in English, while the topic was determined by other work on which I am currently engaged. They are presented alphabetically and, in respect of linear equations, verbatim as far as web-based documents allow.

The English national curriculum[3] for students in the age range 11-14 asserts that pupils "should be able to... manipulate numbers, algebraic expressions and equations and apply routine algorithms". It then goes on to say that the "study of mathematics should include... linear equations, formulae, expressions and identities" and "analytical, graphical and numerical methods for solving equations". This is accompanied by an explanatory note, saying that linear equations "includes setting up equations, including inequalities and simultaneous equations. Pupils should be able to recognise equations with no solutions or an infinite number of solutions". All remaining references to equations in the current document allude to the assessment of students' learning and the unique, among European systems, English tradition of applying levels to students' achievement.

The Finnish national curriculum[4] for grades 6-9 asserts that students, by the end of grade 8, "will know how to… solve a first degree equation".

The Flemish mathematics curriculum[5], expects students in the first grade of secondary education to "solve equations of the first grade with one unknown and simple problems which can be converted to such an equations". During the second grade they will "solve equations of the first and second degree in one unknown and problems which can be converted into such equations".

The Hungarian curriculum[6] for grades 5-8 (upper primary) writes that in year 5 students should "solve simple equations of the first degree by deduction, breaking down, checking by substitution along with simple problems expressed verbally". In year 6 they should "solve simple equations of the first degree and one variable with freely selected method". By year 7 they should "solve simple equations of the first degree by deduction and the balance principle. Interpret texts and solve verbally expressed problems. Solve equations of the first degree and one variable by the graphical method". Lastly, by year 8 students should "solve deductively equations of the first degree in relation to the base set and solution set. Analyse texts and translate them into the language of mathematics. Solve verbally expressed mathematical problems"

In these four examples can be seen not only very different perspectives on a core topic of the lower secondary curriculum. Both the English and the Finnish documents present loose and time independent perspectives on the nature of the topic and how it should be covered. For example, the English document spans three years of education and the Finnish four, yet neither specify when material should be covered. Neither document specifies any particular methods or approaches although the English expects, within general procedural expectations, that students should be exposed

[3] See http://curriculum.qcda.gov.uk/uploads/QCA-07-3338-p_Maths_3_tcm8-403.pdf
[4] See http://www.oph.fi/english/publications/2009/national_core_curricula
[5] See http://www.ond.vlaanderen.be/dvo/english/
[6] See http://www.okm.gov.hu/letolt/nemzet/kerettanterv36.doc

to analytical, graphical and numerical methods. Neither document makes explicit mention, in relation to equations, of problem solving or word problems and the derivation of equations from text. The Flemish document appears more tightly specified although the shift from one year to the next, in respect of linear equations, seems vague. The main difference lies in the explicit expectation of problems to be translated into equations for solving. Lastly, the Hungarian curriculum offers a tightly specified progression over a four year period with methods and problem solving, including word problems, increasingly exploited.

While not wishing to over-speculate, it is interesting to compare such curricular specifications with Hofstede's (1986) dimensions. But first, it is important to locate these nations on his dimensions. When compared with other developed European nations, he presents the British (not just the English) as very low on power distance, high on individualism and masculinity and low on uncertainty avoidance. The Finns are ranked higher than average on individualism with a strong tendency to the feminine. They are also rated low on power distance and uncertainty avoidance. The Belgians (not just Flanders) were also ranked high on individualism – higher than Finland but not as high as England – and substantially higher than both England and Finland on power distance and uncertainty avoidance. They were also ranked among the most masculine of the developed European nations. Unfortunately, Hungary is not represented in Hofstede's analyses as they derived from data collected before the countries of Eastern and Central Europe joined the capitalist world.

So, how do these cultural norms and characteristic patterns of social behaviour find a voice in the curricular presentation of mathematics? It seems to me that the two countries with the loosest curricula expectations, England and Finland, reflect cultures in which power distance, a horizontal culture (Triandis, 2001), and uncertainty avoidance is low. In other words, low power distance could be indicative of a culture in which curriculum developers expect, and are confident, that those responsible for its delivery will do so appropriately. Indeed, Hofstede (1980, p.46) writes, of low power distance cultures, that "people at various power levels feel... prepared to trust people". In terms of the low uncertainty avoidance, a loosely structured curriculum reflects societal norms in which not only is "the uncertainty inherent in life is more easily accepted and each day is taken as it comes" but also a culture in which dissent and deviation are tolerated and people are willing to take risks (Hofstede, 1980, p. 47). The Belgian situation is different. Relatively high levels of power avoidance and uncertainty avoidance may help us to understand why the Flemish authorities produced a more tightly structured curriculum than the English or the Finns. Further, one might speculate that the tightly specified Hungarian curriculum would be a product of a culture in which power distance and uncertainty avoidance are higher than the other three countries under scrutiny. Indeed, it is not inconceivable that a country, relatively recently emerged from more than forty years of Soviet influence, would retain, though possibly now diminishing, high levels of power distance and uncertainty avoidance, or a culture with high levels of verticality (Triandis, 2001). In similar vein, the loosely structured curricula of the English and the Finns may be artefacts of autonomous cultures (Schwartz, 1999), while the more structured curricula of the Flemish, and particularly the Hungarians, could be construed as reflecting more conservative cultures (Schwartz, 1999). Indeed, Flanders, despite recent immigration, remains a strongly Catholic culture, with all that entails in the Weberian sense described above. In short, and although this last paragraph has been somewhat speculative, it is not inconceivable that differences in the presentation of mathematics curricula are consequences of significant differences in the underlying structures of the cultures themselves. In the following I consider how such difference play out in mathematics classrooms.

Culture and mathematics teaching

As indicated above, teachers are proxies for an educational system's values and, in the light of the various models of culture and curriculum discussed above, it would surprising if teachers' actions failed to reflect those models. Indeed, there is increasing evidence that they will necessarily, and possibly unknowingly, operate in ways that reflect those objectives and values. They will privilege certain outcomes and they will exploit specific didactic strategies in ways that distinguish them from teachers in other countries. For example, Hess and Azuma (1991), having observed similarities in the ways in which the organisational structures and procedures of schools in Japan and the US presented barriers to learning, saw teachers in the two countries mediating those barriers in different, but culturally informed, ways. Japanese teachers tended to adopt strategies focused on the development of learners' facilitative dispositions while US teachers tended towards making the learning process more appealing. Such studies have led to a view that teachers in the west behave in ways that distinguish them from teachers in the east. In this respect a number of researchers have emphasised the influence of Socratic and Confucian philosophies on culturally West and culturally Chinese educational traditions (Leung, 2001; Tweed & Lehman, 2002; Watkins, 2000). Indeed, in respect of mathematics, Leung (2001) has proposed six dichotomies between East Asian and Western mathematics classrooms, located within the *Confucian versus Socratic* debate: (a) product (content) versus process, (b) rote learning versus meaningful learning, (c) studying hard versus pleasurable learning, (d) extrinsic versus intrinsic motivations, (e) whole class teaching versus individualised learning, (f) teacher competence in relation to the subject matter versus pedagogy debate. However, such distinctions are inevitably crude and, at times, likely to be inaccurate. For example, Mason (2007) argues that such distinctions are not only clumsy but frequently slide into unconscious stereotyping or even racism.

Consequently it would be naïve to assume that teachers working in culturally similar countries should behave similarly and the evidence would certainly seem to support such an assertion. In addition to the issues raised in previous pages, Sharpe's (1997) commentary, reflecting elements of Weber's (1930) arguments, on the influence of the Protestant and Catholic churches on the development of education in England and France respectively helps us to understand how the very different traditions of mathematics teaching of those two countries, as described by Jennings and Dunne (1996), evolved. Osborn (2004) discusses how societal emphases on the individual, community and nation respectively have informed educational practices and expectations in England, Denmark and France, while Kaiser and her colleagues (1999, 2006) report how differences in the philosophical underpinnings of the English, French and German educational systems influence the school presentation of mathematics. Such evidence is more supportive of differences, at least in terms of what might make a substantive difference to students' learning, in the actions of teachers from different countries than it is of similarities. In the following, I consider how such differences are manifested in classroom mathematics teaching practices.

There is a growing body of research highlighting substantial national variation in the ways in which teachers of mathematics act out their roles. Much of this, following the publication of results of the first TIMSS study (Beaton et al, 1996), has examined differences in the mathematics teaching traditions of the United States and various countries of the Pacific Rim. Such studies have exposed the relative paucity of the US student's experience of mathematics when compared to the cognitively challenge mathematics offered students elsewhere. However, the same studies have also highlighted differences in, say, the mathematics teaching traditions of Japan and China, further foregrounding the folly of assuming that the mathematics teaching of cultures described as Confucian all follow similar models. Inevitably space prevents an extensive review of the classroom-focused literature and so the following section, by way of an introduction to the range of topics and countries involved in such work, focuses on selected European examples. Kaiser et al. (2006) have offered persuasive summaries of the distinguishing characteristics of English, French and German[7] mathematics teaching, particularly in respect of proof and the structural properties of mathematics. In terms of mathematical topics, Hugener et al. (2009) have focused on differences in the ways teachers present the theorem of Pythagoras in Germany and Switzerland, while Depaepe et al. (2005) examined how teachers in England, Flanders, Hungary and Spain conceptualised and present percentages to their students. Robertson (2000) has highlighted substantial differences in the sorts of problems posed to students in France and Scotland. Variation in the mathematics curriculum content of Cyprus and England has been studied by Campbell and Kyriakides (2000), while Pepin and Haggarty (Pepin & Haggarty, 2001; Haggarty & Pepin, 2002) have shown how text book presentations of mathematics in England, France and Germany are culturally determined in ways that not only constrain the opportunities for students to learn mathematics but also confirm the veracity of the various models of culture and curricula discussed above. Such studies are important in reminding us the ways in which systems construct

[7] Japan was also included in their analyses but this commentary addresses only European traditions.

mathematics curricula and teachers work with their students are products of the cultures in which they are located.

To further illustrate the diversity of European mathematics didactics traditions, the following paragraphs summarise some work I undertook recently with colleagues from several European countries. We examined the ways in which teachers present mathematics to students in the age range 10-14 in England, Flanders, Hungary and Spain[8]. The episodes of videotaped lessons were coded against a framework developed iteratively and collaboratively over the course of the year prior to data collection (Andrews, 2007a), where an episode was defined as that part of a lesson in which the teacher's observable didactic intention remained constant. The figures of table 1 show the proportions of episodes coded in each country for each of the seven generic learning outcomes agreed by the project team – full details of these can be found in Andrews (2009a). It is unsurprising, perhaps, that teachers in all four countries privileged the development of both conceptual and procedural knowledge. The major variation lay in the other outcomes. For example, Hungarian teachers placed a higher emphasis on the structural properties (links within and between topics), mathematical efficiency (comparing solutions strategies for elegance and efficiency), problem solving and reasoning than elsewhere. In similar vein, English teachers were rarely seen to encourage structural links or efficiency. Such differences reflect not only differing curricular expectations but long standing mathematics teaching norms.

Table 1. The percentage of each country's episodes in which an generic learning outcome was observed.

	Flanders	England	Hungary	Spain	All
Conceptual knowledge	71.2	78.6	64.1	77.3	73
Derived knowledge	4.5	1.0	6.4	2.7	4
Structural knowledge	17.1	1	39.7	14.7	17
Procedural knowledge	56.8	53.4	51.3	68	57
Mathematical efficiency	12.6	9.7	35.9	14.7	17
Problem-solving	7.2	20.4	30.8	38.7	22
Reasoning	35.1	30.1	44.9	25.3	34
Total episodes	111	103	78	75	367
Total lessons	20	15	18	16	69
Total didactic codes	299	237	320	264	1120
Codes per episode	2.7	2.3	4.1	3.5	3.05

The figures of table 2 show the proportions of each country's episodes coded for ten didactic strategies identified and analysed by the project team. Details of these and their relationships can be seen in Andrews (2009b). Unsurprisingly, teachers, irrespective of their location, explain in similar proportions. However, all other codes distinguished between the didactic practices of project teachers. For example, Flemish teachers were seen to exploit explicit motivational strategies in smaller proportions than elsewhere, while Spanish teachers employed them in over half of all observed episodes. English teachers very rarely questioned (used higher order questions) while Hungarian teachers did so constantly. Spanish teacher coached constantly, that is they offered hints and suggestions to facilitate their students successful completion of the given tasks in more than three quarters of observed episodes, while teachers elsewhere did so in equal proportions. Hungarian teachers invited students to share publicly their solution strategies in almost every episode while teachers elsewhere did so consistently at around the 60 per cent level.

[8] Finland was also included in data collection but ill-health and other factors prevented the data from being coded.

Table 2. The percentage of each country's episodes in which each didactic strategy was observed.

	Flanders	England	Hungary	Spain	All
Activating	23.4	11.7	34.6	13.3	20.4
Exercising	2.7	7.8	5.1	0	4.1
Explaining	52.3	52.4	59	64	56.1
Sharing	61.3	60.2	97.4	61.3	68.7
Exploring	6.3	3.9	0	5.3	4.1
Coaching	38.7	54.4	44.9	76	52
Assessing	19.8	13.6	35.9	1.3	17.7
Motivating	9.9	12.6	46.2	56	27.8
Questioning	48.6	5.8	87.2	70.7	49.3
Differentiating	6.3	7.8	0	4	4.9

Of course, even though such evidence supports the notion that teachers operating within one cultural context behave similarly to others working in the same, there will be variance within each community. In a separate analysis of our project data, cluster analyses were undertaken to determine the veracity of the four cultural models discussed above. Importantly, whereas analyses by nationality rely on predetermined categorisations – the nationality of the teachers involved - cluster analyses allow the data to speak for themselves. That is, any episodes with similar didactic profiles would cluster together to form different groupings from those determined by nationality alone. The analysis (Andrews, 2007b) found that each cluster tended to be dominated by episodes from two countries. For example, there was a cluster dominated by Flemish and Hungarian episodes, another dominated by English and Spanish episodes and so on. Such findings tend to prompt, inter alia, two interpretations. Firstly, within each culture will be teachers whose practices lay at the heart of their country's traditions. Their actions will reflect strongly the cultural influences and expectations of their wider community. Other teachers' actions will lay closer to the boundary of what it is to be, say, a Flemish teacher of mathematics. Significantly, the closer a teacher lays to the boundary of a national didactic tradition, the more likely that teacher is to lay close to the boundary of another. Thus, the clusters, with their mostly bi-national composition, may reflect the boundary of neighbouring cultural traditions, leading to a conjecture that mathematics didactical traditions may lay on a continuum. Secondly, variation within a country's teachers' practices indicate that culture may be only a partial explanation for how the role of a teacher is conceptualised and enacted. Andrews (2010) has conjectured that all teachers operate within three models of curriculum, the intended, the received and the idealised. The first, the intended curriculum, is that proposed by the second international mathematics study and refers to systemic expectations with respect to what teachers are expected to teach and how they are expected to teach it. The second, the received curriculum, alludes to the hidden, beneath articulation practices deriving from the culturally-determined actions of being a teacher in a particular cultural context, while the third, the idealised curriculum, reflects teachers' idealised perspectives on and aims for their teaching of and their students' learning of mathematics that may or may not accord with either the intended or received curricula. The results of the cluster analysis tend to offer support to this particular model with, it would seem, teachers at the boundary of a tradition being, perhaps, in possession of idealised curricula that distinguish them from their more conventional colleagues which, in turn, lessons the impact of the received curricula.

Of course, cultural emphases do not end with the learning outcomes and didactic strategies. in a comparison of Hungarian and English mathematics teachers' beliefs about the subject and its teaching, Andrews and Hatch (Andrews & Hatch, 2000; Andrews 2007c) found substantial variation in the beliefs of English and Hungarian teachers of mathematics, further confirming not only the role of culture in the creation of teachers' beliefs but the veracity of the tripartite curriculum model. They found that English teachers believed their role was teach applications of number as part of their wider goal of preparing students for a world beyond school, while Hungarian teachers focused on mathematics as a problem solving activity from which the skills of logical reasoning are derived.

Interestingly, they found, also, that English teachers felt themselves encouraged to cover the walls of their classrooms with examples of students' work or posters highlighting in colourful ways various mathematical facts and procedures, while such practices were alien to Hungarian teachers who tended to work within classrooms with, essentially, bare walls. In short, the ways in which teachers conceptualise mathematics, as evidenced by the learning outcomes they privilege and the manner in which those expectations are communicated to learners, further highlights how mathematics teaching, in all aspects, is a cultural activity that differs significantly from one country to another.

Conclusions

In this paper I have tried to demonstrate the extent to which teaching and learning are culturally determined activities. Increasingly research suggests that lessons, especially mathematics lessons, "often have a routineness about them that ensures a degree of consistency and predictability". They are "the daily routine of teaching and learning and are often organized in a certain way that is commonly accepted in each culture" (Kawanaka, 1999, p. 91). This sense of predictable routine has been variously described as the *traditions of classroom mathematics* (Cobb et al., 1992), the *cultural script* (Stigler and Hiebert, 1999), *lesson signatures* (Hiebert et al., 2003) or the *characteristic pedagogical flow* of a lesson (Schmidt et al., 1996), where the latter embodies the pedagogical strategies which, through repeated enactment, are typical of country's lesson and beneath the consciousness of most teachers (Cogan & Schmidt 1999). In this manner cultures "shape the classroom processes and teaching practices within countries, as well as how students, parents and teachers perceive them" (Knipping, 2003, p. 282). Indeed, so significant is this hidden role of culture that many of the processes of teaching are so "deep in the background of the schooling process ... so taken-for-granted… as to be beneath mention" (Hufton & Elliott, 2000, p. 117).

Of course, any discussion of culture and its role in education will be incomplete as culture is, as indicated at the head of this paper, a difficult construct to define. Moreover, whether easily defined or not, its manifestations remain problematic. As Schein (1992) notes, culture is multi-layered with only the top layer, the observable behaviours and environments, being visible. The intermediate layer, reflecting the values and beliefs that inform behaviours, is problematic as such constructs remain contested and difficult both to categorise and evaluate. The lowest hides those unarticulated assumptions that influence us all but which are prone only to speculative inference. That said, I hope the above pages have exposed the complex relationship between culture, however it is defined or described, and the processes and practices of education in general and mathematics education in particular. I do not think I have said anything new or ground-breaking but hope to have highlighted, for those colleagues working in mathematics education research, the extent to which their work will be informed by the cultural norms within which they operate.

Therefore, my appeal, to those colleagues researching mathematical teaching and learning, is that they make explicit in the reporting of their work the cultural context in which it was undertaken. Too frequently research is reported with no indication, other than the designation of the authors, of the location of the study. Generalities offered have to be seen within the context of the study. It is foolish to assume that results generalisable to one particular context will be generalisable to another. My closing plea is that colleagues should understand, and make that understanding explicit in all aspects of their work, that the ways in which they conceptualise and undertake research is culturally informed by assumptions, implicit and explicit, about the nature of research and its processes. Bray and Thomas (1995, p. 472) have observed that much educational research "has been undesirably localized in focus", so that "unbalanced or incomplete perspectives have resulted from the lack of an international dimension". I hope that future mathematics education research will avoid this criticism.

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