Trajectory of Charged Particle in Combined Electric and Magnetic Fields Using Interactive Spreadsheets

Popat S. Tambade
Prof. Ramkrishna More College, Akurdi
Pune 411 044, India
pstam3@rediffmail.com

Abstract

The objective of this article is to graphically illustrate to the students the physical phenomenon of motion of charged particle under the action of simultaneous electric and magnetic fields by simulating particle motion on a computer. Differential equations of motions are solved analytically and path of particle in three-dimensional space are obtained using interactive spreadsheet. Spreadsheets can be setup to solve numerical solutions of complex systems. This will make the concept, which so far is left to the abstract imagination come alive for the student, and initiate a deeper understanding of particle motion.
Keywords: Physics Education, electromagnetic fields, spreadsheets, simulation.

Introduction

In recent years, physics educators have started to look more closely at what their students understand about physics concepts. A primary goal of physics education research is to identify difficulties in learning physics through traditional methods and to develop new instructional modes for effective learning (McDermott, 2001). One of the best instructional modes allowing for the development of individual experience is computer simulations. Computer simulations can be the best mode, when the tactile, kinesthetic experience of real objects is impracticable (Arons, 1987). Computer simulations are also useful for providing more extended practice in thinking about a wide variety of examples. It is capable of supplying continual feedback regarding error and correctness and reinforcing the hands on observations when latter have carried out. For proper understanding of physics concepts to the students, teachers can use computer simulations. Teacher should develop computer simulations of physics problems so that it becomes easier for students to understand the phenomena.

Spreadsheets can be a powerful tool in physics teaching-learning. From data analysis and graphing to animation and simulations, Microsoft Excel® is a very versatile program for the teachers and students. The advantage of using a spreadsheet in teaching-learning process is that the programming is streamlined and less time is needed to enter the necessary code. The strong features of spreadsheet are their cell based structure and the simple interface that is easy to use for new users also. The power and simplicity of a spreadsheet is that the data manipulations are held in front of the user in a very direct and accessible manner. In addition, the spreadsheet program itself provides for screen graphics, charts, and easy-data manipulation using large number of functions, on-screen numerical and visual feedback, and fast calculations (Wagner,

2007; Cooke, 1997; Yamani, and Kharab, 2001). Spreadsheets can be used as specific-purpose software as well as for programming. Due to its structure and tools available in it, the amount of programming needed in a spreadsheet is minimal. It is often done by formula editing and copying. This approach helps students to concentrate on analysis and interpretation of results rather than on time-consuming code debugging (Lau and Kuruganty, 2010).

In addition to the features and functions of the Excel, it has programming capabilities called Visual Basic for Applications (VBA). By including VBA with applications such as Excel, Microsoft combined flexibility and ease of programming that Visual Basic offers with the power of application that hosts it. This is a very powerful tool and can be used to solve complex physics problems (Wagner, 2007; Cooke, 1997). In Excel we can develop user defined functions using visual basic for application (VBA) in Excel. Users can use these functions and get their desired output directly (Subedi, 2007).

A spreadsheet program can be used to solve linear and non-linear differential equations and to obtain their visual solutions (Severn, 1999; Lingard, 2003). It may be also used for numerical integration techniques like Trapezoidal rule, Simpsons rule etc. When one solves an equation and analyzes the results, step by step, recording the successive values in the worksheet and plotting the results, then repeating the calculations with different input parameter values and comparing and analyzing the results we could take full advantage of various spreadsheet features (El-Hajj *et al*., 2005). This method is also sufficiently accurate for a large number of practical applications, since Excel stores numbers and performs calculations using 15 digits of precision. The accuracy of the results may be increased by reducing the step size of the integration. Reduction of the integration step size may lead to an increase in the number of iterations, which makes this method slow in simulating large control systems (El-Hajj *et al*., 2002). However, the spreadsheet could act as a good pedagogical tool.

Spreadsheets can be used to obtain numerical solutions of differential equations in physics and their visual graphical representations. Using spreadsheets the data can be manipulated and visual effects can be observed (Webb, 1993; Benacka, 2009; Oliveira and Napoles, 2010). Using spreadsheets animations and simulations can be developed. The spreadsheets can be used to visualize potential surfaces of charges as well as surface plots (Beichner, 1997).

This paper presented spreadsheet implementation for plotting of trajectory of charged particle in electromagnetic fields. Analytical solutions are used to plot the trajectory of charged particle. Scrollbars and naming facility to the cells are used to make the spreadsheet interactive.

Analytical Solutions

When a charged particle of mass *m* and having charge *q* is subjected to motion in combined in electric and magnetic fields, it traces a path in a three-dimensional space that depends upon the magnitudes and directions of electric and magnetic fields, initial velocity of the particle.

The force acting on the particle of charge '*q*' in combined electric and magnetic fields is given by Lorentz's force as

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) \qquad \qquad …. (1)$$

Consider a general case when $\vec{E}$ and $\vec{B}$ are not perpendicular but make some angle as shown in Figure 1 (Takwale and Puranik, 1997).
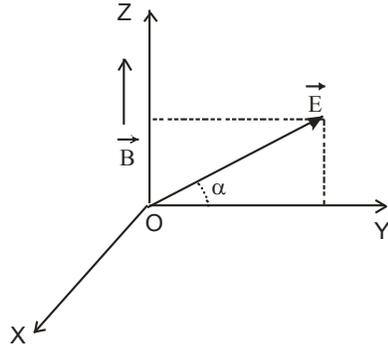

Figure 1. *Electric and magnetic fields*

If we assume that $\vec{E} = E_y \hat{j} + E_z \hat{k}$, $\vec{B} = B_z \hat{k}$ and $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$, then we get three equations:

$$\frac{dv_x}{dt} = \omega v_y \qquad \qquad \dots (2)$$

$$\frac{dv_y}{dt} = \omega \left(\frac{E_y}{B_z} - v_x \right) \qquad \qquad \dots (3)$$

And
$$\frac{dv_z}{dt} = \frac{qE_z}{m} = \text{constant} \qquad \qquad \dots (4)$$

In equations (2) and (3) we let $\omega = qB_z/m$ often referred to as the cyclotron frequency.

The coupled equations (2) and (3) can be solved by differentiating any one of them with respect to time and substituting the time derivatives of velocity from the other. We have

$$\frac{d^2 v_x}{dt^2} = \omega^2 \left(\frac{E_y}{B_z} - v_x \right) \qquad \qquad \dots (5)$$

and
$$\frac{d^2 v_y}{dt^2} = -\omega^2 v_y \qquad \qquad \dots (6)$$

Equations (5), (6) and (4) can be solved to obtain trajectory of the particle in combined electric and magnetic fields. The solutions of these equations respectively are

$$v_x = v_{ox} + \frac{1}{\omega} \left(\frac{E_y}{B_z} - v_{ox} \right)(1 - \cos \omega t) \qquad \qquad \dots (7)$$

$$v_y = \frac{1}{\omega}\left(\frac{E_y}{B_z} - v_{ox}\right)\sin\omega t \qquad\qquad \dots (8)$$

$$v_z = v_{oz} + \frac{qE_z}{m}t \qquad\qquad \dots (9)$$

Integrating above equations we get,

$$x = x_0 + v_{ox}t + \frac{1}{\omega}\left(\frac{E_y}{B_z} - v_{ox}\right)(\omega t - \sin\omega t) \qquad\qquad \dots (10)$$

$$y = y_0 + \frac{1}{\omega}\left(\frac{E_y}{B_z} - v_{ox}\right)(1 - \cos\omega t) \qquad\qquad \dots (11)$$

$$z = z_0 + v_{oz}t + \frac{1}{2}\left(\frac{qE_z}{m}\right)t^2 \qquad\qquad \dots (12)$$

The nature of trajectory depends on components of fields, initial velocity and angle between electric and magnetic fields.

Orthographic Parallel Projection

Suppose a point A in space and a plane ρ that is not passing through A. If we make a line through A perpendicular to the plane, then the intersection of the line and plane is the orthographic parallel projection. If we identify the plane with the computer screen, then we can project 3D objects on it this way (Benacka, 2008).
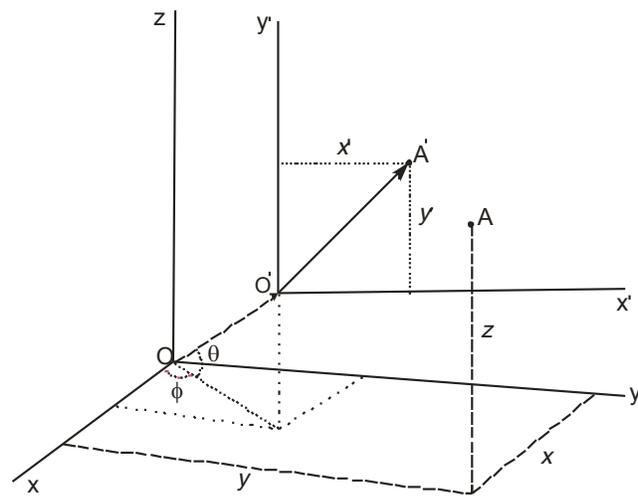

Figure 2. *Coordinate system O' (x', y') on computer screen*

The transformation equations between 3D coordinates (x, y, z) of the point A in O(x, y, z) co-ordinate system to the point A' in 2D coordinates in screen O' (x', y') coordinate system are governed by the equations

$$x' = -x \sin \phi + y \cos \phi \qquad\qquad \text{... (13)}$$

$$y' = -x \sin \theta \cos \phi - y \sin \theta \sin \phi + z \cos \theta \qquad\qquad \text{... (14)}$$

Organization of Spreadsheet

The spreadsheet is prepared using MS Excel® 2007. In Excel it is easy to access to controls such as scrollbars because their implementation requires little or no macro programming (Staples, 2004). The facility of naming a cell or range is available in Excel. Naming a cell allows us to reference it with that name instead of the default name. After we define a name for a cell or range, we can use it in a formula. For example, we may use "charge" as a name to cell "A5". We can use name "charge" in formula instead of cell reference such as "A5". This can make formulas easier to understand. If the named cell contains a formula and we want that formula inside a new formula, naming the cell can make our formula writing easier, without so much nesting. Named cells can be used like bookmarks in a workbook, and named cells can also be referenced in formulas. Naming the cell makes the formula easier because we don't need to remember exactly where the cell is in a workbook and we don't need to get into the whole nesting thing.

The naming facility available in the excel is used to name the cells.

In Sheet 1 the cells named as:

    C4: named as q which takes value of charge.
    C5: named as m which takes value of mass.
    C6: named as Bz which takes value of magnetic field B_z along z-axis.
    C7: named as E which takes value of electric field E.
    C8: named as alpha which takes value of angle α between E and y-axis.
    C9: named as Ey which takes value of E cos α, component along y-axis.
    C10: named as Ez which takes value of E sin α, component along z-axis.
    F10: named as w which takes value of cyclotron frequency $\omega = qB_z/m$.
    H11: named as t which takes value of time which is incremented by dt using scrollbar.

F11, G11, and J11 are named as xo, yo, and zo respectively which take values of initial point.

The input parameters are given through the cells as shown in Figure 3. Scrollbars are used to change the parameters like m, q, B_z, E, α, θ_1, ϕ_1 etc.

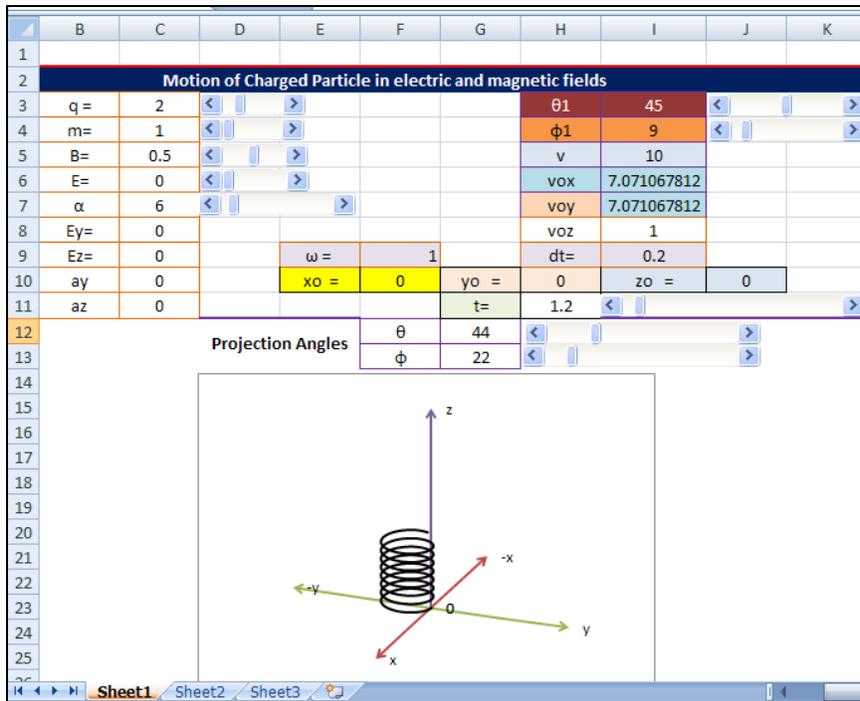The screenshot of sheet 1 of the spreadsheet is shown in Figure 3.

Figure 3. *Screenshot of the spreadsheet for input parameters*

The dataset in the spreadsheet on Sheet 2 is organized in the following way:

A3:A253 is a column that contains the values for t by increment of $dt...$
B3:B253 is a column that contains the values of x obtained using Eq. 10 by using
   $= x0+vox*A3 + 1/w*(Ey/Bz-vox)*(w*A3-sin(w*A3))$ equation in cell B3 and pasting up to
   cell B255.
C3:C253 is a column that contains the values of y obtained using Eq. 11 as like above step.
D3:D253 is a column that contains the values of z obtained using Eq.12 as above.
E3:E253 is a column that contains the values of X obtained using Eq. 13 as above.
F3:F253 is a column that contains the values of Y obtained using Eq. 14 as above.
G3:G253 is a column that contains the values of X_1 which is equal to X.
H3:H253 is a column that contains the values of Y_1 which is equal to Y.
Excel's built-in IF THEN ELSE function is used to generate X1 and Y1values only for those
points X and Y whose time measure is less than a value t that we enter in Cell H11 on Sheet 1.
To obtain X_1 in cell G3 the condition used in cell G3 is

$$= IF (t>=A3, E3, NA ())$$

    The formula shown is interpreted as IF time t is greater than or equal to value in cell A3, THEN reproduce the X-value, ELSE generate the "not available", NA(), value. This condition is pasted up to cell G253.

To obtain Y_1 in cell H3 if condition in cell H3 is used. The condition is

$$= IF\ (t>=A3,\ F3,\ NA\ ())$$

This condition is pasted up to cell H253.

The time t is changed by using scroll bar by the interval of dt which is placed in cell H11 on Sheet 1. This is used to simulate the path of particle.

The screenshot of data table is shown in Figure 3.

	A	B	C	D	E	F	G	H
1								
2								
3	t	x	y	z	X	Y	X1	Y12
4	0	0	0	0	0	0	0	0
5	0.2	0.0008	0.039988	0	0.037045	-0.0054	0.037045	-0.0054
6	0.4	0.006395	0.159808	0	0.146902	-0.02266	0.146902	-0.02266
7	0.6	0.021565	0.359029	0	0.327454	-0.05332	0.327454	-0.05332
8	0.8	0.051053	0.636934	0	0.576333	-0.09888	0.576333	-0.09888
9	1	0.099551	0.992522	0	0.890924	-0.16077	0.890924	-0.16077
10	1.2	0.171684	1.424515	0	1.268373	-0.24039	1.268373	-0.24039
11	1.4	0.27199	1.931357	0	1.705605	-0.33904	1.705605	-0.33904
12	1.6	0.404907	2.511224	0	2.199324	-0.45798	2.199324	-0.45798
13	1.8	0.574756	3.162029	0	2.746035	-0.59839	2.746035	-0.59839
14	2	0.785723	3.881431	0	3.34205	-0.76136	3.34205	-0.76136
15	2.2	1.041848	4.666839	0	3.983505	-0.94792	3.983505	-0.94792
16	2.4	1.347007	5.515428	0	4.666372	-1.159	4.666372	-1.159
17	2.6	1.704902	6.424144	0	5.386473	-1.39544	5.386473	-1.39544
18	2.8	2.119042	7.389715	0	6.139497	-1.658	6.139497	-1.658
19	3	2.592735	8.408667	0	6.921014	-1.94734	6.921014	-1.94734
20	3.2	3.129076	9.477334	0	7.726493	-2.26403	7.726493	-2.26403
21	3.4	3.730933	10.59187	0	8.551314	-2.60853	8.551314	-2.60853
22	3.6	4.400938	11.74826	0	9.390789	-2.9812	9.390789	-2.9812
23	3.8	5.141478	12.94234	0	10.24018	-3.38231	10.24018	-3.38231
24	4	5.954687	14.16983	0	11.0947	-3.81203	11.0947	-3.81203
25	4.2	6.842437	15.42629	0	11.94957	-4.2704	11.94957	-4.2704
26	4.4	7.806331	16.70721	0	12.79999	4.75739	12.79999	4.75739

Sheet1   Sheet2   Sheet3

Figure 3. *Screenshot of the spreadsheet for data table*

Results and Discussion

A method is presented here graphically simulates trajectory of charged particle in electromagnetic fields using modern spreadsheet programs, which are available and familiar to a wide audience of computer users. The graphs are plotted between X1 and Y1 using XY scatter graph in the Chart wizard. The three coordinate axes are plotted by using equations (13) and (14). First the parameters are set using scroll bars. Then using scrollbar for time t the path of particle is simulated. The trajectories of particle for different parameters are shown in the following figures4, 5, and 6. The input parameters that can be changed are electric field, magnetic field angle between electric and magnetic field, magnitude of charge and mass, initial velocity etc.
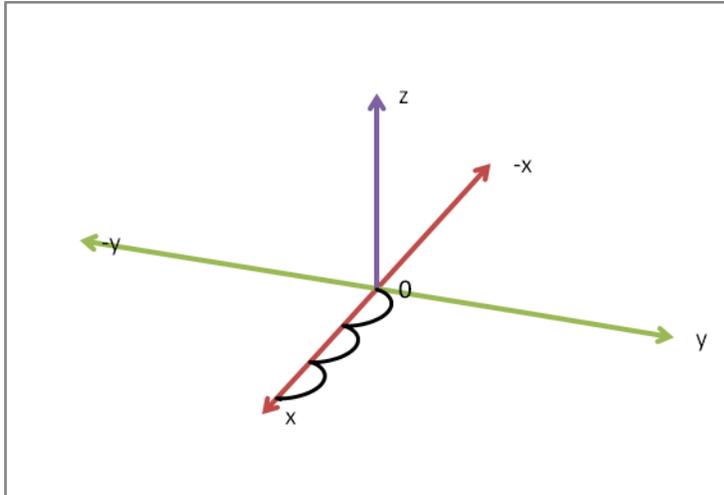
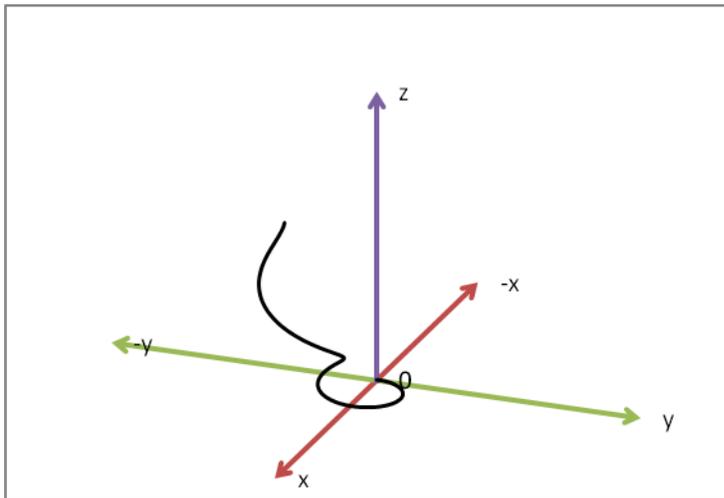**Figure 4.** *Trajectory for q = m = 1 unit, v= 10 unit, vox = 0 unit, voy = 10 unit, voz = 0 unit, Bz=0.4 unit, E = 2 unit and α = 0⁰.*

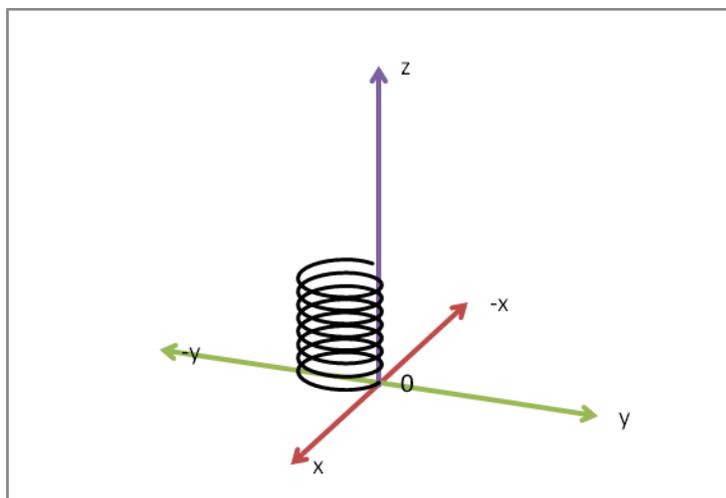**Figure 5.** *Trajectory for q = m = 1 unit, v= 10 unit, vox = 0 unit, voy = 10 unit, voz=0 unit, Bz=0.3 unit, E = 2 unit and α = 6⁰.*

Figure 6. *Trajectory for q = 2 unit, m = 1 unit, v= 10 unit, vox = 7.07 unit, voy = 7.07 unit, voz=1 unit, Bz=0.5 unit, E = 0 unit.*

As part of this learning process, it is useful to actually display the motion of a particle with the dependence on magnetic field, electric field, initial velocity, charge to mass ratio. Understanding of the charged particle motion in electromagnetic fields can be deepened by posing questions such as "What would happen if we change mass of particle keeping other parameters same?" or "What would happen if the initial velocity of the particle is changed?" By using such "what-if" questions, student can test their understanding of concepts and ideas of physical processes. This can all actually be tried with the spreadsheet animation. The use of spreadsheet in this way is supported by constructivist model of learning (Lim, 2006). Before using spreadsheets, the students should be forced to tackle with the problem and writing down detailed predictions through thinking and discussion among themselves. The use of animation then either confirms the correct thinking or forces a re-investigation to revise misconceptions.

The learning can be reinforced when students create the spreadsheets themselves and understand exactly how, what, and why the spreadsheet is calculating before the animation is added. When students learn spreadsheet techniques they can build the spreadsheet, enter the equations, and graph the calculations. As they explore the spreadsheet by varying parameters, students reinforce their understanding of physical phenomena. The excel animations as discussed above appear to help students better grasp the mathematical principles underlying the physical phenomenon and to better appreciate the power of mathematics applied to real world.

Conclusion

Students must know the nature of trajectory of charged particle in electric and magnetic field in order to understand the principle and working of mass spectrograph, cyclotron, betatron etc. A spreadsheet program that has been developed for trajectory of charged particle in electromagnetic fields is menu driven, user friendly, and interactive. It allows the user to carry out a series of numerical experiments using different sets of parameters and to view the effect

automatically in graphical form. These features are difficult to obtain by using a structured programming language such as C/C++. Use of this program in undergraduate classroom and laboratory has resulted in favorable comments by the students. The examples used here demonstrate that spreadsheet programs have proven to be an important pedagogical tool for a variety of problems in physics education. Spreadsheets programs are very flexible, familiar, and relatively easy to use. Spreadsheets can be used to model physical situations involving motion of particle, and thus provide graphical representations to equations. Parameters can easily be changed and their effects investigated. The causal relationships that cause the path of particle become apparent in the spreadsheet model and it can be instructive for the student to make them explicit in graphical form. Students can easily learn to use spreadsheets in physics, and this is good career training outside physics also. With variety of built-in mathematical functions and excellent graphic capabilities, the spreadsheet is a powerful instrument for teaching and learning in many areas of physics. Spreadsheet simulations are effective for students learning because they provide both visual and kinesthetic experiences. They enhance students' ability of using mathematical experiences to study several phenomena of study and have a better understanding of how the input variables impact the dynamics of a problem. These simulations are also beneficial for those students whose logical and mathematical thinking is weak (Lim, 2006). The spreadsheet example presented here, demonstrate that even without the uses of "macros", we can built sophisticated simulations.

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